A Review of BSRLM Research 1995-2002

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Chapter 1

THE FORMAT AND NATURE OF THIS REVIEW

Introduction

A review of research ordinarily would be approached with the ideas of depth and range in mind, a situation where certain expectations are built in and stringent criteria laid down. The research under consideration here, however, is perhaps best described as broad and varied, some of it part of ongoing larger studies, some done in short periods of time, and with participants ranging from teams of people to individual teachers or researchers. Indeed, the work considered here is sometimes at the level of 'first thoughts' about an issue. The fact that the questions explored are so different in terms of nature as well as scale makes it difficult to take the depth of the studies into consideration in a meaningful way. As a result, the content of BSLRM reports and papers will be explored here mainly in terms of the range of topics covered. An attempt has been made to approach the studies in terms of the focus of their content. This has proved to be a difficult exercise and is by no means as tidy an approach as might be desired. The categories, however defined, inevitably overlap. For example, an investigation relating to student teachers of mathematics may well include aspects of pupil behaviour in the classroom but the *student teacher* is the prime area of interest. Similarly, theoretical

considerations are offered in all studies to a greater or lesser degree, but do not form the focus of the work being carried out. With this potential for repetition in mind, every effort has been made to keep any overlapping in terms of focus of studies to a minimum.

Overview of the review process

Four main areas of content have been identified: (a) classroom and methodological considerations, (b) mathematical topics, (c) student teachers and teachers and (d) theories and the application of theory and each of these is reflected in the four main chapters making up the review. A separate category for pupils has not been included since an overview of the studies indicates that pupils as learners of mathematics are studied within the context of the mathematics classroom and the methodologies used in teaching the subject (see Chapter 2) and the focus is not on the pupils as individual learners. At the same time, they appear in almost all studies except those of a purely theoretical or discursive nature. For similar reasons, Chapter 4 (Student Teachers and Teachers) appears to be relatively short. This is because studies in which the sole focus is the student teacher or individual teacher are relatively few, and more frequently studies focus on these elements when considering the classroom context as a whole.

The work begins with the chapter on Classroom and Methodological Considerations. This sets the scene, as it were, for what follows in the review as a whole in that it includes an outline of the development of the field of research in mathematics education over past decades. Each chapter has a slightly different format as, for example, in Chapter 2 studies are grouped into the primary and secondary sectors while this is

not the case in other chapters. However, a common feature within each chapter is the appearance of a 'comment' at the end of sections of the chapter and 'concluding comments' at the end. Main sections of a chapter will occasionally include a detailed analysis of a study or studies in order to help draw out particular points that are pertinent to that area of research. The criteria for the selection of these studies is that they raise issues of a general nature, highlight other questions that can be explored in relation to that content or present an example of a particular strength of BSRLM research. Other studies could equally well have been chosen but clearly it is impossible when dealing with such a large corpus of work, to refer to every study in any detail (although most receive a mention, at the very least). The overall aim is to give an indication of the nature of the areas researchers have chosen to explore, where any emphases may lie and where gaps may occur in terms of content.

CATEGORISATION OF STUDIES

The initial analysis of content of BSRLM research reports from 1995 to 2002 that led to the above categorisation began with some fifteen different categories of study. These have been collapsed into four major categories that are reflected in the titles of the main chapters as shown in the table below. Approximate proportions of studies within each category appear in the table below. It has to be pointed out that the number of studies included in these figures is not precise for a variety of reasons, as for example where the progress of the same piece of research is reported over time and could be counted as a single piece of work (which has not been in the approach taken here).

Grouping	% studies
Classroom and methodological considerations	22%
Mathematical topics	30%
Student teachers and teachers	17%
Considering theory	32%

It may be helpful to offer an example of the difficulty in categorising the work under consideration by taking as an example a study dealing with interviewing techniques. It is essentially an aspect of research methodology but also has theoretical overtones such as considering the evidence for judging why one approach to interviewing in a given research situation is more effective than another. In the end, studies of this nature have been included in the 'Classroom and methodological considerations' category although they may include a strong theoretical element.

Classroom and methodological considerations

Topics within the above group include matters such as teaching methods and factors arising from the use of different methodologies in the classroom, e.g. teacher-pupil interaction, the perceptions of participants of specific situations and the effects of their actions upon each other and in some cases, socio-cultural matters such as gender and multi-cultural factors. Studies that involve the use of ICT are also included here. The studies in this group represent approximately 22% of the total BSRLM output.

Mathematical topics

Interest in researching topics concerned with particular areas of mathematical content such as algebra and geometry almost equals that in relation to theoretical considerations (30% compared with 32%). Studies of this nature are grouped under 'Mathematical topics'. This category also includes aspects of mathematics such as numeracy as well as issues concerned with matters such as assessment.

Student teachers and teachers

Work that is categorised as focusing on 'Student teachers and teachers' would appear to be the smallest category of studies with 17% of the total number. However, it is worth repeating that this is work that relates specifically to matters such as teacher characteristics or actions or the experience of student teachers. Clearly, considerations related to teachers appear in most studies within the other categories

Considering theory

Included in the group that focuses on theoretical matters are studies that have a psychological, sociological or philosophical focus but also subjects that include socio-cultural considerations (e.g. encompassing language issues), approaches to research methodology and specific theoretical perspectives such as employing Vygotskian theory when considering teacher–pupil interaction. They also include theorising undertaken by individuals. This represents the largest proportion of studies in a single category. Work in this category will be approached from three perspectives: the application of theory, theorising or the development of theories, and the critiquing of existing theories.

CONCLUDING CHAPTER

The brief for this review of BSRLM has been a wide one and difficult to identify in a very precise way, as noted earlier. This has particular implications for the concluding chapter where certainly the brief has *not* included the making of recommendations. However, it *has* been taken to include the highlighting of strengths, identifying gaps in terms of coverage and perhaps to note any obvious weaknesses across the total profile of studies that have been reported. In the process, this has, in turn, helped to suggest ideas for further study and to point to possible developments for the future. Having read this review, it will doubtless be the case that others will see possibilities that have not been noted here, which surely must be one of the intended outcomes of this process. As we shall see, the process as a whole has been a fruitful one and indicates that there is much to celebrate within BSRLM.

Chapter 2

CLASSROOM RESEARCH AND METHODOLOGICAL CONSIDERATIONS

INTRODUCTION

This chapter is based on the assumption that the kind of question investigated in any research necessitates a methodological approach best suited to answer the nature of the question being explored. As noted in the previous chapter, many BSRLM studies examine interaction at various levels within the classroom and the levels would, in themselves, necessitate particular approaches. For example, a researcher may be considering dialogue between teacher and pupil that calls for one form of analysis (e.g. Bills 6/98 and Coles and Brown 2/99, 5/02), or they may be considering interaction at a variety of levels, for example teacher-pupil and pupil-pupil, which involves a more complicated methodological approach (e.g. Hardy 3/97, Baker 11/97, Martin 3/95**,** Denvir and Askew 3/01). Given that much of the work carried out is on a small scale, we would not expect to find a large number of studies that set out to explore questions involving a large scale collection of statistical data and the processing that such data requires, although studies of this kind do occur in the body of research (Shayer et al. 6/99, Goulding 2/02)

In order to achieve some sense of the spread of topics in the category broadly described here as 'classroom research', we shall use the differences in methodological approach employed across these studies as the criteria for grouping them, where the methodology used reflects the nature of the question at the heart of a given study. To give some structure to this task, four levels of development in the field of research in mathematics education identified by Koehler and Grouws (1992) are adapted for use here. These levels are hierarchical in the sense that they begin with a single dimension of investigation and move to a multi-dimensional approach. This kind of demarcation can be useful in sharpening our analytical perspective and helping us to consider methodology in relation to the kinds of classroom interaction being explored in the various studies.

The levels of research and the kinds of question focused on at each level are:

Level 1: teacher effectiveness;

Level 2: 'process-product' research (classroom processes);

Level 3: a broadening of Level 2 taking into account factors such as the effect of pupils' characteristics, attitude, gender and race, on the teaching/learning situation;

Level 4: broadening still further by focusing on teaching and learning at the same time.

These four levels will be adopted here in our consideration of BSRLM studies that can be described essentially as 'classroom research', i.e. those that are concerned with various forms of interaction within the mathematics classroom. In the case of Level 3, we shall include work that involves the use of *computers* or *calculators*, treating them as factors that

are focused upon within the teaching-learning situation in order to study how they affect learning outcomes.

As with any attempt with respect to classification procedures, there is a subjective element inherent in the process. The grouping of the studies that follows provides one approach to clarifying the nature of the aspects of classroom issues in mathematics education explored by researchers, but doubtless a variety of possible approaches could have been adopted. However, the levels described above are a useful framework against which to consider them here.

DIFFERENT KINDS OF METHODOLOGICAL APPROACHES

Below is a chart showing the proportion of BSRLM (95-02) studies that have been allocated within the levels adopted from the Koehler and Grouws analysis of kinds of research in mathematics education.

Level	**Focal concerns**	**Proportion**
1	teacher effectiveness	13%
2	process- product (classroom processes)	24.3%
3	taking specific factors in the teaching-learning situation into account	21.5%
4	taking the total teaching-learning situation into account	17.8%

This shows that, of the studies identified in Chapter 1 under Classroom Research and Methodological Considerations, a majority are at Level 2 (24.3%) and deal with classroom processes and the next largest group are

at Level 3 when specific factors affecting teaching-learning outcomes within the context of classroom processes are taken into account. The remainder of the studies are divided between teacher effectiveness and the holistic classroom situation. Studies in the latter category (Level 4) take into account the context, the players within the context and the teaching and learning that takes place, all into account.

The approach adopted in this chapter is to report a representative study from each of the primary and secondary sectors within each level and to give some indication of others that have been carried out. While this approach has not been adopted throughout the whole of the monograph, it is helpful when dealing with classroom studies particularly with regard to giving a fuller appreciation of the extent to which various topics are dealt with in each of the sectors.

Level 1: Studies focusing on teacher effectiveness

Research into teacher effectiveness in the past was concerned with identifying factors that either enhanced or detracted from teaching and learning outcomes in a particular teaching situation. There was an emphasis on measuring outcomes and attempting to quantify the learning that took place. Although recent studies in the BSRLM body of research reflect the trend towards a more qualitative approach, this is not to say that there are no studies in the body of research that use quantitative methods (e.g. Rowland et al. 2/99, Shayer et al. 6/99, Coltman et al. 5/00) but more often than not, these projects would also have a qualitative element to them. Evidence of the effectiveness of teachers in a qualitative approach is generally one of only several embedded aspects of the

teaching/learning situation that emerge from such studies as opposed to being the specific focus.

Primary sector

A study of the type where an aspect of teacher effectiveness emerges from qualitative research is one in which the use of investigational activities in primary schools were explored. Houssart (11/99) taped discussions with 26 teachers to explore their views about the suitability or potential of such an activity with the classes they taught. The investigational activity was for use with primary pupils of a range of abilities and the analysis of teachers' views indicated a frequent use of the word 'pattern' in their discussions, where 10 specifically referred to pattern and 4 spoke of related issues.

Houssart found that there was a tendency for the teachers to associate pattern with higher ability pupils. There was also an indication that some teachers felt that spotting pattern was something that could be taught while others felt that it either happened or did not happen. In this example, two potentially important factors have been identified with respect to the judgements teachers make in the teaching and learning of mathematics: firstly, that the belief is held by some teachers that only more able children are able to detect pattern in a sequence and, secondly, that some teachers consider that if a pupil is unable to spot pattern, nothing can done about it. If a teacher holds either of these beliefs, clearly it will limit severely the opportunity for pupils who are not considered to be at the top of the ability range, to progress in their mathematical understanding. The study is an example of the effect and the potential power of such judgements made in the classroom. The notion of pattern is

seminal in the development of mathematical understanding and the disadvantage of inadvertently limiting a pupil's potential for such understanding is clear.

Other examples in the primary sector at this level of methodological approach include work by Back (RME 2000) who explored the way in which teachers induct 6 and 7 year olds into mathematical discourse and Bills (RME 2000) whose concern was the influence of teachers' presentations on pupils' mental representations. Another approach to assessing teacher effectiveness is reported by Adhami (5/00) where peer evaluation of whole-class teaching was the focus. In this instance, teaching is referred to as "a dynamic optimisation process" and the importance of shared agreement about the professional development of teachers being informed by shared norms is emphasised. Williams and Ryan (2/02) explore the notion of 'argumentation space' between teacher and pupils and productive lines of argument in the context of dialogue between them while Anderson and Boylan (5/00) explore the relationship between teacher questions and pupil anxiety during numeracy teaching sessions.

Secondary sector

The example taken from the secondary sector is one whose specific focus was the effect of teachers' judgement making in which Watson (11/96) focuses on teachers' judgement making in the informal classroom assessment of pupil achievement. The method used in such a study might once have been to quantify the use of the word 'confident' or 'confidence' by a teacher in relation to a particular pupil's understanding of a mathematical topic, to quantify the achievement of the pupil in that

topic and then, in some way, to measure the relationship between the two. Watson, however, approached the situation by using informal interviews with primary and secondary teachers based on core questions and analysing the resultant transcripts. These interviews were held at the end of their teaching sessions in which the researcher acted as a support teacher, and teachers were asked to elaborate or exemplify their comments from time to time. The assumption had been made that meanings were shared with respect to the notion of 'confidence' in the context in which it was being discussed, which was not the case. Watson gives us some insight into the way the analysis was carried out when she talks of 'layering' by examining paragraphs, moving on to sentences then to words in stages of refinement in the analysis.

The recurrence of the use of the word 'confidence' in a variety of situations from this process identified it as being an important factor in teachers' assessment procedures at this informal level and led Watson to consider the concept in relation to assessment in greater depth than may otherwise have happened. The study as a whole provides a further example of how the unexpected can arise as a result of this methodological approach and why one word to describe research of this nature is 'illuminative'. Teacher and researcher have worked side by side and have shared the common experience. The researcher's job is to act as an agent in helping to 'illuminate' what the teacher was thinking or intending when using certain words or taking certain actions, noting the consequences of these and then jointly, reflecting on the outcome.

The BSLRM archive contains several examples of studies of this nature at secondary level where researchers set out to identify specifically the effectiveness of a teacher or of teachers within the teaching-learning

situation. The focus of Morgan's (5/96) work was to evaluate the effectiveness of teachers in assessing GCSE coursework and Byatt (5/96) investigated whether or not the conjecturing atmosphere aimed for by teachers within mathematics classrooms observed had, indeed, been achieved. An instance of a teacher carrying out a study of their own effectiveness is that of Hall (11/01) where his aim is to listen to the voice of his pupils to obtain firsthand feedback about their reaction to the mathematics teaching they receive. Coles (3/01) reports a studying which there is a similar emphasis on the teacher's 'listening' within the classroom.

