# Prompting for progress: Using teacher voice for the implementation of Realistic Mathematics Education with low achieving mathematics students

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In teaching a group of low-achieving 14 and 15 year old students, I struggled to motivate them because they could not see the point of doing their General Certificate of Secondary Education examinations (GCSEs, taken at the end of compulsory education aged 16) when they were only going to get the lowest grades. The work of Freudenthal and the Realistic Mathematics Education Group (RME) at the Freudenthal Institute provided an alternative way of doing things, allowing students to take away more flexible mathematical ideas to use beyond the classroom. My research question was, ‘How can we promote the use of more sophisticated methods by students?’ 4 teachers taught a series of lessons to their classes using the idea of realistic situations. From our group ‘teacher voice’ discussions a common theme was when to intervene to take conceptual thinking on, what we called ‘prompting for progress’.

**Key words: teacher voice; teaching mathematics; Realistic Mathematics Education**

## Introduction

I am a secondary school mathematics teacher and interested in ideas and techniques that have direct impact in the classroom. This project began because of a problem. I had a low achieving class who were just beginning their GCSE (General Certificate of Secondary Education) studies. These students had predicted grades of Es and Fs where most colleges, sixth forms and many employers now require a C grade minimum for entry. They lacked motivation because, as they put it, “What’s the point?” They wanted practical skills that they could use in day-to-day life not a GCSE grade that was too low to be useful for college or sixth form entry. They didn’t understand how Pythagoras’ theorem or algebra was relevant or useful for them. I wanted to find something that could do both – give them practical skills but also give them skills to help them succeed in their school mandated GCSE examinations.

## Literature Review

As I began my project, the debate surrounding effective teaching strategies for low achieving mathematics students was centred on whether constructivist, contextualised or explicit instruction was best. The NCTM (2007, pp. 1-2) claimed that “Systematic and explicit instruction” was the most effective. Barnes (2005, pp. 46-49) advocated a constructivist approach with “more focus on relational and conceptual understanding as opposed to learning by rote and memorisation”. Finally, Baker, Gersten, & Lee (2002, pp.60-67) recommended “explicit teacher-led and contextualised teacher-facilitated approaches”.

As part of my reading on the subject, Realistic Mathematics Education (RME) came up as being particularly useful for working with low achieving students.

"Realistic" seems to imply a reliance on context-based problems, but it is a slight mis-translation from "the Dutch verb zich REALISE-ren that means to imagine" (Heuvel-Panhuizen, 2005, p. 2). "Realisable" may be a more accurate term. RME uses models to gather intuitive ways of solving a problem. These models are based on ideas or concepts that students can "realise" and so can discuss and share ideas about. For example, a model might be a map to start a discussion about distance, time and speed, or it might simply be a diagram of a rectangle to start discussion around area (as we will see later). Once the initial ideas have been gathered, a series of prompts are used to develop this informal method into a more sophisticated or formal one. These may take the form of abstract questions – 'vertical mathematizing', or context problems – 'horizontal mathematizing', (Treffers, 1993). At its core is a focus on discussion and the careful presentation of problems.

RME began in the Netherlands, was used in the United States of America and then brought to the UK in a trial by Manchester Metropolitan University. As part of this study, teachers reported "an improvement at the bottom end of the ability range", that "activities interest the pupils and so engage them in the lesson" and, "pupils are willing to have a go at solving a problem" (Searle & Barmby, 2012). However, using RME raises some key concerns. First, the amount of time it took to investigate and work though a topic (Romberg, Meyer, Gutstein, Keys, & Teaf, 2001). More emphasis on discussion and sharing ideas means that progress is slower which caused concerns as courses like the GCSE often have tight timings. Another concern was how teachers could best promote pupils' progress to higher levels of understanding and more sophisticated methods of working (Dickinson & Eade, 2005, p. 2).

## Pilot

I carried out a pilot study to see if RME could work with my class. I audio recorded and then transcribed a series of lessons about area.

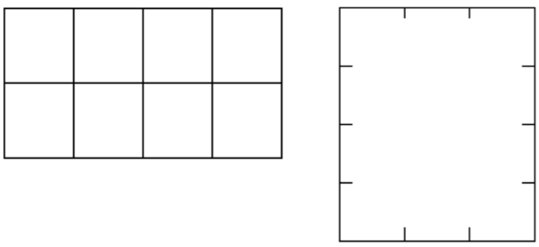

Figure 1: Area Lesson Model

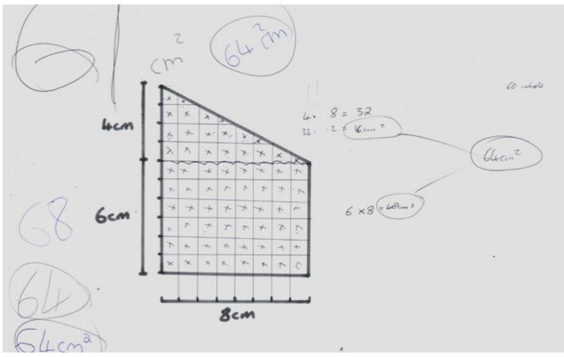


Figure 2: Group work – trapezium

I used Figure 1 as my model to begin the topic. This is the discussion that emerged:

> Teacher: What do we know about area?
> Student D: **Count the boxes**.
> Student L: The inside of the shape. It's the measurement of the inside of the shape.
> Teacher: So what is the area of this shape here?
> Student A: 8 [the others agree].
> Student L: and the other one is 12.
> Teacher: How do you know?
> Student L: I counted the squares.
> Student A: An easy way is to **count in threes**…or **4 x 3**…or 4 and 4 is 8…

These students have illustrated to the class three methods for solving this problem already – counting the squares, counting in strips of three or multiplying the two dimensions together. There is also a clear mixture of ‘informal’ and ‘formal’ methods. This is a topic they have seen before and so, through discussion, are pooling their ideas and memories.

> Teacher: What could I do here? [Gesturing at the second image]
> Student A: I know it’s **4cm x 3cm which equals 12**
> Teacher: How do you know?
> Student A: Can I come up? [I confirm] If I **draw in the lines**…

This part of the discussion shows one of the students clarifying the connections between the different methods and sharing this with the class. We then moved onto a second, similar problem, this time with no lines drawn in to help, just the dimensions labelled.

> Teacher: Can anyone tell me a method to find the area here?
> Student J: You could do **5 x 3**
> Student P: You could draw it in your book and then use the **squares in your book**

This fragment is of two other students discussing different methods to solve the problem, based on the previous discussion. The students went on to solve a variety of problems on finding areas of squares, rectangles and triangles using the method they were most comfortable with, whether that was counting squares or multiplying.

To see if these students could extend the use of their methods to more complex problems, I presented them with a series of questions on finding the area of a trapezium or a parallelogram. Figure 2 is the work produced by a class discussion on how to find the area of this trapezium. In the left of the picture, you can see the various answers the students put forward, using a combination of multiplying and counting squares methods. On the right is my summary of the discussion had after prompting them towards the multiplying method. The students were then given a variety of problems to work through themselves. I found that, even if the student was using one of the informal methods such as counting squares, they could see what the ‘next step’ was and could aim for that. In Figure 3, the student has used multiplication to find the area of the centre of the shape and then resorted to the counting squares method for the partial squares in the triangles at the end. They can see that the multiplying method is faster, but have a method to fall back on if the multiplying method fails them. They were comfortable moving between different methods and were encouraged to “have a go” (Searle & Barmby, 2012).

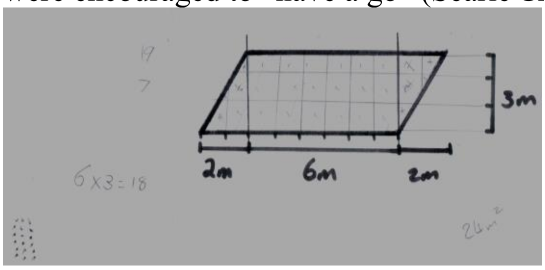


Figure 3: Student work – parallelogram

## Research questions

My pilot study indicated that RME techniques could be effective for low achieving students and led me to form the following research questions for my main study:

1. Does use of RME benefit low achievers beyond my classroom alone?
2. How can we promote the use of more sophisticated methods by students?
3. How can we support teachers in becoming more confident delivering material this way?

## Structure of the project

I decided to place discussion groups composed of colleagues at the centre of my research. Student voice is frequently utilised in schools to varying degrees of success, but teachers are not given many opportunities to get together to share best practice or ideas, as meetings are so often taken up with administrative issues. “Using a group whose knowledge or expertise you can tap can be a fruitful and time-saving means of obtaining information” (Open University Course Team, 2001, p. 174). In addition, an issue highlighted in the use of RME in schools was the need for improvement in teacher education linked to RME (Wubbels, Korthagen & Brockman, 1997). From a practical point of view, it also gave me access to more classrooms without having to carry out observations that can be nerve-wracking for the teachers involved.

## Review of research questions

I am going to focus on the second research question for review, how can we promote the use of more sophisticated methods by students? This was highlighted as an area for development in previous studies using RME (Dickinson & Eade, 2005, p. 2) and is key for teachers’ confidence in their delivery of the material. Three key themes emerged from the discussion transcripts and student work.

