

Deep and shallow approaches to learning mathematics are not mutually exclusive

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From time to time, students are characterised as having a deep or shallow approach to learning. A deep approach to learning tends to attract more approval than a shallow approach, at least in the West. Students on a university-based Foundation course to prepare them for undergraduate studies were divided into those likely to have a deep approach (26) and those likely to have a shallow approach (18). Their performance in a test of problem solving in an aspect of applied mathematics was compared. Contrary to expectations, the test scores and interviews with the students indicated that those with a deep approach did not benefit when asked to apply their learning in new contexts, and those with a shallow approach were not markedly disadvantaged. It is suggested that, at least amongst learners, neither approach is likely to be entirely self-sufficient, but should be seen as acceptable starting points of potential routes to success. Although a small scale study, mathematics tutors should be able to relate the findings and suggestions to their own experiences and practices.

Keywords: Deep and shallow learning; older and younger students

Introduction

For many international students who study undergraduate degrees in the UK, a one-year foundation programme often bridges the gap between judgements of their academic knowledge and proficiency and university admissions criteria. In addition, politicians want to widen the participation of UK citizens in higher education. Again, because they may not meet conventional admissions criteria, a first step is to join a foundation course. One consequence is that older and younger students can find themselves in the same class. Experience has taught us that this can be beneficial (Mathias, Bruce & Newton, 2013). Nevertheless, differences in age and culture can produce differences in approaches to learning. This brings challenges for the tutor and highlights the importance of identifying specific barriers to learning in such groups in order to teach them effectively. This paper examines some older and younger students' approaches to learning in an aspect of applied mathematics and relates it to their test performances. The overarching aim was to use this to inform the teaching of students who have such approaches to learning mathematics.

Some underpinning notions

That students adopt different approaches to learning is well-known. For instance, students with a shallow approach tend to treat information as unconnected facts to be memorized; those with a deep approach construct relationships and build meaningful mental structures. Since its inception, this dichotomy has produced some interesting observations. For instance, older students have been found to adopt a deeper approach and seek meaning more than younger students (Richardson, 1995). Similarly, Dittmann-Kohli and Baltes (1990) found that older students are more capable of

exhibiting the interpretative, contextualised and relativistic conceptions of learning which reflect a deep approach to learning. At the same time, the approach to learning can be shaped by culture and educational experience, something which is found amongst both Confucian and Western heritage students (Dennehy, 2015). The former were more inclined to favour a shallow approach than the latter.

More recent studies, however, have found variations of the two approaches and how they operate (Case & Marshall, 2004), and some reject the dichotomy in favour of a continuum stretching between the two (Volet & Chalmers, 1992). Moreover, in practice, it is doubtful that a deep approach is necessary for all learning, as when acquiring general matters of fact which enable action in the topic under study (Beattie, Collins & McInnes, 1997). Haggis (2003) even questions the relevance of learning approaches and argues that memorization can be a pre-cursor to understanding. For example, a tendency has been found for Asian medical students to employ memorization as an initial process in moving toward understanding, rather than memorized information being the end point (Tavakol & Dennick, 2010). This suggests that approaches to learning should not be seen as tunnels to different destinations but as different starting points on paths which could, eventually, converge at the same place. The implication is that having a shallow approach need not be as mentally disabling in the long term as may be assumed. Equally, a leaning towards a deep approach may lead to a neglect of facts, rules, procedures and a facility with algorithms which could hinder the application of learning in any context. Understanding is also not an all-or-nothing matter: we tend to develop, increase, revise and replace it as experience and knowledge grow (Newton, 2014). Having a deep approach is not a guarantee that this will be achieved sufficiently to ensure success in an imminent examination. On this basis, it is not a matter of one approach being better than the other but that blends of both approaches could be mutually supportive and beneficial. Older and younger students (and those from different learning cultures) may have different approaches to learning, but the approaches are not always poles apart and may, over time, converge. It may even be that the optimum approach for student success, particularly in time-limited examinations, is a suitable blend of approaches tuned to psychological needs and course goals. In this case, teaching would need to reflect and support that.