Comment

The more qualitative approach to research into teacher effectiveness in the studies above exemplify the richness in potential of a methodology of this nature when used with particular kinds of questions in mind. It is not the main purpose of such a procedure to look at teacher effectiveness in terms of specific gains in pupils' knowledge, but one that involves the observation of teaching and learning as it is happens. Romberg (1992) would call this the 'symbolic paradigm' (also known as interpretive or phenomenological) where *symbolic* relates to the symbols used in creating meaning and he describes it in these terms:

> 'This paradigm is common within such disciplines as sociology, political science, and anthropology. In education, this perspective translates into the belief that knowledge is situated and personal, that pupils learn by construction as a consequence of experiences, that the job of teaching is to create instructional experiences for students and negotiate with them intersubjective understandings

> gained from those experiences, and that the organization and technology of the classroom and school are arranged so that all of the experiences can be rich and meaningful.' (p55)

In the case of Watson's study, the meaning established between pupil and teacher perhaps in reality, was not shared. The pupil appeared 'confident' in a given situation and the teacher continued to apply this label to the pupil in other mathematical situations where confidence may not have existed.

However, in both of the above cases (Houssart and Watson) we are talking about the actions of single teachers being observed in the classroom and the question arises of whether one can extrapolate from the evidence gained. In short, can one generalise from the results? Stephens (1982) has described a traditional view of generalisability as *horizontal* because it allows the likelihood of repetition of the findings to be quantified and similar results would be expected in similar situations as the one studied (assuming reliability and validity). He describes a second contrasting kind of generalisability as *vertical* where the researcher goes beyond the immediate situation being studied and links factors within it to more general considerations of a theoretical nature. Theories are suggested in relation to the findings of the research. Quite clearly what we are looking at here is vertical generalisability.

The results of the studies by Houssart and Watson show that teachers may not always be aware of the potential powerfulness of the ideas that form the basis of their thinking and actions. Houssart's results identify misconceptions on the part of some primary teachers about pupils' ability to learn to spot pattern and to develop an understanding of it. Where

Watson's results are concerned, it is questionable, for example, how many teachers are aware that the superficial evidence of confidence they detect in pupils in a given situation may lead them to think that the pupil understands the mathematics they are doing and hence are judged to need little more in the way of support in that particular topic. The real importance of any research is the extent to which it informs practice. A great advantage of studies of this kind is that they involve the collaboration of teacher and researcher and this makes any results immediately more accessible. The teacher involved knows the value of the outcomes and their relevance to their professional practice and ideally, it is the kind of information derived from research of which we would want all teachers to be aware. Equally important is the fact that the research throws up new questions and may lead to new theories to be tested. It has already been pointed out that there are many studies of this nature in the BSRLM archive and studies of this kind must be seen as one of its strengths.

Level 2: Process-Product (Classroom processes)

Methodologies at this level set out to answer questions relating to the variety of processes that are included within the mathematics teaching-learning situation such as grouping of pupils, resources used and interaction between teacher and pupils, and pupils and pupils. Most studies of this nature also fall within the symbolic paradigm where the meaning of selected activities within the classroom is being scrutinised.

Primary sector

Back (RME 2000) studied examples of transcripts of teacher-pupil talk within primary mathematics lessons. Her purpose was to explore the relationship between the different levels of discourse that were taking place, i.e. in classroom discussion, mathematical discussion and the discussion of 'school mathematics'. Analysis focused on talk but also took into account factors such as intonation and gesture from time to time. The latter was seen to be relevant because the aim was to discover how teachers use classroom talk to encourage the pupils' mathematical thinking as well as their interest. Back gives a detailed account of how the analysis was approached and of her search for a joint construction of mathematical meaning between the teachers and their pupils as well as evidence in pupils' talk of their mathematical understanding using Bruner and Vygotsky in her underlying theoretical rationale. She concludes that 'During the course of their lessons, teachers interpret the offerings of their pupils and pupils interpret the offerings of teachers, often rephrasing each others' comments to fit their own their own frames of reference' and as a result, the mathematical content is seen to be 'collaboratively constructed jointly by teacher and pupils.' (ibid. p44) Back's work is an example of how research that focuses on either dialogue or group discussion can be useful in showing the way in which teachers can offer a 'template' for the meaning of a mathematical idea; there is give-and-take with the pupil until they both agree on what that meaning is and in terms that are valid and understandable to them both.

A further example at this level with Year 5 pupils is reported by Bold (7/02) in which the teacher-pupil and pupil-pupil language used in the context of learning probability was recorded and analysed.

Secondary sector

An example of research from the secondary sector at this level is an investigation by Edwards and Jones (11/01) into the use of exploratory talk in a secondary classroom. Based on the notion of this type of language used between peers in the classroom and exploratory talk as described by Mercer (1995), the purpose of the study was to record the talk of pupils of equal ability while observing them within collaborative working groups in a mathematical learning situation. This was, in part, to explore the neo-Vygotskian perspective that students in a group do not have to be of unequal ability in order to learn in a collaborative situation. Students' views were taken into account by interviewing them and asking them about their perceptions of the following when working in small groups:

- The purpose and benefits of working in small groups;
- Teacher expectations of appropriate student behaviour in such groups;
- Characteristics of small groups for them to be successful;
- The extent to which both individual and group accountability exists in these situations;
- Aspects of stability of these groups.

Three groups of students were interviewed: (a) a class of low attaining 15-16 year olds; (b) a class of high attaining 15-16 year olds; (c) a class of middle attaining 12-13 year old pupils. Analysis of the results produced 25 different categories, five of which are reported in the paper.

While the data indicated support for some of the conditions identified by others as necessary for effective collaborative learning situations (e.g. knowledge of peers improved confidence in expressing views and offering solutions), evidence did not support other previous findings. An example of the latter was the fact that there appeared to be little differentiation of views between pupils and those of their teacher, based on the attainment level of pupils.

Other examples at this methodological level include pedagogical research taking place at the higher education level where the 'classroom' takes on a different connotation. Kent and Stevenson (2/98) worked with first year undergraduate chemistry students and studied their progress within a mathematics laboratory situation and Hegedus (5/96) transcribed the 'think aloud' work of two undergraduates as they worked on calculus problems. Their work also serves as an example of the breadth covered within the BSRLM umbrella of participants and their interests.

Comment

The studies cited above are further examples of qualitative methods used at a different level of research and again are representative of the symbolic paradigm (see above) where classroom dialogue and discussion become the focus of the study, and a major concern is with the use of language within the social context of the classroom. The work of both Back (RME 2000) and Edwards and Jones (11/01) exemplifies the importance of language when considering classroom processes and the interpretation of events within the mathematics classroom. In the former case, there is a concern with the give and take between pupils and teacher and there is mention of collaborative construction of meaning in the

process and data are gathered through observation. In the latter case, the collaboration being focused upon is between peers and their views about such an approach in the mathematical learning situation are sought directly through interviews. In the study by Back, the symbolic focus is upon the "intersubjective understanding" of the participants, while in the case of Edwards and Jones the focus of exploration is upon the arrangement of a learning situation to discover the extent to which it may be "rich and meaningful" (Romberg 1992 op.cit.).

Level 3: Accounting for the effects of specific factors

This is the kind of research situation where in the past, a specific variable or variables might have been controlled and studied in order to probe their effect on the teaching-learning situation. Once again we shall see evidence of how a more qualitative approach has been adapted to this kind of research question and a range of alternative ways to investigate problems of this nature has been opened up.

Primary sector

Work by Barrington et al. (11/97) provides an example of research at level 3 where the specific factor being taken into account is the effect of pupils' ability within a particular learning situation. The focus was on teaching numeracy to low attaining KS2 pupils. In this project, rather than using an error analysis approach, the researchers study what the pupils bring to the learning situation in the way of understanding of mathematical operations. The intention was to broaden the view of numeracy from one where it is seen as consisting of mathematical objects

upon which pupils act using various processes, to the situation where they recognise that the objects themselves are the result of such processes. Attention is diverted to how the objects are derived and their meaning is developed. INSET sessions emphasising this approach were held for 15 teachers involved in the project who worked with pupils with special needs in a normal classroom setting. After a period of teaching using the approach, teachers were asked to identify pupils within three ability levels and these pupils were subsequently all given the same numeracy task to carry out. The factor of concern was the ability level of the pupils and whether the achievement of the low attainers would improve with the new methodology. One of the conclusions drawn was that in order to help low attaining pupils, it may be more effective to focus on mathematical objects and to teach them particular strategies that they can use with these objects and ultimately develop and adopt more flexible ways of working with them. The more able pupils already appear to have a repertoire of strategies from which they can choose. In another study Newstead et al. (2/96) focus on children's use of language when doing division problems.

Sutherland et al. (5/02) report the use of ICT in teaching geometric concepts to 10-11 year olds as part of a wider study of the ways in which new technologies can be used in classrooms to enhance pupils' learning. An important role of the teacher in this context is identified as encouraging pupils to become aware of what they observe by speaking and writing about it. There is an emphasis within the project on the importance of the teacher-researcher partnership.

Secondary sector

In a study of the use of hand-held computers with a geometry class of 12 year-olds, Gardiner (11/97) explores the effects of mediation by both the teacher and the computer screen. He approaches the situation from a Vygotskyan perspective and examines how students reached conviction/proof in this context. The situation is described as 'a ZPD defined as the area in which spontaneous and scientific concepts interact'. The data consisted of transcripts of pupil-pupil and teacher-pupil interaction and after analysis, the interchange of different levels of visualisation, conviction and mathematical argument were noted. The conclusion drawn was that this type of learning situation involving the use of computers helps to consolidate pupils' meaning-making in the area of construction and proof. It also provides a situation in which it is possible to study the way in which visualisation can move from the pupil's 'inner screen' (Mason 1991) to paper diagram, to electronic screen and back again.

Many studies at this methodological level exploring the effects of the use of ICT in the mathematical teaching and learning situation are reported in the BSRLM corpus (e.g. Sivasubramaniam 11/98, Penteado 2/99, Crisan 6/99, Chae and Tall 6/99, Gates 6/99, Perks 2/02, Parker 2/02, Pope 2/02, Godwin and Beswetherick 11/02). However, there are examples of research where the focus is different such as that of Rowlands et al. (2/98) where they explore the use of concept questions in teaching A-level mechanics and Pope (11/02) who reports on the use of origami in teaching geometry.

Comment

While it is difficult neatly to characterise research as falling into specific patterns, the studies by Barrington et al. (11/97) and by Gardiner (11/97) are both examples of a methodological approach falling more within the critical paradigm described by Romberg (1992) than within the symbolic paradigm. Romberg states that the basic assumption of a critical paradigm is that it is possible "through thought and action" to improve some aspect of the world in which we live (p55) In the case of mathematics education research, the purpose would be to enter into the teaching-learning situation with the deliberate intention of improving it in some way and to consider the result of such intervention. Romberg goes on to state that:

> "The task of inquiry is to illuminate the assumptions and premises of social life so that individuals come to know themselves and their situation, understand the scope of the boundaries placed upon their affairs, and offer arguments that are appropriate to the dominant culture or institutions." (ibid. p55)

This again transfers to the classroom situation where both pupils and teachers are coming to 'know themselves', to understand better the situation in which they find themselves and to extend its boundaries in terms of mathematical understanding by a variety of appropriate means. The teachers in the study by Barrington et al. (11/97) have their boundaries extended and in turn, broaden those of their pupils. With the introduction of hand-held computers into their mathematical learning situation, the pupils in Gardiner's study have similarly reached a deeper understanding of construction and proof in geometry. Gardiner's study is one of many in the BSRLM archive dealing with the effects of introducing computer technology into the mathematics classroom. In showing how a metaphor such as that of the pupil's 'inner screen' can be

used to good advantage and built upon, he raises further questions such as, for example, is there an optimum point in the sequence of the visualisation process at which the teacher's intervention might be more beneficial than another.

Level 4: Taking the total teaching-learning situation into account.

Research studies which deal with the total teaching-learning situation use a phenomenographic methodological approach and are representative of the symbolic paradigm. Longitudinal studies tend to fall within this group and there are several examples of work based on such studies within the BSRLM archive such as Adhami et al. (5/95), Coles and Brown (2/99), Shayer et al.(6/99), Brown et al. (11/98), Denvir et al.(11/99)) and Brown (2/00). The example from the primary sector given below is a study carried out by an individual researcher and the secondary example is part of a longitudinal study. Studies of this kind capture something of the culture of a classroom reflected by the classroom processes within it (Nickson 1992). In the case of mathematics as with other subjects, different approaches to teaching will generate different learning environments that will affect learning outcomes. Research of this nature is usually concerned with finding out what these effects are and how they are generated, and generally involves a wide range of methodologies which may include the use of questionnaires, pupil data, classroom observation, testing and interviews with pupils and teacher.

<u>Primary sector</u>

Bills (2/98) provides an example of an holistic study in which the teaching, classroom interaction and pupils' learning are all taken into account in data gathering. The aim of the study was to investigate images formed by primary pupils in the context of their interaction with representations of two-digit numbers. The data were gathered from field notes of lessons that included teachers' words and actions, the equipment used and the tasks set. They also included factors such as pupils' responses and reactions to the situation including their physical actions and their questions, and pupils' answers to the researcher's questions as to what they think as they do the set tasks. Tapes were made of semi-structured interviews with individual pupils and notes were made of their physical activities in their descriptions of images they had as they did the tasks. Bills concludes that four steps are required in order for a pupil to develop a concept which include remind, recollect, recognise and realise. Simple recall of what he refers to as 'representation-specific procedures to representation-specific questions' by the pupil is not enough. (ibid p44)

There are several other instances of primary research at this methodological level including that of Price (6/99, 11/01) who is developing a model of the relationship between teaching and learning mathematics in the primary classroom and Barwell (5/00) who uses discursive psychology in the analysis of the mathematical learning of EAL pupils.

Secondary sector

Molyneux-Hodgson and Sutherland (11/96) report on the exploration of students' ability to convert units of measurement in vocational science, as part of an on-going project. The focus of concern is the extent to which the mathematical practices students undertake within this context are influenced by the ongoing science activity. The methodology includes classroom observation, analysis of course materials, individual interviews and diagnostic tests. The researchers found that in some cases, students who had limited success in the preliminary diagnostic test were more successful within the science context than their counterparts, and concluded that the science situation may have helped to supply a supportive structure for the students. They found that the converse was also the case, where students who had done well in the diagnostic test appeared to have no resources to draw on when having to do the mathematics within the given context.

A report by Johnson et al. (11/97) provides an example of a snapshot of a longitudinal study at this methodological level, while the work of Nardi (6/97) and Jaworski et al. (2/99) take the approach beyond the secondary sector in their work within the Undergraduate Mathematics Teaching Project. Members of the research teams involved in the longitudinal studies referred to at the beginning of this section regularly present papers bringing the membership up-to-date with these projects.

Comment

The studies mentioned that fall into this fourth methodological level clearly require the use of a wider range of methodologies to account for a greater range of processes taking place within the classroom, which in turn, lead to an exploration of their effects on learning outcomes. In his

development of the idea of images and representations young pupils have and use when developing an understanding of two-digit numbers, Bills recognises the need to record gestures and physical movements since younger children resort to such behaviour in a learning situation. This is built into the methodology so that counting on fingers, physically moving blocks and so on can be related to the images they form. Without gathering descriptive elements of this kind in the data, an important element of the way young children develop concepts and think mathematically would be missed, an approach also found to be appropriate where A-level students were the focus of the study (see Rowlands, Graham and Berry (2/98) above).

The study by Molyneux-Hodgson and Sutherland presented here is a small part of a much larger whole and is indicative of how one important bit of data can be captured from one of the many aspects of the study. The fact that students who apparently can perform necessary calculations out of context but find it difficult within a scientific context suggests that their understanding of the mathematical concepts involved remains at a superficial level. This conjecture can either be tested further or at the very least, the possibility of its occurrence will be flagged in future work. The same applies to the converse of the situation that has been exposed within the study.