### *Use of discussion*

> M: We were starting with the first exercise which is bar charts where the book just gives a rough bar chart and shows that to the kids and I said ‘what is wrong with that’. I said ‘that must be a silly child that has done that bar chart’ so we started then discussing what is wrong with it. We spot what is wrong with it. So they came up with several ideas which I asked them to write in their books which they have done… and then after that I said ‘alright, now that we have spotted all the things that are wrong, do the bar chart correctly’. Which they did in their book and following that they started to answer the questions straight after the bar chart which is quite good, and in the second lesson we were using what they did for composite bar charts so that they had to apply what they had said, that they had kept the width of the bar chart to be the same [shows work] and they kept the bars… the names within the width of the bars so that is a certain consistency from the first lesson. (Second Discussion)

This teacher explains how discussion in his lesson was particularly helpful in allowing students to develop methods and then retain and apply those methods to other, more complex problems later on. This is similar to my observations from the pilot study.

### *2a. Use of students’ ideas*

> H: But what was really nice was people started out by drawing on the squares [onto the triangles], and then were quite lazy and decided actually we couldn’t be bothered to draw out the squares and we’re going to do it by writing the method without drawing the pictures so actually we ended with this, by the time they’d made the step up, which was actually quite a sophisticated written method.
> (Final Discussion)

Pythagoras’ Theorem is a difficult concept to introduce relying entirely on students’ own ideas and it took very careful framing of the ideas. The students began by finding the areas of squares drawn on the sides of right-angled triangles and then spotted the link between the areas before working backwards to find the missing side. This student has moved on from the initial, informal method of drawing pictures, to what resembles a more formal written method (Figure 4). Not all of the class accomplished this, but they could see how the student's written method related to their pictures and so knew what the 'next step' was for them once they were more comfortable.

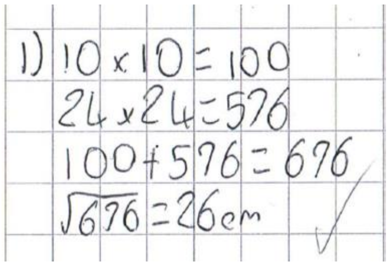


Figure 4: Student work – Pythagoras

### *2b. Use of students' ideas*

> G: A visual prompt to get a discussion going can be very, very helpful, and you can use it for asking questions, getting them to ask questions and discuss around that. The next bit of that, telling them versus their ideas, what I find is people are very reluctant at low ability level to think for themselves, to have ideas… so you end up feeding the ideas that they could start to have which can be frustrating.
> (Final Discussion)

Using student's own ideas was not always successful and we echoed the findings of Dickinson and Eade (2005, p. 2) that this was the area that the teachers involved felt they needed the most support with. What happens when the students' ideas do not come naturally? The discussion in the teacher voice groups was helpful in allowing us to share ideas and models for this.

### *3. Prompting for Progress*

> H: I think it's how we support them making that step-up. So… going from the pictures to their written methods [in the Pythagoras example] how they were happy doing it, they just did it. How do I then support the rest of the class in making that step as well? Because they have a method that works so why would they want to? And that's sort of, the issue being that lots of them now are very happy with that method and that's great, they can answer exam questions and for a C grade topic, that's fine. But to get them developing mathematically, should I maybe be looking to push them towards using the written method?
> A: do those students need to be pushed to using an abstract method?
> H: that's what I was starting to think about. I can see where it would go next and some have developed that idea naturally, do I then go along and say "right, everyone try it like this" at the risk of losing some of them? Or do I say "try this, because it's going to be more efficient" or do I say "actually, do you know what? They've seen the written method, it's up to them if they make the step or not."
> A: that comes back to the very difficult question of "why are we teaching this to them?" Do they need to know an abstract method? (Final Discussion)

This idea of an 'abstract' method is an example of 'vertical mathematizing' (Treffers, 1993, p.94). As the teachers became more familiar with the style of teaching and began to realise the importance of these prompts to the development of the methods being used, the teacher discussion often focused on this 'prompting for progress' (as we referred to it). We developed from discussion of how we use prompts to should we always use prompts. Does not prompting students limit them unfairly to an informal method that is less flexible? Or, by encouraging them to go through vertical mathematizing do we risk undermining their confidence in their informal methods?

## Areas for development

Discussion after my presentation at BSRLM raised three areas of continuing interest. Firstly, can use of context in adult mathematics education be considered differently? For example, could a 'ladder against a wall' problem be more helpful for older students learning Pythagoras' Theorem as they could more easily see the relevance? Is this just a return to the argument about teaching using a context problem as opposed to a 'realistic' or 'realisable' problem? Secondly, what were the issues surrounding school uptake of RME? Dickinson and Hough (2012, p.21) highlighted four emerging issues from trials of RME in secondary schools in the UK. These are: progress and assessment, preparation for GCSE, pupils experiencing a mix of approaches and development of the use of RME. Preparation for GCSE and development of the use of RME echo what I found through my study. Thirdly, could there be symmetry between the RME techniques used by the teachers working with students in the classroom, and those I am using in my teacher voice sessions with my colleagues? RME is based around the use of discussion with a focus on careful presentation of problems to help develop initial informal ideas into something more sophisticated. In our teacher voice sessions, we were using students' work to start a discussion that took teacher's initial reactions and began to form them into something they can use in the classroom.

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