The aim of this study was to explore the extent to which students with a leaning towards shallow or deep approaches to learning in mathematics, were enabled or disabled by their approach in mathematical problem solving. The purpose was to inform supportive teaching practices and formative feedback.

The study

The one-year Foundation Mathematics programme offered at this university currently includes two modules: Core Foundation Mathematics in the first semester and Mathematics Application Combined in the second semester. The first module and beginning of the second module are the topics, mostly algebra, taught to support learning in the rest of the second module. Mathematical proficiency in these topics is tested in a two-hour, written examination (Test 1). The main part of the second module is about kinematics and dynamics and calls for solving problems involving Newton's Laws of Motion. Problem solving abilities in novel contexts are tested via a two-hour examination (Test 2). In order to solve the problems, the students need to understand the situation, set up an appropriate mathematical model, and manipulate its elements.

The Force Concept Inventory (FCI) is a diagnostic test comprising 30 questions which grew out of the research of Hestenes, Wells & Swackhamer (1992). The questions target 28 common misconceptions in Newton’s Laws of Motion to assess students’ understanding. All students took the FCI test before and after being taught, the scores are indicators of conceptual understanding in Newtonian physics. Due to the wide range of student ability and background, it was anticipated that some students would have been taught these topics before and, perhaps, learned it well. A score of 60% in the FCI test is regarded as being the ‘entry threshold’ to Newtonian physics at university level; students scoring below 60% may be considered to have an inadequate grasp of the subject for progression into some STEM degree programmes (Hestenes & Halloun, 1995). Only those students with FCI scores less than 60% on the pre-test formed the participants in this study.

Over the three years of the study, the classes comprised ‘mature’ UK students (25 years of age or more). These formed the older group of students. Also in the class were younger students (aged 17-19 years of age; some being from cultures which have been described as fostering shallower learning, such as China (Clark & Gieve, 2006). According to the studies described above, the older group are more likely to adopt a deeper, relational approach to learning while the younger group are more likely to favour a relatively shallow approach to learning. The numbers of students in the study are shown in Table 1. Ethical approval and agreement of the students was obtained to use Test 1 and 2 and the FCI scores anonymously.

Table 1. Number of participants (scoring less than 60% on the FCI test: OS = Older students, YS = Younger students).

Observation of students working in class added qualitatively to our interpretation of the data. Six older students and four younger students, chosen at random, also agreed to be interviewed, primarily to clarify our understanding of the data. One older student was interviewed a second time to give a view on the conclusion.	Year	OS	YS
	2011-12	12	10
	2012-13	5	4
	2013-14	9	4
	Totals	26	18

Results and discussion

Table 2 shows the mean FCI scores for the two groups of students. On average, both groups had higher FCI scores after teaching ($t(OS) = 7.10, p=0.00$; $t(YS) = 2.61, p=0.14$) indicating an increased conceptual understanding. The older students’ improvement, however, tended to be greater than that of the younger students. This is in accordance with expectations: a lower increase in understanding is what would commonly be predicted for these younger students, who, according to the stereotypical view, might be expected to have a shallower approach to learning, perhaps with a bias towards memorizing procedures to apply in exercises. The older students, on the other hand, seem to have given more attention to underlying, relational conceptual matters. One such student said that:

Life experience can give you more images to think about how things move, what happens when you do things to an object, this helps you to think about the wider concept.

Table 2. Mean scores (%) on the pre- and post-FCI test (OS = Older students, YS = Younger students; SD = standard deviation).

They also showed sufficient interest in conceptual matters to engage with one another in heated debate, as when

	FCI before teaching		FCI after teaching	
	Mean	SD	Mean	SD
OS	37.48	10.36	63.04	14.68
YS	31.31	13.21	44.28	14.20

two of them eventually arrived at an understanding of why objects continue to move forward when dropped from an aeroplane: ‘You roll forward, not backwards, when you jump off a moving train: Do you?’ This kind of thinking could help them in their interpretation and modelling of problem situations.