GENERAL COMMENTS

The strategy identified at the beginning of this chapter to help in our considerations of BSRLM classroom research was to focus on the kinds of methodology used within the studies undertaken. It is important to bear in mind that the work considered does not necessarily represent a profile

of all of the mathematics education research in the UK but is a representative sample of the breadth of interests served within BSRLM. The following comments are intended to make some general points about the kind of methodologies used by the membership.

The analysis of classroom studies against a background of different levels of research (Romberg 1992) has provided a structure that gives an indication of the kinds of studies, methodologically speaking, that have been carried out. This in turn, provides a sort of profile of the different kinds of classroom investigations engaged in by researchers. It is perhaps not surprising that so many are concerned with classroom processes (Level 2) and with studying the effect of a particular factor within the teaching-learning situation (Level 3). The kinds of questions explored in such studies lend themselves to ethnographic methods in data gathering and these approaches will fairly often involve the mathematics educator in *in situ* research that can arise from part of their daily work situation. It is likely primarily to be the availability of time that determines what and how studies are carried out together with situations where large demands in terms of funding are not made. For many it must seem like making the best of very little in terms of resources. Given this situation, BSRLM members produce an impressive corpus of work.

Methods used within an ethnographic approach to research

The 'why'

Several larger studies have used a full range of data gathering activities but most have used a selection from audio/video recording classroom situations and activity, field notes from classroom observations, surveys, questioning pupils as they work, interviews with pupils and/or teachers, and questionnaires. There may be a tendency, because such methods on the whole may appear familiar and therefore easy to use, not to give careful consideration to (a) why they are being used and (b) how they will answer the questions being asked. Some studies give detailed consideration of both, some give one but not the other and many give neither. While to some extent this is acceptable because of the unevenness in the depth of studies, it would be useful for the reader to know something of the rationale for the selection of a particular method. If the study were a small part of a larger whole then it would be helpful if this were made clear (although this does happen in most cases where this applies). Equally, however, if it is an individual working on his or her own in an isolated situation, it would be good practice to get into the habit of matching questions being explored with methods being used, and it would also help to make the research more accessible to the reader.

The 'how'

There is much that could be said about each of the methods mentioned above in the adoption of an ethnographic approach. For example, books have been written about using interviews when undertaking educational research (e.g. Kvale 1996) and it is not easily undertaken if it is to be

done in a meaningful way. There is, for example, a difference to be drawn between structured interviews and clinical interviews and it makes a difference to the reader to know which has been used. Once interviews have taken place, it is equally valuable to know how they have been analysed, for example whether they have been coded and how. The same situation exists with respect to how analysis of field notes, tapes and observations of all kinds are carried out. It is interesting to note that a discussion group has been established within BSRLM focusing on interviewing as a methodology that obviously will offer a good support structure for those using interviews for the first time.

A similar argument applies with respect to observation techniques. It is always helpful when interpreting research, to have some detail, however brief, about what the main foci for observations are and how they are recorded. If it is a question of going with the flow of what is happening in a classroom as opposed to sticking strictly to an observation schedule, this is important in understanding the data collected. Such detail helps the reader to get a grip on how a question is being pursued and what is in the researcher's mind.

BSRLM is an appropriately sympathetic forum for new researchers to be introduced to such ideas by more seasoned researchers. It is a question of at what stage, from the point of initially engaging in research onwards, the importance of adopting the habit of a disciplined approach in reporting findings should begin. There are details to be addressed that are particularly important in developing good habits of practice when reporting empirical research. It is likely that the above detail may be lacking in some reports because of the restrictions that are necessarily laid down for the submission of papers in the tri-annual reports of

proceedings so the launching of a new annual proceedings containing peer-referenced papers is highly welcome (see Chapter 6).

Chapter 3

MATHEMATICAL TOPICS AS A FOCUS OF STUDY

INTRODUCTION

It was noted in Chapter 1 that, when taking the number of studies reported in an area as a reflection of the degree of interest in research related to it, the interest in mathematical topics is almost equal to that in research related to theoretical matters (30% of the former and 32% of the latter). Research into the various mathematical topics are grouped below within five areas: (a) algebra; (b) shape/space/geometry; (c) statistics and probability; (d) numeracy; (e) assessment. Although assessment is not strictly a mathematical topic, it is included here since interest in ideas related to it is usually specific to mathematical content (although not always, e.g. Winter 11/97, Godfrey and Aubrey 6/99). Studies that fall outside the five categories identified here are mentioned at the end of the chapter.

ALGEBRA

Not surprisingly, most studies concerned with the teaching and learning of algebra or pre-algebra are carried out at secondary level. The research covers a variety of areas which tend to fall within two main clusters. The

first grouping contains studies relating to (a) what teachers do with their pupils and while they clearly contain algebraic content, the focus of study is not the conceptual content of lessons being taught and learned. The second grouping of studies relates to (b) the teaching of particular topics within algebra where matters concerning particular content are addressed. Some studies do however, focus on (c) pre-algebraic concepts at primary and secondary level and will be referred to separately.

What teachers do with their pupils

Research in algebra covers a variety of issues that fall outside a straightforward consideration of how the subject is taught. An example of an ongoing study gives a flavour of the kinds of issues that are explored (Brown and Coles 6/97, Coles and Brown 11/98, Brown and Coles 11/99, Winter 2/00, Coles 3/01). Their concern is with the development of a 'metacommunity' within a classroom where pupils are engaged in doing algebra. Reporting of the research begins with a reflection of what it is like for teachers and researchers to work together (Coles and Brown 6/97) and continues through to the reporting of a case study of change in teacher behaviour which focuses on 'listening' (Coles 3/01). The research initially draws on the work of Kieran (1996) and although the project focuses strongly on pupils learning algebra, a major thrust of the work is to help teachers to reflect upon their interactions with their pupils and to develop theories as a result of this analysis. The aim is not to lose sight of the detail of pupil actions and their algebraic development in the totality of the classroom situation. A characteristic of the study is that theories are developed throughout in response to the data that is collected as it progresses.

Other studies in this group are represented by work such as a study by Hitt and Lara-Chavez (6/99) where the focus is on an exploration of pupils' ideas connected with the concept of limit and that of Vile and Polovina (6/98) who discuss the use of graphical reasoning in the development of logic and in which the study of graphs is considered from a semiotic perspective. Healy and Hoyles (5/95) report a study of the use of a computer where an essentially visual approach to solving algebraic equations is adopted. In a similar study, Malabar and Pountney (3-6/97) explore the interaction between visualisation and symbolisation in connection with computer-generated representations of functions.

Teaching specific algebraic topics

Pirie (3/95) deals with the fundamental concept of equivalence in a study in which she focuses on the use by low attaining pupils of the equals sign when working with equations. The context of the study is the teaching of linear equations to a low achieving group of secondary pupils during which they are encouraged to see the equation with unknowns on both sides as a single entity rather than as consisting of two sides. This is seen as a vital point in the development of pupils' understanding of algebra that has been labelled as the 'didactic cut' where arithmetic begins to merge into algebra. In the study, the pupils are encouraged to see the equals sign as a 'fence' and Pirie focuses on pupils' misconceptions about the role of the equals sign that have to be re-learned by many at this pre-algebra stage. Observations of the pupils in the study indicated that they were successful in solving the equation by viewing it as a static entity, a 'mathematical object' in which the equal sign was part of the whole and not taken a signal for action between two separate parts (Kieran 1992). The aim was that pupils would come to understand that equivalence

would be maintained between the two sides of the 'fence' and how this could be achieved.

Further studies dealing with the teaching of algebra come from Bills and Rowland (11/96) who explore the use of generic examples in the teaching of generalisation and proof and a theoretical paper in which Barwell (7/01) considers the 'modelling' pupils perform in solving word problems.

Pre-algebraic concepts

In a study focusing on pre-algebra at primary level, Houssart (11/99) reports an analysis of interviews with pupils about a set task the focus of which is pattern and the generation of the Fibonacci sequence. (This study is reported in more detail in Chapter 4). Johnson (5/96) draws on research related to the pre-algebra field when considering the pedagogy of algebra at ITE level and identifies five kinds of activity connected with it which emerge from an analysis of the algebraic pedagogy within one institution. The extent to which students opt into each of the five areas depends upon whether their focus is primary or secondary. Johnson's (5/96) work also takes into account the beginnings of algebraic thought at primary level although the main focus of this work is secondary.

Other aspects of algebraic understanding

Several researchers are concerned with the exploration of pupils' and students' understanding with respect to particular areas of algebraic understanding. Nardi (5/00, 3/01) explores undergraduates' understanding

of the concept of limit and other aspects of the concept of images of function as well as the development of the use of symbols (Nardi and Iannone 11/00). Further work on pupils' symbol sense includes research by Sharma (11/00) and Pope and Sharma (11/01). Barwell (11/00) focuses on EAL pupils' solutions to word problems as does Blanc (6/99) and Hoyles and Küchemann (11/00) explore pupils' proof responses in carrying out algebraic and geometric tasks.

Comment

Major considerations in the wider research field related to the teaching and learning of algebra tend to cluster around five areas: pre-algebra, solving equations, solving word problems, symbolisation and the use of technology (Nickson 2000). Each of these areas is dealt with in the corpus of BSRLM studies dealing with algebraic topics.

An understanding of algebraic concepts underlies much of the problem solving process and it happens to be one in which UK pupils have not been particularly successful in international comparative studies (TIMSS 1997). Perceptions of, and beliefs about, algebra will affect how it is taught and there are many misconceptions on the part of both teachers and pupils about the teaching and learning of the subject (see for example Gray and Tall 1994, Furinghetti and Paoloa 1994, Bills 1997). An example of the fundamental nature of the kind of misunderstanding that can occur is reported by Pirie's (1995) work dealing with equations (see above). The two approaches generally used in teaching linear equations are identified as relating linear equations to 'real life' situations or 'undoing' a series of operations within the equation to solve for x. This approach focuses on the development of the structural nature of algebra

(Kieran 1992) and moves away from the procedural aspect with which pupils are most familiar from their arithmetic experience and which, the body of research suggests, many teachers continue to use in the introduction to solving linear equations (e.g. Chaiklin and Lesgold 1984, Kieran 1992). Growth in understanding that leads from arithmetic to algebra is inherent in moving from procedural to structural operations (Sfard 1991, Kieran 1992) and is vital to algebraic understanding. Pirie's study offers an example of an attempt to address the structural as opposed to procedural aspects of algebraic thought. A second study indicating a move towards the structural in algebra is that of Bills and Rowland (11/96) (see above) where they consider the role of generic examples in teaching algebra in leading to the development of 'structural' generalisation skills of student teachers of mathematics.

Much research dealing with algebra indicates an interest in the procedural as opposed to the structural aspect of the subject. Bearing in mind the importance of the pre-algebra stage of development and the need to bridge the gap between arithmetic and algebra, clearly the more we can learn about how to help pupils to establish that bridge is extremely important. An emphasis in research that moves more towards the structural end of the arithmetic-algebra bridge (as exemplified above with respect to concepts such as equivalence and symbolisation) could be fruitful. Equally, the fact that few studies address pre-algebraic issues at primary level suggests a need for earlier intervention; however, Johnson (5/96) and Houssart (2/99) have made a beginning here.

GEOMETRY/SHAPE AND SPACE

An active Geometry Working Group exists within BSLRM whose reports appear at the end of each issue of the publications of research appearing in each volume. The work of this group is not wholly reflected in the following analysis of studies in this area as often their reports consist of matters such as feedback from attendance at other conferences related to the interests of the group. However, the fact that the group exists and is prolific in its reporting should be acknowledged when considering the work of BSLRM in this area of mathematics. It should also be noted that several studies appearing in other chapters focus on geometry in classroom research or other contexts.

The proportion of studies focusing on geometrical topics in the primary and secondary sectors is about one to four, respectively. Wimbourne (3/95) explores the nature and development of the idea of pattern and how it leads to the abstraction of structure and from there, to classification. Other studies include the transition from informal explorations to formalised processes in pupils' geometric learning (Jones 5/95), the relationships between socio-cultural theory and the ideas of conviction and proof in studying dynamic geometry (Gardiner 11/97) and a study of geometric thinking in out-of-school contexts (Magajna 6/97). Research by Olivero and Sutherland (11/00) and Olivero (RME 2001) reports the outcomes of a classroom experiment using Cabri-géomètre to support the processes of conjecture and proof and Fujita (11/01) considers an historical perspective of the use of practical and experimental tasks in teaching geometry. Küchemann and Hoyles (5/02) explore the variety of reasons pupils give in approaching a three-step geometric task.

In a study focussing at primary level, Anghileri and Baron (3/97) explore the development of the notion of relationships by children in the early years through the use of three-dimensional blocks, with a particular emphasis on the children's use of language in describing spatial concepts. More recently, Coltman et al. (5/00) have reported a study of adult intervention in the learning of spatial concepts by young pupils using three-dimensional blocks.

Comment

It is appropriate to refer here to a report from the BSRLM Geometry Working Group (Jones and Fujita 11/01) for an insight into the current state of the teaching and learning of geometry and the directions future research might take. They suggest in this report that new pedagogic methods are vital, since

> the failure of existing pedagogic models for geometry means that across many countries important aspects of geometry (such as work in 3D) are omitted, there is an over reliance on teaching methods that rely solely on memorisation, and there is little experience of new pedagogic tools, especially recently developed computer software such as dynamic geometry.

A study by Pallascio et al. (1993) has shown that there appears to be a sequence of kinds of competencies and operations that take place within a geometric exercise working with 3-D objects, and the nature of the pupil activity at each stage (see Nickson, 2000 pp 80-1). A sequence of this kind provides an example of a kind of approach that could supply the beginnings of a structure for a pedagogic model. In it, for example, the 12

and 14 year olds move through the following sequence of operations: transposing, generating, structuring, transposing, classifying, determining, generating. (The first type of transposing is from the linguistic into the physical and the second is from the physical to drawing.) What are the actions and inputs on the part of the teacher to help pupils make the move from one kind of an operation to another? Is there one transitional stage of operations more difficult than another and, if so, how can pupils be helped through this? The researchers found that it is important to involve pupils in activities that would lead them to carry out the operation of *determining* which is seen as the activity of reasoning in relation to a geometric model and its drawing. This is when pupils become involved in 'defining the elements or parameters that are determined by the geometric limitations that apply to a spatial structure in order gradually to lead them to deductive reasoning.' (Pallascio et al. 1993, p13) The importance of teacher intervention to ensure that pupils are given the opportunity to undertake such activity is clearly vital and the study identifies an important stage at which this should take place. Some of these areas are already being addressed within the BSRLM corpus and much of the work involves the use of ICT.