Although the FCI test indicated that the older students’ grasp of Newtonian concepts was better than that of the younger students, they were significantly outperformed by the latter in Test 2, the test of their application, (Table 3, $t = 3.35$, $p=0.002$). This implies that conceptual understanding alone is not sufficient for successful problem solving, at least in timed tests. Test 1 scores offer a clue to what else is needed. Test 1 aimed to assess the skills in pre-requisite mathematics to service the problem solving work in the second module. The younger students also outperformed the older students in these skills ($t = 4.22$, $p=0.00$).

Table 3. Test 1 and Test 2 scores (Mean scores (%)) (OS = Older students, YS = Younger students; SD = standard deviation).

	Test scores (%)			
	Test 1		Test 2	
	Mean	SD	Mean	SD
OS	69.27	16.13	49.46	20.48
YS	86.22	10.49	68.67	17.39

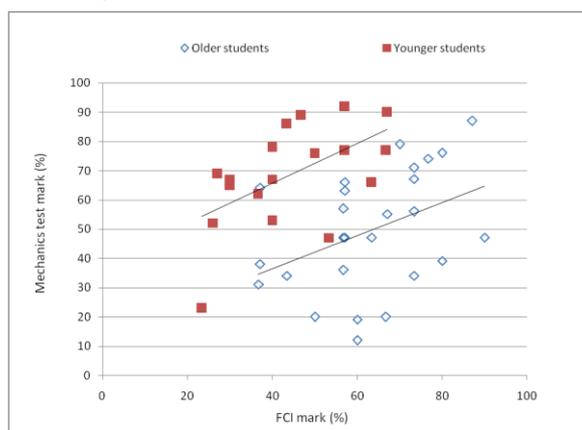
This is not to say that understanding is a waste of time. Wildy and Wallace (1992) did a longitudinal comparison of mathematics learning in which the

emphasis for one group of young students was on procedures and algorithms and, for another, it was on understanding why these worked. Over time, the second group increasingly outperformed the first.

Clearly, understanding can be of long term benefit. Figure 1 depicts the relationship between FCI scores and Test 2 scores (OS: $r = 0.41$, $p<0.05$; YS: $r = 0.55$, $p<0.01$). Although the YS scores were, on average, higher than those of the OS, both sets of scores show that higher Test 2 attainment tends to be associated with greater conceptual understanding. While understanding alone may not be sufficient for success, a little understanding, in conjunction with good memorization and manipulative skills, seems to go a long way in problem solving. For instance, a young Chinese student felt he had an advantage in having less need of a calculator.

For some calculations I can do very quickly but UK people have to use calculator: it may be a Chinese’s advantage.

Figure 1. Relationships between the post-teaching FCI scores and Test Two scores for OS and YS.



On the other hand, an older, UK student felt that ‘Most people (older students) are confident at conceptual understanding ...’ but are frustrated by their lack of skill.

Concluding remarks

A lot of attention tends to be given to deep and shallow approaches to learning, generally with the assumption that the latter is a bad for you and should be discouraged. In practice, approaches to learning are unlikely to be sharply dichotomised and some students strive more than others to acquire understandings. In the same way, these students vary in their stock of useful and necessary fact-like knowledge and the ease and competence with which they use it. Neither approach is sufficient in itself to guarantee student success, at least in mathematics. Instead, the two approaches combine to produce each student’s level of success, and, over time, both could lead to understanding with competence. Prosser and Trigwell (1999) suggest both deep and shallow approaches ‘should be considered to be simultaneously present in the student’s awareness’, rather than ‘independently constituted’. For example, some students tend to learn through a four-stage process: (1) memorizing, (2) understanding, (3) applying and (4) questioning or modifying (Tweed & Lehman, 2002). Haggis (2003) contends that the Western model, which professes to emphasise deep learning, involves teaching that mainly represents an academic’s view of understanding and knowledge acquisition which is at odds with reality. Cooper’s (2004) study of Chinese and Australian accountancy students also provides evidence that the shallow learning associated with the Chinese tradition of memorization can deepen understanding and achieve high levels of academic performance. While we might want students to know and understand everything, in reality, modules (and life) are too short and human behaviour is such that, instead, we promote some combination of kinds of learning. As mathematics teachers, we should decide what constitutes an appropriate combination of deep and shallow learning, then encourage and help students to reach it. Haggis (2003) expressed doubts about the value of the notion of learning approaches. More moderately, it is certainly unhelpful and unrealistic to see it only in terms of extremes with pejoratives attached to one and praise to the other.