Apart from a focus on pedagogy, Jones and Fujita (11/01) list other important areas where research is needed which include, amongst others, the role and impact of practical experiments and the age at which different geometrical concepts should be taught. These areas would seem to be two where some investigation at primary level might be fruitful and lead to some exploration of how the success in teaching spatial concepts at this level might be extended to work at secondary level. While the van Hiele (1986) theory of development of geometric thinking suggests levels of thought within the development of geometric thinking, these are not

linked to ages. At the same time, the work of Fuys et al. (1988) and Gutiérrez et al. (1991) challenge the notion of attaining such levels and moving on from one to another and suggest rather, that pupils oscillate between levels before progressing to a higher level. Other work (e.g. Pallascio et al. 1993) has highlighted the importance of real-world experience and 'action' in the development of geometric concepts in the young. This parallels Jones and Fujita's call for practical experiments in geometric learning and is supported by the work of Anghileri and Baron (3/97).

Of all the mathematical topics that form the focuses for these studies, perhaps geometry is the area where ICT plays the most prominent part. As one of the issues identified as of current importance by CoPrIME, the emphasis of interest in the use of ICT in this context suggests that this is a particular strength within BSRLM.

PROBABILITY AND STATISTICS

In one of two studies concerned with probability, Ojeda (11/98) focuses on primary pupils and their interpretation of the idea of chance in dealing with particular mathematical tasks and goes on to explore secondary pupils' work with probability and the use of fractions. In a second study the focus is on how teachers are taught probability during their training courses for teaching in Mexican schools and the way it does not conform with what happens in the classroom (Ojeda 6/99). Threlfall (5/00) considers the strength of evidence of the effectiveness of assessing whether or not young children understand the concept of probability and Afantiti-Lamprianou and Williams (2/02) report on the development of a scale for assessing probabilistic thinking and representativeness. Further

work relating to probability is reported by Watson and Collis (1994) and Pratt and Noss (1998).

There appears to have been little investigation in relation to statistics. In their study involving the use of computers and a spreadsheet-based approach, Ainley et al. (3/97) focus on the generating of graphs and involve primary pupils in the gathering of data, tabulating it and finally, representing it in graphical form. This is an example of how ICT can provide a useful way into the teaching of the early stages of statistical concepts.

Comment

There has been little research relating to probability and statistics in the mathematics community generally since it has been introduced into curricula in recent decades (Nickson 2000) and this fact is reflected in the relative dearth of related work in the field within the BSRLM corpus of research. This has been ascribed to some extent, to teachers' insecurity with the subject. However, given the way in which statistics pervades so much of modern life and the existence of a real need for individuals to be 'statistically aware', it is an area that is due more attention that it receives. The work of Ainley et al. (3/97) is an example of the relative accessibility, using a computer, of some of the processes underlying statistics such as data collection, tabulation, representation and interpretation of data, that form a basis for the development of basic statistical concepts. It is the kind of approach that might readily be built into the teaching and learning context of the classroom.

NUMERACY

There are many studies involving different aspects of number (including calculation) that appear across other categories of research reported in this volume, for example in classroom studies and theoretical studies. Most of these deal with the development of children's understanding of number operations (Womack 6/97, Beishuizen 3/97, Steinweg 11/98, Foxman 2/99, Price 6/99, Pike and Forrester 11/96). In some cases, the focus is more specific such as the study by Caterall and Sangster (5/00) who examine the assumption that early number knowledge directly supports the learning of money concepts, and discuss where positive and negative transfer can arise.

There are also studies particularly concerned with aspects of numeracy within the BSRLM corpus which include several reports of various aspects of the Leverhulme Numeracy Project. In one such study, Denvir et al. (11/99) discuss the effects of homework on primary pupils' gains in numeracy which is one aspect of the five year project. In studies outside the project, Lawson and Lee (5/95) explore links between literacy and numeracy and how secondary pupils use language (both oral and written) to 'think through' relevant concepts. Barrington et al. (11/97) concentrate on the meanings pupils bring with them to the classroom in order to study the development of numerical ability of low attaining pupils at Key Stage 2. Pinel and Pinel (11/00) explore the concept of benchmarking within progression in numeracy and Paine (11/00) reports on work exploring the relationship between pupils' ability to hold numbers 'in their head' and their numerical ability.

Threlfall (5/00) focuses on the notion of strategies within the National Numeracy Framework (DfEE 1999) and examines the notion of 'strategy'

in relation to the way children carry out mental calculations. His particular concern is whether strategies should be taught and he argues that mental calculation, by its nature, is not always strategic and if strategies are taught as advocated in the NNF, this will not always have the desired positive effect. Underlying his argument is the fact that there is no direct evidence that children decide how they are going to do a mental calculation before they carry it out; it is only after the event that they can describe what steps they took to arrive at an answer. Mental calculation, he concludes, is not, therefore, strategic.

Comment

The progress of the Leverhulme Numeracy Project since its inception is well documented in BSLRM research and aspects of it appear in different chapters of this monograph. The work in relation to this project is one example of how BSLRM researchers are kept up-to-date with major studies related to recent mathematics educational policy (in this case the Numeracy Strategy, DfEE 1999). The forum presented by BSRLM for ongoing discussion of these matters by teachers and researchers from all levels of education must be unique and the perspectives each gains from the other can only be invaluable in broadening the scope of understanding of all of the participants. The scale of the Numeracy Project, however, has not precluded further exploration of aspects of numeracy on a smaller scale. The studies reported above (and those which appear in other sections) give an idea of the richness of the topic.

ASSESSMENT

Several research studies are related to assessment issues, almost all of which deal with assessment in the secondary sector. Some explore aspects of national tests such as Wiliam (2/96) who examines the gap between attainment at GCSE compared with that at A-level and Berry et al. (11/98) whose focus is to discover whether pupils gain more marks from routine parts of questions on A-level tests. Lowe (11/98) examines whether questions on KS3 tests actually distinguish between procedural and conceptual understanding as was intended when the test were originally established and Pope (11/98) reports a study of teachers' views of Key Stage 3 tests.

Two studies relate theoretical considerations to assessment issues. Winter (11/97) reports on the use of teachers' intuition in the assessment of pupils' mathematical achievement while Day (6/98) considers activity theory in relation to an integrated approach to teaching and assessment. In one of three other studies at secondary level, Hunter (10/95) analyses pupils' understanding of the use of a letter in algebra through tests using two different types of items. Morgan (5/96) explores tensions that arise within teachers' assessment generally as well as coursework in particular and Hilton and Rowland (2/99) consider the use of standardised tests in assessing the mathematical learning of pupils with mild learning difficulties. Watson (5/95) focuses on teachers' judgment-making in the classroom and Smith (5/00) examines the idea of 'using and applying mathematics' as an assessment construct.

The mental calculation methods used by 11-year-olds and gathered from responses to questions in the APU survey (1987) are reanalysed by

Foxman (11/01) using categories devised by Beishuizen and Anghilieri (1998). Another study at primary level by Godfrey and Aubrey (6/99) was part of an international comparative study assessing the mathematical development of primary pupils.

Comment

It would be understandable if the excessive influence ascribed to assessment and the heavy time commitment that teachers currently have to face in the UK were to result in some antipathy towards research in relation to assessment matters generally. However, the different aspects of assessment referred to above indicate that, in spite of this, there is some interest in a range of issues of relevance and importance in the field. Morgan's (10/95) work on the assessment of coursework is widely referred to in the assessment literature (e.g. Hughes et al, 1996) and is one of few studies that have addressed the issues involved in judging pupils' investigative work in mathematics. Forms of assessment such as teachers' judgements made within the classroom situation are also touched upon and the strong potential for influencing pupils' mathematical development that such actions have is recognised (e.g. Watson 5/95). Misunderstanding as well as understanding, and bad as well as good habits of practice, can inadvertently be reinforced. There is a growing literature that examines many issues related to assessment (e.g. the work of AEREA and the Journal of Educational Assessment) and recent developments in AS- and A-level assessment suggest it is an area which will demand some research vigilance.

GENERAL COMMENTS

Several factors come through as particular strengths within the BSRLM corpus of work related to specific mathematical content. A major feature is the continuity provided by presentations over the years relating to projects as they develop. The work in algebra by Brown, Coles and Winter is one such example and the Leverhulme Numeracy Project another, but equally important is the forum for tracking the development of the work of individual researchers such as that of Ojeda in the area of probability or Gardiner in geometry.

The number of studies within each of the topics suggests that there is a greater degree of interest in the exploration of the teaching and learning of algebra and geometry than other areas of mathematics. The numbers also reflect a greater interest focused on the teaching and learning of specific topics at secondary level than at primary level. This, of course, reflects the nature of mathematics taught in each of the sectors and the recent increased emphasis on numeracy at primary level. There is evidence of the fact that children in UK primary schools have well developed spatial abilities and succeed well comparatively in this area in international tests (TIMSS 1997) but it would be a great pity for this strength to be allowed to flounder because of the dominance of another part of the mathematics curriculum.

Mathematical content plays a part in almost all of the research carried out by BSRLM members. However, it may be worth pointing out once again that issues relating to subject content are also addressed in other chapters relating to classroom research and theoretical issues.

Chapter 4

STUDENT TEACHERS AND TEACHERS AS A FOCUS

INTRODUCTION

The greater part of BSRLM teacher research concerns the interactions of the teacher in the context of the classroom (see Chapter 2). There are, however, some studies that have a particular focus on various characteristics or actions of student teachers or teachers, and it is these that will be dealt with in this chapter. These studies will be considered against a background of seven areas in mathematics education identified by the Committee for Professors in Mathematics Education (CoPrIME) as of importance at a national level. These are: (a) the mathematical knowledge of teachers; (b) progression from novice to professional; (c) interactive teaching skills; (d) post-14 skills; (e) teachers as researchers; (f) expectations (standards of achievement) and (g) ICT. Teacher education in the context of this chapter refers to initial teacher education (ITE) and not the continuing professional development (CPD) of teachers.

A rough estimate of the number of studies relating to teacher education and practising teachers in the primary and secondary sectors suggests an

almost equal degree of interest between the two. However, the numbers focusing on teacher education as opposed to practising teachers between the two sectors is significantly different. About three times as many studies deal with teacher education in the primary sector compared with the number at secondary level, while in the secondary sector a greater proportion of studies focus on practising teachers. Although the overall total number of studies here is relatively small (about 54 studies), this disparity in emphasis points to a contrast in priorities at the two levels. At primary level there is more concern with the mathematical knowledge of individuals and the characteristics they bring to the teaching/learning situation while at secondary level, interest appears to be directed more towards pedagogy.

TEACHER EDUCATION

The research concerned with Initial Teacher Education falls into three main areas: student teachers' mathematical knowledge; factors that influence them such as their beliefs and anxieties; theoretical and pedagogical issues.

Student teachers' mathematical knowledge

With the establishment of the National Curriculum and more recently, the National Numeracy Strategy, the mathematical knowledge of primary teachers has come under close scrutiny. This has attracted work in this area, one example of which is an extended study of student teachers' mathematical knowledge undertaken by Rowland et al. (6/98). Their study begins with an audit of the relevant knowledge of primary PGCE

student teachers on their entry to the course. The results of the audit are ultimately linked with the students' success in teaching number on completion of their course as they are followed through to their final teaching practice (Rowland et al. 2/99, RME 2000). Statistical analysis carried out on data gathered by the end of the project produces evidence that suggests that "the successful teaching of mathematics is not guaranteed by subject knowledge alone" (p 15). One of the main conclusions drawn is that the effectiveness of a student teacher is the result of "a complex matrix of knowledge and personal qualities" and they provide a list of five of these one of which is the student's "willingness and….intellectual capacity to 'go back to first principles' regarding the mathematical content (however seemingly elementary) that she was about to teach". (ibid.)

Pinel and Pinel (11/99) also explore primary students' mathematical knowledge. Their study began with an audit and based on these results, addressed the gaps and students' weaknesses in mathematics. The strategies developed to do this included face-to-face workshops with input sessions, multiple choice, self-assessment and distance learning materials

At a different level, Jaworski et al. (2/99) explored the collaboration between mathematics teacher educators and university lecturers in the teaching of first year mathematics to undergraduate mathematicians and Hegedus (5/96) reports on an investigation into the role of metacognition in relation to undergraduates' study of calculus.

Student teachers' beliefs/influences/anxiety

Examples of work in this area include a study by Brown et al. (11/98) who identify primary student teachers' conceptualisation of mathematics and the teaching of mathematics, and subsequently track their evolution as learners of mathematics to teachers of mathematics, throughout a BEd course. In a further study, Brown (2/02) explores the identity of student teachers as they progress through their first year of mathematics teaching in primary schools and investigates the way in which they come to reconcile personal aspirations with various external demands made upon them. In another study at primary level, Green and Ollerton (6/99) examine the roots of mathematical anxiety in primary student teachers and in a study at secondary level, Smith (6/99, 5/00) reports work in relation to student teachers' beliefs and the influence of secondary mathematics teachers on them.

Promoting pedagogy/theory for teacher education

A variety of interests are addressed in this category of teacher education research.

Johnson (5/96) examines the activities within primary and secondary ITE courses with respect to the teaching of algebra (see chapter 3) and in doing so, she identifies five areas of algebra-related content in ITE courses:

- Students' own algebra learning and knowledge;
- Knowledge of issues surrounding algebra and its learning and teaching;

- Knowledge of tasks or activities and materials available for learning algebra;
- Pedagogical skills in the context of teaching algebra;
- Reflection and self-awareness on the part of students of their confidence in their knowledge of algebra and issues involved in its teaching and learning.

Johnson argues for the need for a model for working with students learning the pedagogy of algebra that incorporates these five areas. While the issues themselves may be included in teacher education curricula, their interconnection is not apparently always made explicit.

Blanc (3/95) explores how student teachers approach investigative problems and identifies the strategies they develop in the process. In doing do, he draws on the notion of the didactic contract formulated by Brousseau (1986) where a set of rules are defined by the relationships between the content taught, the teachers and the pupils in the mathematics classroom. Chilakamarri (2/98) examines the gap between investigational methodology as approached in teacher training institutions and compares it with what happens in investigational work in schools.

Two studies touch upon the use of reflective practice in PGCE teacher training, one by Edwards (3/95) and a second by Martin (3/95). In Martin's work, a major focal point is to question the extent to which the potential for reflective writing is not realised because the contexts of tasks and assignments set for students do not connect meaningfully with their personal theories and experience. They may have difficulties in (a) how to reflect, (b) why they should reflect and (c) why the content is significantly important to be reflected upon. This has clear implications

for approaches to methodology in teaching student teachers. Woodrow and Jarvis (3/01) identify the different learning preferences of undergraduates and PGCE students according to their ethnic origin and compare students of mathematics with those of other subjects.

Further work in this area includes the development by Baker (11/97) of a model of mathematics as social practice and a model of a pedagogy for teacher education by Prestage and Perks (11/99, RME 2001). Womack (6/98) reports a study in which students focus on the invention of signs for operations with the aim of trying to make explicit the intuitive skills of young children in similar situations and he also explores the strategies used by students to solve addition and subtraction problems mentally (Womack 6/98).

Comment

The work of Rowland et al. (RME 2000) concerning student teachers' mathematical knowledge is an example of how BSLRM research reports occasionally plot the progress of a single study and its development over a period of time. Reports related to this study appear over two years (Rowland et al. 6/98, Rowland et al. 2/99) and culminate in a final paper in Volume 2 of the Society's annual proceedings (Rowland et al. RME 2000). The study also raises issues that are relevant to other areas of BSRLM research insofar as it explores questions raised in the past about what makes for successful mathematics teaching. Examples of research that has dealt with this question over the years include the work of people such as Shulman (1987) and Fennema and Franke (1992), all of whom recognised the fact that successful teaching is dependent upon the intersection of several factors. Fennema and Franke sum it up in this way:

> Teachers have to take their complex knowledge and somehow change it so that their students are able to interact with the material and learn. This transformation is not simple, nor does it occur at one point in time. Instead, it is continuous and must change as the students who are being taught change. In other words, teachers' use of their knowledge must change as the context in which they work changes. (p162)

Students have to be willing to question what they do, given each new set of circumstances, and reconsider it in the light of new demands. Building such stages of thought into teacher education programmes is clearly important. The confidence of student teachers in their ability to take specific mathematical concepts back to basic principles has been found by Rowland et al. (RME 2000) to be an important factor in successful teaching; this in turn suggests that it would be valuable to explore ways to engage student teachers in such actions during their courses, and to find ways of using these actions to build their confidence. This would be one example of helping teachers change what they know as described by Fennema and Franke (1992) above, by re-examining their knowledge at the level of basic principles and adapting that knowledge in the light of the demands of the needs of the pupils they teach.

Of the seven areas of governmental or national importance identified by CoPrIME at the beginning of this chapter, no one topic within BSRLM research in teacher education stands out as being of greater interest to researchers than others. The two topics dealt with more than once are the mathematical knowledge of student teachers and the adoption of an inter-subjective teaching style in the context of investigational work. The

remainder of the work dealing with teacher education shows some concern with matters such as student perceptions of mathematics, of themselves as learners and of what it is to be a mathematics teacher, as well as theoretical considerations in relation to teacher education. This includes a core of researchers who have an interest in building theories of models of mathematics teaching within ITE courses (see Chapter 5).

It is clear that some of the issues within the CoPrIME list do not fall within the concern of teacher education as, for example, the teacher as researcher or post-14 issues. On the other hand, the use of ICT as a means of delivering mathematics to be learned in classrooms within teacher education courses is within the sphere of concerns and while doubtless guidance in using ICT already takes place, there could be 'comfort zones' and 'risk zones' to be studied in this context (Penteado 2/99). Awareness of expectations in terms of pupil achievement is another issue not addressed in the context of teacher education.

TEACHERS

The concerns of studies in this chapter focus more on the individual teacher's actions. These studies form a relatively small proportion of all studies (approximately 5%). These are in contrast to studies adopting the more holistic perspective of Chapter 2 where concern is with an overview of the interaction of teacher and pupils in the classroom or those in Chapter 3 which are devoted to BSRLM research clearly also involving teachers but which focus on teaching particular mathematical topics. The five groupings below indicate that there is a fairly wide spread of interests represented in this small category of work involving teachers that fall

outside these two main topics. These include work related to (a) novice teachers, (b) pedagogy, (c) the role of teachers as assessors, (d) the use of ICT and, (e) teachers as researchers.

Novice teachers

The term 'novice teacher' is taken to refer to one who has completed a teacher education and in the early years of teaching. Georgiadou-Kabouridis (11/00, 11/01) reports a three-way exploration of the interaction of headteacher, researcher and a novice primary teacher where a teacher adopts a new approach to his teaching, dropping the traditional transmission approach, and follows this up with later work dealing with support for novice teachers.

Studies with a pedagogical focus

Various approaches to the teacher's role within a particular pedagogy in the mathematics classroom have been the focus of several investigations. Amongst these is a study of the role of the Socratic method as a strategy in teaching A-level mathematics (Rowlands et al. 6/98)). In a project undertaken by Jaworski et al. (2/99), teachers are led to reflect on their own actions in the classroom and by gathering relevant evidence, the aim is ultimately to develop a theory to support mathematical pedagogy. Back (RME 2000) undertook the analysis of transcripts of primary school mathematics lessons in order to gain evidence about the conjectures made about teachers' approaches to their teaching while in a study by Bills (RME 2000), the concern is with exploring the influence of teachers' presentations on pupils' mental representations.

In a study carried out by Anderson and Boylan (5/00) they explore primary pupils' reactions to having to answer teachers' questions in class individually, in particular during the numeracy hour, when apparently such questions can lead to pupil anxiety. Another study on a similar theme at secondary level reports pupils' responses to the different ways in which teachers ask questions (Boylan and Lawson 11/00). Houssart (5/00) studies another aspect of pupils' responses in which children from a lower attaining group are found to respond positively in class discussion but not to perform as well on tasks that have to be completed.

The understanding of the theory underlying the Cognitive Acceleration in Mathematics Education (CAME) by teachers participating in the project is explored by Goulding (2/02). With a combination of Piagetian and Vygotskyan theory underpinning the approach taken in the project, teachers were asked to give some explanation for the gains made by their pupils and appeared to attach weight to different elements of underlying theory. It was judged that more weight is attached to Vygotskyan as opposed to Piagetian elements of it.

Teachers as assessors

Morgan (5/96) suggests that in considering the assessment of coursework, it is important to explore the 'ways in which creativity and divergence may be valued'. In comparing the assessment of the same piece of coursework by eleven teachers, she identified seven teacher-assessor positions and found that different reading strategies could lead to different interpretations of text and ultimately, to different evaluations on the part of assessors. In another aspect of teacher assessment, Hadjidemetriou and Williams (3/01) examine teachers' knowledge of

pupils' errors and misconceptions in graphical reasoning. Teachers were invited to record the order of pupils' difficulties on a scale and to suggest misconceptions pupils may have had, and were found to have misjudged pupils' difficulties.

Use of ICT

A considerable body of work referred to elsewhere in this review incorporates the use of computers as part of the teaching/learning situation. However, in some instances the computer and/or software in themselves are the actual focus of the research. This is the case in studies by Crisan (6/99) and Penteado (2/99) where examinations are carried out of the interaction between ICT and the teacher's professional knowledge in terms both of mathematics and technology. Penteado (2/99) identifies a comfort zone and a risk zone in teachers' work with computers. Other investigations of the use of ICT include a study by Gates (6/99) using NUDIST and Gardiner's (11/97) exploration of mediation between the pupils and the screen by the teacher in a dynamic geometry learning situation. Chae and Tall (5/00) used computers and oscillators in a study that found evidence to support the hypopthesis that graphic representations have an important role in conceptualising the notion of period doubling in chaos theory. In other words, it would appear that graphic representations produced by computers and oscillators are not only visual but conceptual as well.

Teachers as researchers

The increase in the amount of teacher involvement in classroom research has resulted in a related interest in the role of teachers as researchers. In

some cases, a particular mathematical topic is the focus of teacher-related research as is the case with Coles and Brown (11/98, 2/99, RME 2001) and Brown (11/00) who are studying algebra classrooms with the aim of creating a classroom culture that gives the topic a purpose and enables it to become more meaningful to pupils (see Chapter 2). Jaworski (5/00) discusses the kinds of enquiry teachers might engage in and its contribution to the development of teaching. This is essentially an exploration of the interface between research and teaching and she concludes that with their participation in the process, teachers make a contribution to the vision of the research community.

Mohammad (3/01) explores the learning partnership established between teachers and teacher educators/researchers in classroom situations and concludes that responsibilities evolve as the relationship between the two evolves and the needs of the teacher in particular emerge.

Comment

Primary/secondary differences

A comparison of topics in the student teacher/teacher categories suggests that researchers are drawn to a wider range of interests in the secondary sector than is the case with the primary sector. In the secondary sector, there is at least one study dealing with each of the key issues identified in the CoPrIME list, which is not the case in the area of primary research. The majority of secondary studies deal with novice-to-expert issues or interactive teaching skills while primary studies show concern with teachers' mathematical knowledge. As noted at the beginning of this

chapter, the emphasis at secondary level is on practising teachers while at primary level interest is more with ITE issues

The studies involving students in the primary sector that relate to their mathematical knowledge are a reflection of the importance attached to this question and the measures put in place in recent years by government to assess the level of mathematical knowledge of student teachers in this sector. Although wider issues in student teacher research are explored in other studies (e.g. the role of socio-cultural factors dealt with by Baker (11/97) and teachers' and pupils' beliefs about mathematics by Brown et al. (11/98)), there is a gap with respect to the perceptions and expectations of teachers and student teachers in connection with their pupils' mathematical learning. A considerable body of evidence exists with respect to the effects of teacher expectations of pupils' performance ranging from the seminal work of Rosenthal and Jacobson (1968) through to reviews of such work by Brophy (1985). However, in more recent years this has evolved into an increased interest in teachers' beliefs, particularly with respect to mathematics, how it is taught and their role as teacher which clearly, in turn, will affect their expectations (e.g. Thompson 1984, Groves and Doig 1998, Askew et al. 1997). Smith's (6/99) study quoted above is an example of such work.

Work by Douady (1997), building on Brousseau's (1988) notion of the didactical contract, highlights the importance of considerations in this field. Using the notion of the didactical contract as a theoretical framework, she describes the different roles mathematics can play in the teaching-learning situation in the following way (paraphrased by Nickson 2000):

1. knowledge is what is at stake for the teacher but not necessarily the pupil;

(concern is with pupil expectations about school and the proportion of them for whom knowledge is not the focus)

2. knowledge is not at stake for the teacher or the pupil;

(a situation where mathematical content is presented in a mechanical algorithmic way and neither teacher nor pupil have any ownership of it but survival for both parties is assured);

3. knowledge is at stake for some pupils but not the teacher;

(the teacher's perspective is that of (2) above but some pupils are genuinely interested and want to learn about mathematics in a meaningful way);

4. knowledge is at stake for both teacher and pupils

(the ideal situation but it does not guarantee effective learning but demands informed decision-making on the part of the teacher).

Nickson (2000) pp 153-4

Expectations will depend on which of these roles is in play, and beliefs become involved at the level of the perceptions of the players in the classroom context. The potential effects of a mismatch between teachers and pupils are evident. By identifying the ways in which this can happen, a theoretical framework of this nature helps to refine and inform research about aspects of teacher-pupil expectations (as is evidenced in the reference to Brousseau (1986) and the notion of the didactical contract by Blanc (3/95) above).

GENERAL COMMENT

Research in teacher education and studies that focus on individual teachers is not as easy to undertake as other kinds of enquiry. Although it may appear to be a straightforward option from the point of view of accessibility insofar as it involves mathematics teacher educators in their workplace, it is more difficult to arrange to undertake investigations in schools because of time, money and in some cases, accessibility. For example, the notion of giving teachers time to reflect on what they are doing in their individual classrooms perhaps does not come across as a major priority to some. Below are two aspects of one direction in which research in mathematics education is beginning to evolve.

Communities of enquiry

A substantial document has recently been published by Group 3 of FRAME (Formulating a Research Agenda for Mathematics Education) in which issues related to 'Mathematics Teaching and Teachers' Professional Education and Development' are addressed. This has been based on four areas: Teachers' Knowledge (TMK); Initial Training of Mathematics Teachers (ITT); The Continuing Professional Development of Mathematics Teachers (CPD) and Developing Communities of Inquiry and Critical Intelligence (DCI). With respect to initial training, it is suggested that factors such as personal independence and autonomy have been sacrificed to pressure exerted by government led agendas, which in turn has meant teacher educators with traditionally broader educational expectations for their students, now have to adapt their professional stance to fit against this narrower, more restricted background of their students. It seems likely that this has contributed, in turn, to the relative

narrowness of the research agenda for student teachers in the primary sector as well as the paucity of studies in the field.

The FRAME document notes that worldwide research projects in the development of teaching in mathematics education tend to encourage models of critically reflective practice leading to the development of communities of enquiry together with critical intelligence in them. This type of research is well illustrated by the work of Coles and Brown referred to earlier within the group of studies categorised as involving the teacher as researcher. Coles and Brown's (11/98) initial paper relating to their ongoing study includes a reflection on what it is like for teachers and researchers to work together. In the long term, this project is concerned with the development of a culture of algebraic thought and activity within the classroom; however, the reflections on the initial stage on the role of teacher as researcher indicate the fruitfulness of such a partnership and the results of critical reflection:

> The sense of 'becoming a mathematician' and the stress I have put on writing is not part of the school culture – however there is evidence of students across the range of achievements beginning to work in a 'global meta-level' way. ……Mechanisms that have supported this seem to be my own 'meta-commentary' on the events of each lesson, the emphasis on students' written and verbal communication and the self-generative, self-checking nature of the activities in lessons.
>
> Coles and Brown 11/98 p22

As well as positive outcomes in terms of classroom learning, the study in its entirety is a very good example of the benefits of collaboration over

time between a teacher and a colleague for whom research is part of his or her professional life. The fact that the BSRLM community as a whole gains from it is an added bonus to the profession as a whole, both in terms of development of individuals concerned and in terms of the building up of a working understanding and partnership between these two areas of mathematics education. (More studies of this kind are touched upon in Chapter 2.)

Critical reflection

Skovsmose (1994) deals with the whole notion of critical theory in relation to mathematics education in detail in the context of critical education as a whole. He equates the term *mathemacy* with literacy and postulates that:

> *If mathematics has a role to play in critical education, similar to but not identical with the role of literacy, then mathematics must be seen as being composed of different competencies, a mathematical, a technological and a reflective competence. But especially: Reflective knowing has to be developed to provide mathemacy with an element of empowerment.*
>
> Skovsmose (1994) p117 (author's italics)

While the meaning of the word 'critical' in conjunction with various aspects of mathematics education may be interpreted differently by various users, the notion of the importance of a reflective element attached to its meaning seems to be generally acceptable. Skovsmose refers to 'empowerment' as a result of reflective knowing and the FRAME document takes a similar direction when referring to the work of

Wells (1999). In his case, Wells places importance on the collaboration of teachers and researchers and on critical reflective practice so that what transpires in an essentially 'social' situation between the participants is continuously reconceptualised and developed. The product of critical reflection or the reconceptualisation that takes place would, in this case, be for the benefit of learners of mathematics. In both the case of Skovsmose and Wells, the emphasis within the process is social. For Skovsmose (1994) this is between the individual (pupil) and society as a whole, while for Wells it is between the individual (pupil) within a particular community (teachers).

The whole notion of critical theory and the various types of critical action that arise from it, have overtones that may not be considered particularly relevant by some mathematics educators or by those whose job it is to oversee mathematics education at a national level, i.e. various government agencies. However, this is a direction in which research is taking us and it would seem to be an especially fruitful one in the way in which it can bring together mathematics researchers and practising teachers into a partnership. It will be a challenge to find ways to continue to pursue these partnerships that have already begun on several fronts.

Research priorities

The lists of research priorities identified by CoPrIME and FRAME appear on the surface to be somewhat different. This is not surprising since the former list deals with priorities in research in mathematics education as a whole while the latter is concerned with priorities related to teachers and teacher education. The former identifies seven areas and the latter only four. The one area they appear to have in common is

teachers' mathematical knowledge. However, closer scrutiny suggests that more than one area from the CoPrIME list is included within a single category in that of FRAME. For example, aspects of interactive teaching from CoPrIME would doubtless be included in Continuing Professional Development (CPD) of FRAME, as would moving from novice to professional and possibly post-14 skills and ICT. The teacher as researcher would fall within Developing Communities of Critical Inquiry (DCI). The one area that does appear to stand out as not being common to both is the notion of teacher expectations seen in terms of pupil achievement as identified by CoPrIME although arguably this hopefully would also form part of CPD concerns.

In the next chapter, we shall consider BSRLM studies that explore theoretical issues in relation to mathematics education.