At this point, we must acknowledge that the study was not of large numbers of students and it took some three years to accumulate the data offered here. This, of course, means that we cannot be sure that the findings apply everywhere that there are older and younger students studying mathematics. Nevertheless, we believe that the study has highlighted a matter worthy of attention and one which could affect how the subject is taught. Tutors must relate a study’s context to their own situation and consider how it might affect their teaching.

In this context, we feel that these groups of students could learn from each other. As might be expected, the groups tended to look inwards for interaction and security, however, it could be helpful for them to see other approaches to learning and the benefits of them, perhaps by sharing their thinking with others so that the value of understanding a key concept and, at the same time, an appreciation of the worth of skilled manipulation of mathematical ‘facts’ is evident. This process may be supported by giving a little time to highlight when certain kinds of learning are particularly useful – there seems little point in keeping secret the need for an appropriate blend and what that blend should be. Perhaps we should see students’ approaches to learning as acceptable starting points and then help them progress from

these. The message that it is not one kind of learning or the other that is worthwhile, but a mutually supportive combination of both that is important for student success at this level in mathematics.

References

- Beattie, V., Collins, B., & McInnes, B. (1997). Deep and surface learning: a simple or simplistic dichotomy? *Accounting Education*, 6(1), 1-12.
- Case, J., & Marshall, D. (2004). Between deep and surface: procedural approaches to learning in engineering education contexts. *Studies in Higher Education*, 29(5), 605-615.
- Clark, R., & Gieve, S. N. (2006). On the discursive construction of 'the Chinese learner'. *Language, culture and curriculum*, 19(1), 54-73.
- Cooper, B. J. 2004. The enigma of the Chinese learner. *Accounting Education*, 13(3), 289-310.
- Dennehy, E. (2015). Learning approaches and cultural influences: a comparative study of Confucian and western-heritage students. *Journal of Further and Higher Education*, 39(6), 818-838.
- Dittmann-Kohli, F., and Baltes, P.B. (1990). Toward a neofunctionalist conception of adult intellectual development: wisdom as a prototypical case of intellectual growth. In Alexander, C.N., and Langer, E.J. (eds.), *Higher Stages of Human Development: Perspectives on Adult Growth*. New York: Oxford University Press. 54-78.
- Haggis, T. (2003). Constructing images of ourselves? *British Educational Research Journal*, 29(1), 89-104.
- Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force concept inventory. *The physics teacher*, 30(3), 141-158.
- Hestenes, D., & Halloun, I. (1995). Interpreting the force concept inventory. *The Physics Teacher*, 33(8), 502-506.
- Mathias, J., Bruce, M. & Newton, D.P. (2013). Challenging the Western stereotype: Do Chinese International Foundation students learn by rote? *Research in Post-Compulsory Education*, 18(3), 221-238.
- Newton, D.P. (2014). *Thinking with Feeling*. Abingdon: Routledge.
- Prosser, M. & Trigwell K. (1999). *Understanding Learning and Teaching*. Buckingham: Open University Press.
- Richardson, J. T. (1995). Mature students in higher education: II. An investigation of approaches to studying and academic performance. *Studies in Higher Education*, 20(1), 5-17.
- Tavakol, M., & Dennick, R. (2010). Are Asian international medical students just rote learners? *Advances in Health Sciences Education*, 15(3), 369-377.
- Tweed, G., & Lehman, R. (2002). Learning considered within a cultural context: Confucian and Socratic approaches. *American Psychologist*, 57(2), 89-99
- Volet, S.E. & Chalmers, D. (1992). Investigation of qualitative differences in university students' learning goals. *British Journal of Educational Psychology*, 62, 17-34.
- Wildy, H. & Wallace, J. (1992). Understanding teaching or teaching for understanding. *American Educational Research Journal*, 29, 143-156.