Chapter 5

CONCERNING THEORY

INTRODUCTION

Research that focuses specifically on theoretical matters forms the largest category of research in the BSRLM corpus, with approximately one third of all studies falling into this group. There may many reasons why studies of a more theoretical nature are so frequently undertaken; for example, while they may be costly in terms of time and thought, they are usually not so costly in financial terms. However, the nature of theory is in itself one of the reasons it is such a strong focal point for research. Cohen and Manion (1980) note a perspective of theory identified as essentially scientific, in which the production of theory is seen as the ultimate aim. They refer to Mouly (1978) who suggests that theory helps us to make sense out of a particular phenomenon but they go on:

> More than this, however, theory is itself a potential source of further information and discoveries. It is in this way a source of new hypotheses and hitherto unasked questions; it identifies critical ideas for further investigation; it discloses gaps in our knowledge; and enables a researcher to postulate the existence of previously unknown phenomena.
>
> Cohen and Manion p17

The purpose of considering theoretical studies here is to exemplify some of these characteristics, two of which will be considered. Firstly, developments in existing theory within a discipline or disciplines may raise new ideas that can fill an existing gap in the explanation or understanding of a particular phenomenon. Alternatively, empirical data may suggest a framework that adds to the clarity of our understanding about the phenomenon. In both such cases, theorising takes place. Secondly, theories about a phenomenon may be applied in situations in which it has not been considered before. This can raise issues about the relevance of that phenomenon within different contexts and 'test' its validity by opening it up to further investigation.

Cohen and Manion (1980) also note that the status of theory varies according to the area of knowledge in question and that some theories

> like educational theory, are only at the early stages of formulation and are thus characterised by great unevenness. (ibid.)

Research in mathematics education is at an even earlier stage than research in education generally and in the relatively short period of its existence, it has grown at a considerable rate. In spite of this (or perhaps because of it), theories proliferate. This rapid development emphasises the need for a very important aspect of theories and theorising and that is, that they must be scrutinised and challenged, and their merits judged against existing theory.

With the above thoughts in mind, the BSRLM studies considered in this chapter will be grouped in three categories: (a) those that focus on the application of a particular theory; (b) those concerned with developing a

theory or that engage in theorising about a particular issue, (c) those that offer a critique of a particular theory. Only one or two studies will be described in each category since the intention here is to illustrate the range and nature of theoretical concerns of BSRLM researchers.

APPLICATION OF A PARTICULAR THEORY

Any research involves the application of theory in the sense that it builds on previous theories or ideas, and generally speaking, more than one theory is invoked. Sometimes, however, while referring to several theorists, studies may focus more strongly on one than any other or indeed, they may set out to probe one particular theory and rely little on other theories to help in the investigation. The studies included in this section explore the use of a specific theoretical idea or perspective as a focus of the work. In some cases, these studies take a specific theory and, as it were, 'test' it in the teaching-learning situation. In others, a theory takes precedence over others in informing the teaching-learning situation and a main aim is to explore the extent to which it illuminates a mathematics teaching-learning situation. The application of a particular theory most often takes place within a classroom context and clearly these studies could equally well have been included with studies of that kind (see Chapter 2). However, those quoted below have been selected because they give an indication of the nature of researchers' theoretical interests in the teaching and learning context.

The concept of 'procept'

Hunter (10/95) explores the idea of procept in the understanding of algebraic expressions with Year 9 secondary pupils by means of using test items to explore strategies used by them in solving algebraic equations. 'Procept' is the concept at the centre of a theory developed by Gray et al. (1997) relating to the development of pupils' algebraic thinking and the notion of the 'proceptual divide'. At the centre of the theory is the role of symbolisation where symbols can represent a process or a concept. The example of the expression *2x+4* is used to illustrate the notions of procedure, concept and procept. If *2x+4* is thought of in terms of procedure, the expression has meaning only when *x* is given a value. When *2x+4* is viewed as a concept, it has meaning as a whole and a student is able to visualise it as a graph or, in other words, to understand the potential for manipulating the expression algebraically. A student with proceptual understanding is able to see the expression both in terms of a procedure and as a concept and is able to switch between the two in a way that clearly demands some flexibility.

In the study, test items were devised in order to discover, amongst other things, the way in which students viewed the use of letters, as well as how they viewed expressions as a whole, i.e. to evaluate whether they perceived items proceptually. Open and closed versions of items were used to do this and students were asked to explain their answers to the questions. Although these test items were described as something of a blunt instrument for the purpose, it was possible to draw some conclusions about the degree to which there were indications of proceptual understanding on the part of some of the pupils, of some of the questions. The concept of the proceptual divide is also used by Pitta

(5/96) in the interpretation of differences in communicating imagery between low and high achievers in mathematics.

Vygotskyan theory

Rowlands, Graham and Berry (2/98) used what they describe as the 'experimental-developmental' method of Vygotsky when studying the effects of the introduction by the teacher of concept questions into an A-level mechanics teaching situation. This involved the planned intervention of the researcher-teacher into the zone of proximal development (ZPD) of students with the purpose of observing the effects resulting from the intervention. The intuitive beliefs of students were challenged by the teacher-researcher and the ensuing dialogues between teacher and student were analysed. The purpose of studying the student responses was to determine the effects of the different kinds of question on their understanding. The report takes us through the development of two particular students' thinking as a discussion progresses with the teacher, and cognitive conflict signified by students' reactions is noted (e.g. perplexed expressions, silence, fidgeting). One-to-one informal interviews were held with twelve students who had been instructed in this way. Data gained from them added considerably to the detail of what was going on in the minds of students and helped to identify two possible laws that guided their reasoning in this context.

Gardiner, Hudson and Povey (RME 2000) also apply Vygotskyan theory and the notion of the zone of proximal development (ZPD) to explore a particular mathematical teaching-learning situation. In this case, the theory is used to illuminate the total teaching situation and the roles of all the participants are explored. The researchers study a relatively new ICT

teaching and learning environment using a dynamic geometry program, with the purpose of examining the contribution of the technology and the teacher as mediating factors within the ZPD of the learners. They suggest that from the three classroom episodes, when considered from the theoretical perspectives such as the dialectic of proof and the interplay of intuitive and scientific ideas, a framework emerges for a classroom approach using the technology. A main factor of this is the importance of the mediating role of the teacher in an ICT learning situation as well as that of the pupils and the technology itself. Day (3/01) also uses Vygotskyan theory in considering mediational activity in a classroom teaching context.

Constructivism

Byatt (3/97) reports the beginning of a study which draws on constructivist theory, the aim of which was to develop an approach to teaching mathematics based on conjecture and discourse. The discussion reported centres on factors that affect the creation and maintenance of a conjecturing atmosphere in classrooms of 11-16 year olds. He describes how he uses the constructivist notion of 'coming to know' through a process of discussion with pupils that involves explanation and conviction. The important factors to be considered in 'coming to know' were seen to be the articulation of 'why' questions and 'because' statements. These, in turn, lead to a discussion in which subject matter is negotiated and refined. From the evidence gathered, Byatt identifies one important aspect of the classroom environment as situations where teachers do not take advantage of the possibilities offered for extending the contributions of pupils in class. Other key issues to emerge were gender, peer pressure and discipline.

Other studies

Other studies exploring particular theories in practical teaching and learning situations include those of Brown (11/96) and Hitt (11/98) who explore the relationship between psycholinguistics and the learning of mathematics. Hitt argues the case for the role of semiotic representations in understanding and promoting mathematical learning at high school level. In a different area of interest, Pendlington (6/98) uses Kosslyn's theory of image generation with children at primary level to investigate their use of imagery in arithmetic and the extent to which external images match the internal processes of individual learners.

Comment

The examples given here clearly do not include all studies within the BSRLM body of work that refer to the idea of procept, Vygotskyan theory, semiotics, constructivism or indeed, any of the topics touched on above. They have been chosen as examples of how particular theories attract researchers' attention and helped them to answer questions of a quite specific nature. With respect to the work involving the notion of procept, the researcher has the opportunity to probe the validity of a concept and try to discover the extent to which it is helpful in our understanding of what happens within the pre-algebra teaching and learning situation. In the case of Vygotskyan theory, the studies show a concentration on two different aspects of the theory, the role of mediational tools and the notion of the pupils' ZPD, and show that aspects of this theory help to illuminate what goes on in a mathematics classroom. Semiotic theory is used as a background against which to view

a particular teaching and learning situation. The role of theory, in these studies, is more one of adopting a particular theoretical framework to help make sense of a situation and less a situation where new theories are likely to be generated as a result of its application (although both may happen).

DEVELOPING IDEAS AND THEORISING

The main focus for many BSRLM researchers is to build up their own theories in the exploration of questions raised within the context of mathematics education. Theorising of this sort takes place at a variety of levels. This section mainly concerns two studies that are chosen to exemplify contrasts in the way in which theories emerge. The first is an instance of a large empirical study involving a team of researchers and the second is an example of individual theorising. In a sense, each represents the ends of a spectrum of particular kinds of research activity although in each case, theory is the result.

A large scale study

Askew et al. (RME 2000) report the results of a research project that is an example of a large scale study although the work discussed is only one part of the whole. The researchers are members of a team involved in a longitudinal study of the teaching and learning of numeracy (the Leverhulme Numeracy Research Programme). The aim of this part of the project is (a) to gain more understanding of how particular numeracy teaching practices impact on pupils and (b) what the implications are for the teacher attempting to undertake whole class numeracy teaching while

at the same time responding to individual pupils. The project as a whole aims to identify factors that lead to low attainment in numeracy and assess ways of raising attainment. Data are being collected over a five year period, from two cohorts each of Reception and Year 4. Part of this process involves the development of a theoretical framework to satisfy two particular aims: firstly, to understand 'critical points' in progression in primary mathematics and secondly, to account for how classroom practices influence standards of attainment. In doing so, the researchers draw on the work of Sfard (1998), Lave and Wenger (1991) and Rogoff (1990), amongst others.

The researchers have formulated two models from an analysis of data gathered from 'numeracy events' within classrooms, based on the assumption that classroom events are goal-directed. One of these provides an interpretative structure for numeracy events and the second, a similar structure for numeracy orientations (applicable either to teacher or pupils). At the outset of the project, the focus was upon the teacher when studying classroom events but as the project progressed and data was analysed, the need for a shift from the teacher's perspective to that of the pupils was indicated. As the researchers put it, while the daily mathematics lesson is now commonly viewed at policy level in the UK as an 'objective event' and may appear to be offering 'the same experience to all pupils', this has been found not to be the case. (Askew et al. RME 2000 p74).

This project indicates the potential this kind of study has for the generation of new hypotheses and to identify critical areas that may previously have gone unnoticed, as suggested by Cohen and Manion (1980). The work reported above shows how ideas evolve and are

adapted as data are gathered. In this case, the assumptions made at the outset appeared not to be in tune with the classroom situation and the emphasis was switched from one set of participants to another. What is emerging from the process is the development of a theoretical framework to be used as an analytical tool in the promotion of effective teaching and learning of numeracy.

Another example of work on this scale where new theories emerge within a long term project is the work of Adhami et al. (5/95), Johnson et al. (11/97), Adhami (5/00) and Shayer et al. (6/99). They report on a project in which they build on the work of Vygotsky, Piaget and others to develop a theory of instruction which aims to give a theoretical interpretation of (a) the nature and effect of teacher intervention in lessons and (b) pupil-pupil interaction within class.

Individual theorising

An example of theorising at an individual level is provided by the work of Lins (2/96). He is concerned with the production of meaning and his theory involves three elements, author, text and reader, and in the development of his ideas, he draws upon the work of Derrida, Vygotsky and Walkerdine. He suggests that as a result of someone saying something, the recipient (whether hearing or reading what is said) believes that this is essentially a demand that meaning should be given that enunciation or text. Lins points out that every author needs a reader and every reader needs an author, and the outcome of this fusion is communication. He suggests that failure in communication is a sort of 'accident'. He states that:

> " 'Knowledge' becomes a commodity, which can be stored and passed over to someone else. Institutionalised teaching becomes part of productive systems. Whenever reproduction is required, faithful communication is a good thing. Such teaching constitutes communication." (ibid. p4)

He goes on to suggest that the processes of meaning production, i.e. being an author or being a reader, are very close and he introduces the notion of an *interlocutor* to refer to both (jointly) as a sufficient mechanism for the convergence of meanings between the two. His proposal is that we rethink teaching to take account of the author/reader and view it as a process through which interlocutors ultimately become internalised. (ibid. p6) But he notes that:

> "If pupils are to internalise interlocutors, and if teachers want to become interlocutor to pupils, teachers should at least try to know *why* pupils are saying what they are." (ibid. p6 author's italics)

Thus justification has to become central to teaching not only in the role of proof, but as part of eliciting the meanings being produced. In other words, it becomes important for pupils to be able to articulate why they believe what they believe and for teachers to guide them to a situation where they have to do this overtly. There are clearly important implications for teaching and learning implicit in such a view of the production of meaning. However, this is a small glimpse of theory building which began at an earlier stage (see Lins 1994) and doubtless will evolve further over time.

A range of individual or small scale studies

Further examples of theorising by researchers include Duffin and Simpson (5/95) who describe their development of a theory of understanding and also explore the idea of methodology as a 'way of working' (Duffin and Simpson 11/96). Hardy (3/97) brings sociological considerations to bear on the mathematics classroom in her consideration of the power relations between mathematics teachers and the discipline of the mathematics curriculum. Cotton (11/96) explores the notion of social justice in the mathematics classroom while Woodrow (2/96) considers democratic education within the context of mathematics education. Lerman (11/96) explores problems associated with carrying out socio-cultural research and examines the theoretical underpinnings of work of this nature. Mason (11/01) considers mathematics as a constructive enterprise and, in particular, what happens when learners are given different degrees of freedom and constraint in tasks they are asked to carry out. Topics of a more philosophical nature include Sam and Ernest's (3/97) consideration of values within mathematics curriculum. The affective aspects of behaviour and their role in the learning of mathematics are explored by Daskologianni and Simpson (11/99) who examine sixth formers' attitudes to mathematics by building on research into behaviour such as beliefs in mathematics education.

Theorising also occurs in relation to methodological matters. Watson (2/98) extends earlier work in which she relates theoretical considerations to the use of anecdote in research. O'Reilly (2/96) focuses on how meaning can be constructed from a number line in the dynamic medium of Boxer and compares such constructions within and outside the medium. Stevenson (7/01) studies the use of new technologies in teaching

Newtonian mechanics and the effects of the direct manipulation of teaching and learning environments. Finally, Prestage and Perks (RME 2001) propose a model for subject knowledge in the context of teaching others how to teach mathematics.

Comment

Again, the studies cited here give a flavour of the range of interests and levels at which research within the BSRLM community is carried out. Theorising is at the centre of all the studies referred to above, and as is the case with BSRLM research as a whole, some represent an initial foray into considering a particular phenomenon or situation from a theoretical perspective while others are continuing to evolve over time. The range of examples noted here of how theories are generated includes examples of answering new questions arising from previous research, identifying new concepts and hypothesising, all of which were identified at the beginning of this chapter.

There clearly is a wealth of interest in matters theoretical within BSRLM over a wide area of subjects. It is of interest that much of the individual theorising is of a sociological or psychological nature with not a strong representation of studies dealing with philosophical aspects of mathematics education. However, this probably reflects the nature of the current predominant theories within mathematics education such as the work of Vygotsky and constructivist theory in all its manifestations.

CRITIQUING THEORY

The unevenness of educational theory in general has already been noted and it was suggested at the beginning of this chapter, that theory related to mathematics education may be more uneven than is the case in other areas of education because of its relatively short existence. This adds to the strength of the necessity for a critical perspective when considering theory in the mathematics education context. There are those who would agree that theories within the discipline are sometimes adopted with less consideration than they are due and lead to less than favourable results (Nickson 2000).

The studies in this section are examples of critiques by individuals of various aspects of mathematics education theory who consider their topics in the light of experience and current developments. The section begins with a critique of constructivism as an example of the kinds of argument put forward in other studies. Other studies are then noted.

Constructivism

Debate about current theories related to mathematics education is exemplified by Rowlands' (6/99) consideration of constructivism. In this he raises philosophical issues in relation to a constructivist interpretation of how learning takes place, issues that he considers have been ignored by proponents of the theory. In doing so he refers to the work of von Glasersfeld (1995) and Staver (1998) and the focus of his criticism is the attack on realism and hence on the correspondence theory of truth, which he views as implicit in constructivism. Rowlands (6/99) asserts that the correspondence theory is untenable

> when it comes to the way science explains the physical world.
>
> (Rowlands 6/99 p76)

However, unlike von Glasersfeld, he does not accept that this means that the external world is 'unknowable'. Rather, scientific theories are the way in which the external world becomes 'knowable' through experimentation and as he says:

> *If the physical world were other than it is, then the concepts of science would be different to what they are."(ibid* author's italics)

Rowlands sees the apparently widespread adoption of constructivism by mathematics educationalists as having considerable potential for adverse effects on the future of research in mathematics education. In particular, he views it as downplaying

> the very subject matter (science and mathematics) that educationalists are meant to have a keen interest in". (p77)

This is clearly undesirable, politically.

Other studies offer further critical perspectives on constructivism as, for example, Rowlands (11/96) when he examines the way in which many constructivists invoke the work of Vygotsky (referred to earlier in this chapter).

Other disciplines and constructs

Other researchers are concerned with current thought in relation to mathematics education and analyse various aspects of it in some depth. Huckstep and Rowland (11/99), for example, examine the construct of creativity in relation to mathematics and mathematical activity while Rogers (11/97, RME 2000) explores the parallels between the historical development of mathematics and the development of concepts by individuals. The role of semiotics in mathematics education is considered at some length (Vile 2/96, 6/97, Vile and Polovina 6/98) and Rowland (5/95) examines mathematical generalisation in the context of inductive reasoning. Rowlands also considers relativism in mathematics education (11/98), and develops further his critique of the socioculturalists' interpretation of Vygotsky's zone of proximal development in relation to mathematics education (2/99).

Rodd and Barber (6/98) examine the relevance of research in contemporary philosophy of mathematics to research in mathematics education. Methodologies are considered by Reyes (11/99) who discusses the theoretical underpinnings of a phenomenographic approach in mathematics education while Harries et al. (2/99) report a comparative study of primary textbooks and the important integral part they play in much of methodology. They focus, in particular, on the way in which images are used in the UK and compare these with similar textbooks in four other countries.

Comment

These studies indicate an appropriately critical stance with respect to their subjects and are concerned to clarify and elucidate their place within mathematics education. The consideration by Rowlands (6/99) of the philosophical issues related to constructivism is an example of the level of debate that takes place within the BSRLM forum and this kind of challenge to current theory is an indication of the healthy exchange of views in which members are able to engage.

GENERAL COMMENT

Theoretical research has been approached from three different perspectives here. Firstly, we have looked at the application of specific theories to evaluate them in, as it were, 'fresh' research contexts where the outcomes have been to offer a new insight into an existing theory, exploring the extent to which it may need adjustment in different contexts and thus produce a 'new' version of the existing theory. Secondly, we have identified studies that offer new theories with respect to aspects of the teaching and learning of mathematics. It is clear from the work cited here that there is no lack of theorising at this level within the BSRLM community. Research of this nature will always challenge existing ideas and raise new questions. These will, in turn, result in further research that will explore the validity and applicability of these new ideas and so the cycle of competing theories resulting in new theories is continued (Popper 1972). Finally, we have considered the critiquing of particular theories that have become established to greater or lesser degrees within the community of research in mathematics education. Existing theory has

been vigorously scrutinised, in some cases with respect to content and its validity within the mathematics teaching and learning situation, and in others with respect to the way in which theory has been interpreted and applied. In both cases, the result is of a theoretical nature in which ideas may have been discarded or added but a theory still remains.

In a discussion of different approaches to research in mathematics education, Davis (1992) writes:

> ….very little research in mathematics education has focused on the actual ideas in students' minds or on how well teachers are able to identify these ideas, interact with them, and help students improve on them.
>
> Davis 1992 p732

Since this was written in 1992, however, and with the more general adoption of interdisciplinary approaches to research, Davis' statement does not reflect the situation as accurately as it might have then. In Chapter 2 we noted that by far the greater proportion of BSRLM classroom studies are at what was characterised as Level 2 or above, and entailed investigating problems that dealt, more than any other area, with classroom interaction and pupil-pupil and teacher-pupil contact. Davis (1992) suggests that by adopting research methodologies that are moving in this direction, we:

> seek to create a *culture* of mathematics rather than allow it to remain merely a subject that is "taught".
>
> Davis 1992 p731

The nature of the theorising we have touched upon in this chapter indicates that there exists a wealth of ideas within BSRLM related to mathematics education research that view mathematics education in the light of such a culture.

In the next and concluding chapter, we shall consider the role of theory further when we consider the BSRLM archive as a whole.

Chapter 6

AN EFFECTIVE RESEARCH FORUM

INTRODUCTION

The purpose of this chapter is to draw together strands from the overview of research studies presented in the previous chapters. It is not part of the brief for this review to make recommendations. However, in the process of identifying existing strengths or gaps within the body of research, it is inevitable that the implications of factors within a given situation will become evident and such observations will noted throughout the chapter as they arise.

There are obvious aims of a body such as BSRLM, the main of these being to engage in research and to disseminate results. While it can be time consuming to consider what is meant by 'research', it is important to consider the activity against a background of shared meaning and to this end, Bishop's (1992) identification of three components to "qualify as research in mathematics education" are helpful:

- Enquiry: which concerns the reason for the research activity. It represents the systematic quest for knowledge, the search for understanding, and gives the dynamism to the activity. Research must be *intentional* activity.

- Evidence, which is necessary in order to keep the research related to the reality of the mathematical education situation under study, be it classrooms, syllabuses, textbooks, or historical documents. Evidence samples the reality on which the theorizing is focused.
- Theory, which recognizes the existence of values, assumptions, and generalized relationships. It is the way in which we represent the knowledge and understanding that comes from any particular research study. Theory is the essential product of research activity, and theorizing is, therefore, its essential goal.

p711

There are diverse factors that emerge as strengths within BSRLM that to some extent overlap and thus make it difficult to do justice to the whole. However, what follows are those that have stood out strongly and deserve particular note. These are:

- The accessibility and inclusiveness of BSRLM as a research forum;
- The involvement of teachers in the research process;
- The work related to the use of ICT in the teaching and learning of mathematics;
- The extent to which theory and theorising play a part in the proceedings and output of the society.

Each of these areas is considered below and critical factors noted.

ACCESSIBILITY AND INCLUSIVENESS OF BSRLM

Positive aspects of accessibility

BSRLM as a professional organisation dedicated to mathematics education at a national level is admirable in its inclusiveness and accessibility. It provides a platform for practitioners for testing their thoughts, airing their professional problems, sharing knowledge and keeping up-to-date with current research issues. As well, it occasionally provides an international perspective with frequent visiting researchers or graduate students from abroad providing contributions that can only broaden the perspective of all concerned. The accessibility of the society is characterised by the content of studies that may be seen as lying on a spectrum from those of (i) initial thoughts to (ii) those of considerable depth.

(i) The 'initial thoughts' end of the spectrum is highly important to the society as a whole. This is when an issue may be addressed once only by an individual who essentially is sharing a problem with professional colleagues or who may be reflecting on actions either taken or observed in a classroom. They may never have addressed a professional group before or committed their thoughts to paper, but the fact that they feel comfortable in doing so within the BSRLM forum can only be seen as a positive feature of the society.

(ii) Mathematics educators and researchers at the other end of the spectrum are either those for whom research is part of their professional brief or those who have made research a major part

> of their professional development. What is of particular benefit here is the opportunity afforded these researchers to present results of ongoing research, test their theories with their colleagues and engage in debate over a period of time. It also provides those with an interest in a specific mathematical topic (e.g. geometry, pupils' mathematical language, etc.) to develop this interest and explore it over time.

This inclusiveness involves members of the profession from primary teachers through to mathematics educators in higher education.

A tension within the spectrum

While this breadth of interests and the accessibility it provides is a strong point of the Society, there is some need for caution. The spectrum identified above can also be seen as representing a tension within BSRLM between breadth and depth of interests of the research body as a whole. The three factors identified earlier (Bishop 1992) as constituting research in mathematics education are enquiry, evidence and theory. A proportion of the studies included in BSRLM reports could be categorised as "the search for understanding" within enquiry and while obviously intentional, have not perhaps achieved a degree of a systematic approach that constitutes research in the usual acceptance of the term. The tension arises from the fact that this level of enquiry must co-exist with deeper, systematic, long-term pursuit of a question. However, no matter how strained the tension may become, it is imperative that it be sustained because it is at this level that novices are likely to enter the fray. The reports of proceedings of the society include many studies of this sort, which should be seen as a major strength of the overall output.

A commendable move to accommodate the more rigorous demands of the wider research community has been made by the launching in 1999 of an annual proceedings. The publication, Research in Mathematics Education, contains peer-reviewed papers and is stringent in the demands it makes upon its contributors. The early volumes of the annual proceedings provide a valuable record of a sample of the developments of the work of researchers that appeared in earlier reports of informal proceedings of the society and contain some of the main thrusts of the interests of BSRLM membership. In some respects, this publication represents a 'coming of age' for BSRLM and can be seen as a culmination of the dedication and many years of work that this community of mathematics educators has contributed to mathematics education.

The need for a primary focus

The output of research reports suggests that a large majority of the membership is from the secondary sector which is not a surprising imbalance, given the specialist nature of teaching mathematics at secondary level. However, it suggests a situation where the potential is being missed for a strong interface between primary and secondary mathematics educators, whether practising teachers or researchers at other levels. The gap between mathematics education at the primary and secondary levels in particular, has historically been a difficult one to bridge. However, BSRLM would appear to be an ideal situation for drawing professionals with interests in the primary sector into the research process and for sharing those interests and pursuing problems with their secondary colleagues. It is clear that expanding membership in

this way is not an easy task and strategies for doing so are difficult to identify. One example of how this might happen would be to have an annual conference given over entirely to mathematics research interests at primary level, to share these interests and work but at the same time, attempt to draw in new members. The success of this kind of strategy clearly would depend heavily on the commitment of the whole of the membership of BSRLM. A first step in acknowledging this importance of a primary focus has been the recent establishment of a primary working group.

THE INVOLVEMENT OF TEACHERS IN THE RESEARCH PROCESS

There are factors of a nature implicit in the kind of forum represented by BSRLM not necessarily delineated by descriptions of the research process such as that given above. For example, there are the effects of engaging in the activity upon the participants and the impact on their professional development. Particularly positive contributions to research in mathematics education of this nature emerge from the work of BSRLM. These are: (a) the role played by BSRLM in the induction of teachers into the research process; (b) contributing to the continued professional development of teachers by providing the opportunity to keep abreast of current theory and new methodologies.

The induction of teachers into the research process

Many of the studies referred to in this review have involved teachers directly in the research process. BSRLM as a forum is in a privileged

position in this respect having, as it does, committed teachers within its membership but also other members who have ongoing links with teachers in carrying out their classroom research. However, it is important not to take this situation for granted since it presents one that is not easily achieved. Tooley and Darby (1998), in their report on educational research to OFSTED, include evidence stating that:

> at the end of the day, the hard facts are that a minority of practitioners who are interested in the outputs of research either cannot see its relevance to their increasingly over-specified modus operandi, or simply have no time to engage with it.
>
> (Tooley and Darby 1998 p39)

Note that the statement refers to practitioners who *are* interested in the results of research, a factor which heightens the importance of the need for a constant awareness of the potential that arises from the contact between teacher and researcher at a variety of levels within BSRLM. The problem of relevance will always be present, even *within* the membership.

Continued professional development

Another important outcome that arises from teacher involvement with the research process is the nature of the contribution that involvement can make to the professional development of the individual teacher. In reviewing research into the effectiveness of mathematics teachers, Brown and Borko (1992) note that one of the limitations of research in the area of the continued professional development of teachers in the past has

been the fact that studies have not been longitudinal. They state that, in considering research into teacher effectiveness:

> There is little evidence to indicate whether the changes reported in these studies are long-lasting or only temporary.
>
> Brown and Borko 1992 p233

Much of the classroom research reported here involves teachers directly with the theoretical rationale underlying the intentions of a research project from its inception. This level of inclusion may be one way to optimise the effects on teachers' actions in the classroom of the total research process, but at the same time, it is important to maintain a degree of realism about the potential effects of their involvement in it. Oja (1980) reports the results of studies related to Action Research on Change in Schools. The purpose of the research was to build on phases of teacher development identified in the work of Loevinger (1980), Kohlberg (1969) and Hunt (1971) and to gain a better understanding of ways to assist teachers to develop professionally. Oja (1980) identified three phases of professional development, the third of which is described as follows:

> Phase (3). Applying newly acquired skills and theory to the teachers' own classroom setting with consistent on-going supervision in small groups and advising in individual conferences.
>
> (Oja, quoted in Grouws, p231)

Teachers who are directly involved in the research process are engaged in the acquisition of new skills and are introduced to theoretical

considerations as suggested above. Whether involved with BSRLM either as members of the society or active participants in the research process, they are exposed to a culture of research that is more likely than not to make a positive contribution to their professional development. This may be only, at the very least, at the level of recognition that there are others in the profession who are deeply interested in how they, as teachers, achieve what they do in the classroom. One of the tasks of BSRLM is to nurture this context and to ensure that it does not disappear.

ICT IN THE TEACHING AND LEARNING OF MATHEMATICS

Writing about the redefinition of mathematical boundaries as a result of the presence of the computer, Noss (2002) states that:

> This is [the computer's] ability to offer alternative means to express mathematical relationships, novel kinds of symbolism, and innovative ways to manipulate mathematical objects: in short, the emergence of new mathematical cultures. The computer points to new ways to say mathematical things, as well as new mathematical things to say. (p45)

He goes on to suggest that in using the computer appropriately, it becomes possible to attain understanding in using it, in other words, to gain knowledge in the process.

> We do not have to make a choice between knowledge and pedagogy. (ibid.)

If we accept this as the case, we are acknowledging a depth in the role of technology in teaching and learning mathematics that has yet to be generally accepted or exploited. Noss' point is that

> We need new pedagogies to teach old knowledge in accessible ways, but we need to consider how technologies can help us build new curriculums as well. The limitations of old technologies have hung round the necks of mathematics classrooms for two thousand years, shaping what it was possible to teach, what it was possible to learn. (ibid.)

A BSRLM example: Geometry

There are several examples of how the possibility of new pedagogies is being explored within the work of BSRLM members, one such being the work of the Geometry Working Group. Some of the work of this group has already been noted, as have the conclusions of a report by Jones and Fujita (11/01) with respect to the use of new pedagogic tools in geometry (see Chapter 3). Geometry is the topic in which ICT figures most prominently within BSRLM research reports and where its use is being explored and studied in ways suggested by Noss above. The use of Cabri-géomètre in the developing of the notion of conjecture and proof, and other work showing the importance of intervention on the part of the teacher within a learning situation when computers are being used, are two such examples. These are instances of the use of technology in ways that support the points made by Noss (2002) about not having to choose between knowledge and pedagogy. Here pupils are using technology to generate and develop mathematical thinking for themselves and new pedagogies are being developed.

Other areas of mathematics

The strong identification of the use of ICT by a particular topic group (the Geometry Working Group) within BSRLM provides a model for what could happen in connection with other areas of the mathematics curriculum whether in terms of content, pedagogy or other matters. This is an existing strength that could be used to inform research in other topic areas. Using algebra as an example, while there is some controversy about the benefits of using technology in the teaching and learning of algebra generally (Nickson 2000), this relates largely to its use with respect to teaching functions where there is a potential for rigidity and it has been suggested that the technology comes between the learner and the mathematical objects that have to be dealt with directly (Pimm 1995). This suggestion in itself presents a challenge and suggests an area open to investigation with respect to ICT and the teaching and learning of algebra. Is it possible to approach the teaching of mathematical objects using ICT in such a way that allows the learner to engage with the mathematics involved in a meaningful way, generating their own mathematical objects rather than be presented with them? This is the kind of situation that Noss (2002) is suggesting can happen. There are other areas of algebra that need to be explored in a similar way, such as promoting understanding of the notion of equivalence and in modelling problem situations, something which is needed at both primary and secondary levels. Algebra is only one of many other areas of mathematics that continue to present a pedagogical challenge where the use of ICT is concerned.

THEORY AND THEORISING IN THE WORK OF BSRLM

The major part played by theoretical considerations within the work of BSRLM has been referred to earlier, with approximately one third of reports focusing on theory in one way or another. Theorising and theory are mentioned in the second and third components of research in mathematics education identified by Bishop (1992) at the beginning of this chapter. In the first instance, he notes the need for evidence to supply a focus for theorising, and then suggests that theory is the "essential product of research activity" (p711); both of these appear prominently in the studies published by BSRLM. There are strengths of the former kind indicated where researchers gather empirical evidence from classroom situations and develop theories from this but there are also a considerable number of studies where researchers are building on the theories of others or critiquing them in such a way as to inject some new perspective which alters existing theory and leads to the beginning of a new theory.

Gathering evidence for theorising

Given the difficulties facing educational research generally both in terms of funding and time commitment, the output of BSRLM classroom-based research is strong. A variety of approaches are used in which the cooperation between researcher and teacher is optimised and the gathering of evidence and subsequent theorising are shared. Added to this, it is arguable that theorising may come more easily to the researcher than to the teacher concerned and this is one of the challenges of the nature of this work. It is the receptiveness of the researcher to ideas that are generated by teachers in the classroom in these situations that opens the potential for the collaboration to be a positive one in this respect.

There is difficulty in knowing at the outset of a project how great the commitment of a participating teacher may be, as noted earlier, and it is possible that both researcher and teacher may give a lot of time and energy to an activity before discovering that neither is really comfortable with what is going on. However, one of the strengths of BSRLM researchers seems to be the degree of success they achieve in gaining the commitment they do from teachers with whom they work. Evidence of this lies in the many ongoing classroom studies that are reported over several years where clearly the collaboration has been successful and the research enterprise would appear to be equally valued by teacher and researcher alike.

Building on theory

Romberg (1992) writes about the scholarly traditions that inform research in mathematics education and notes that "These traditions are based on different ideologies and reflect different intellectual histories"(p60). Referring to the work of scholars in the USA, he goes on to suggest that

> Since these scholars view educational problems through their own cultural lenses, it is often difficult to accommodate such works to American purposes." (ibid.)

This does not appear to be the case where researchers within BSRLM are concerned.

An important factor reflected in research activity of the membership is apparent in the way in which emphases have changed and been adapted with respect to theoretical considerations over the period covered in this

review (1995-2002) and these theories are drawn from an international field and not by any means entirely UK based. For example, studies over the years show the development of a broader, adaptive approach to constructivism as a way of interpreting how pupils learn in the classroom, to an increased concern with acknowledging the ways in which more interpersonal factors affect the learning situation. Frequent reference to activity theory of Vygotsky is one example of how researchers accommodate this aspect of learning in the classroom situation and the work of Foucault, Brousseau, Derrida, Sfard and others has also been invoked. This is one indication of the way in which the work of BSRLM continues to grow and develop, and shows how researchers are concerned to adapt existing interpretations and look for new perspectives to help to interpret the events they are studying in the classroom. The theories invoked are rich by virtue of the fact that they are drawn from a wide international community of mathematics educators that indicates a positive, outward looking attitude towards mathematics education research generally.

Critiquing theory

In writing an historical overview of research in mathematics education from the end of the 1970s onwards, Kilpatrick (1992) states:

> More importantly, practitioners were increasingly becoming key members of the interdisciplinary groups needed to help research link the complexity of practice to theoretical constructs. The techniques and concepts used by anthropologists, sociologists, linguists and philosophers proved helpful in that task. (Kilpatrick 1992 p31)

The work of the BSRLM membership indicates that this interdisciplinary approach has been embraced wholeheartedly and that, very importantly, the new demands of such an approach in research have not been neglected. There is within BSRLM a healthy probing and questioning of new, current and past theoretical positions across a spectrum of disciplines that inform current studies in mathematics education, and an overview of these suggests a healthy degree of ongoing and lively debate amongst the membership where such matters are concerned. A suitably critical stance with respect to constructivism is only one such example. This kind of rigorous approach to scrutinising theory must be a prerequisite in order for any research forum to fulfil its professional obligations. In the present research climate characterised, as it is, by this inter-disciplinarity in interpreting classroom events, and with the direct involvement of the teacher, new demands are made with respect to the professionalism of researchers. It is vital that the new expectations that arise from these new demands are accommodated. At the same time, we must remember that at the heart of the enterprise of *mathematics* education research is the concern with effective teaching and learning of the subject, and the relationship between sociology, linguistics and the other forms of discourse drawn upon must be firmly linked to *mathematics* education.

CONCLUSION

In adopting the outward looking approach it does, it is clear that BSRLM is making its own particular contribution to research in mathematics education at a national and international level. This approach is made all

the stronger by the way in which the Society draws practitioners from all strands of mathematics education into its fold. All contributions are treated as equally important to the considerations of the forum. Although, as noted at the beginning of this review, it does not claim to represent the whole of the UK mathematics education community, the very existence of BSRLM is evidence of the strength of the total community of which it is a part. The clear determination within BSRLM to be involved in the exploration of the teaching and learning of mathematics at all levels of education gives it a position as a strong focal point nationally.

It seems appropriate that the final word should rest with members of BSRLM. Introducing Volume 3 of the Society's annual proceedings, Research in Mathematics Education. Jones and Morgan (RME 2001) write:

> The first thing to observe is that the improvement of the experience of mathematics learners in all contexts is a central concern….The concerns of the authors are also in many cases related to current priorities for teachers, curriculum developers and policy makers – priorities that may be driven by developments in educational policy, by debates about the nature of the mathematics curriculum, by increased availability of new technologies. (Jones and Morgan 2001 p4)

This is an admirable summing up of what the work of BSRLM as a whole is about and which this review has been concerned to celebrate from an appropriately critical perspective.

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March 1995

Blanc, P: *Investigating the mathematics learning of student teachers: explorations and discoveries*

Edwards, C: *Planning for mathematics teaching: perceptions of experienced and trainee teachers*

Martin, S: *Promoting reflective practice amongst mathematics novice teachers: Are the perceived benefits related to novices' learning styles?*

Pirie, S: *The equation, the whole equation and nothing but the equation*

Wimbourne, P: *Pattern v structure*

May 1995

Adhami, M, Johnson, D and Shayer, M: *Cognitive acceleration through mathematics education: an analysis of the cognitive demands of the National Curriculum and associated commercial schemes for secondary mathematics*

Duffin, J and Simpson, A: *Characteristics of understanding*

Healey, L and Hoyles, C: *Linking the visual and the symbolic: a microgenetic analysis of students' evolving approaches to generalising problems*

Lawson, C and Lee, C: *Numeracy through literacy*

Rowland, T: *Inductive reasoning*

	Watson, A: *Evidence used by teachers to assess pupils' mathematics in the classroom*
October 1995	Hunter, M: *Analysing proceptual thinking through test items* Morgan, C: *Learning to write mathematically*
February 1996	Lins, R: *Struggling for survival: the production of meaning* Newstead, K, Anghileri, J and Whitebread, D: *Language and strategies in children's solution of number problems* O'Reilly, D: *Constructing meaning for number* Vile, A: *Is this a sign of the times? A semiotic approach to meaning-making in mathematics* Wiliam, D; *Is the gap between GCSE and A level bigger in mathematics than for other subjects?* Woodrow, D: *Democratic education – does it exist – especially for mathematics education?*
May 1996	Hegedus, S: *Analysing the metacognitive behaviour of undergraduates in calculus* Johnson, S: *Algebra with student teachers* Morgan, C: *Teacher assessment – tensions and contradictions*
November 1996	Bills, L and Rowland, T: *Examples, generalisation and proof* Brown, T: *Mathematics, language and Derrida*

Cotton, T: *Flipping the coin: models for social justice in the mathematics classroom*

Duffin, J and Simpson, A: *A co-spective way of working*

Lerman, S: *Some problems in research on mathematics teaching and learning from a socio-cultural approach*

Molyneux-Hodgson, S and Sutherland, R: *Mathematics in the practice of vocational science*

Pike, C and Forrester, M: *The role of number sense in children's estimating ability*

Rowlands, S: *The Vygotskian perspective and the radical verses of the social constructivism debate*

Watson, A: *Problematising confidence: is it a helpful concept?*

March and June 1997

Ainley, J, Nardi, E and Pratt, D: *Making connections through active graphing*

Anghileri, J and Baron, S: *Working with 3D shapes - poleidoblocks*

Beishuizen, M: *Two types of mental arithmetic and the empty numberline*

Brown, L and Coles, A: *Interpretations of a classroom vignette or what does reading about someone else's theories-in-action do for you?*

Byatt, S: *A conjecturing atmosphere in 11-16 mathematics classrooms: factors affecting its creation and maintenance*

Hardy, T: *Tales of power: Foucault in the mathematics classroom*

Magajna, Z: *Researching geometric thinking in out of*

	school contexts Malabar, I and Pountney, D: *Interactions between visualisation and symbolic senses* Nardi, E: *Coping with the requirements for rigour: the novelty of university mathematics* Sam, L C and Ernest, P: *Values in mathematics education: what is planned and what is espoused?* Vile, A: *What can semiotics offer mathematics education?* Womack, D: *Investigating children's intuitive understanding of number operations by formalising their mental strategies*
November 1997	Baker, D: *Mathematics as social practice: an investigation into primary student teachers' responses to mathematics* Barrington, R, Hamilton, C and Harries, T: *Numeracy and low attainers in mathematics* Gardiner, J: *Dynamic geometry in the classroom* Johnson, D, Adhami, M and Shayer, M: *Does 'CAME' work? Summary report on pahse 2 of the Cognitive Acceleration in Mathematics Education (CAME) Project* Rogers, L: *Ontogeny, phylogeny and evolutionary epistemology* Winter, J: *The role of intuition in the assessment of mathematics*
February 1998	Chilakamarri, V: *Investigatory approaches in mathematics*

	teaching – implications for teacher training in India
	Kent, P and Stevenson, I: *A computer-based mathematics laboratory for undergraduate chemistry students: a preliminary evaluation report*
	Rowlands, S, Graham, T, and Berry, J: *Modelling, strategic questioning and the laws of student reasoning in A level mathematics*
June 1998	Bills, C: *Relations between teachers' representations and pupils' images*
	Day, C: *An activity approach to teaching and assessment*
	Pendlington, S: *Primary children's imagery in arithmetic*
	Rodd, M and Barber, M: *Convention or reality*
	Rowland, T, Heal, C, Barber, P and Martin, S: *Mind the 'gaps': primary trainees' mathematics subject knowledge*
	Vile, A and Polovina, S: *Possibilities in Pierce's existential graphs for logic education*
	Womack, D: *Teacher trainee students' understanding of operation signs*
November 1998	Berry, J, Maull, W, Johnson, P and Monaghan, J: *Routine questions and A level mathematics grades*
	Brown, T, Hanley, U, Jones, L and McNamara, O: *Primary student teachers' understanding of mathematics and its teaching: a preliminary report*
	Coles, A and Brown, L: *Developing algebra – a case study of the first lessons from the beginning of Year 7*
	Hitt, F: *The role of the semiotic representations in the*

learning of mathematics

Lowe, A: *Do KS3 SATs test for qualitatively different forms of mathematical understanding?*

Ojeda, A-M: *The research of ideas of probability in the elemenatray level of education*

Pope, S: *Key Stage 3 statutory Assessment 1998 – report on an ATL/ATM/NATE joint study*

Rowlands, S: *A critique of relativism in mathematics education: the need for an objectivist perspective if we are to facilitate cognitive growth*

Sivasubramaniam, P: *Distributed cognition, Computers and the interpretation of graphs*

Steinweg, A: *Children's understanding of number patterns*

February 1999 Coles, A, and Brown, L: *Meta-commenting: deleoping algebraic activity in a 'community of inquirers'*

Foxman, D: *Untaught mental methods of 11 year olds: data from the 1987 APU survey*

Harries, T, Sutherland, R and Winter, J: *A consideration of the way images are used in primary maths texts*

Hilton, C and Rowland, T: *What do mathematics tests test?*

Houssart, J: *Teachers' perceptions of good tasks in primary mathematics*

Jaworski, B and Potari, D: *Characterising mathematics teaching using the teaching triad*

Jaworski, B, Nardi, E and Hegedus, S: *Undergraduate*

mathematics teaching project: a methodological report of work-in-progress

Penteado,M: *Mathematics teachers and the use of computers within the classroom*

Rowland, T, Martin, S, Barber, P and Heal, C: *Primary trainees' mathematics subject knowledge: an update report*

Rowlands, S: *Have socioculturalists turned Vygotsky on his head?*

June 1999

Blanc, P: *Solving a non-routine problem: what helps, what hinders?*

Chae, S D and Tall, D: *Aspects of the construction of conceptual knowledge: the case of computer-aided exploration of period doubling*

Crisan, C: *The interaction between the use of ICT and mathematics teachers' professional knowledge base for teaching*

Gates, P: *Mapping discursive structure. Working with habitus, discourse and ideology to explore the politics of mathematics teaching*

Godfrey, R and Aubrey, C: *Assessing early mathematical development*

Hitt, F and Lara-Chavez, H: *Limits continuity and discontinuity of functions from two points of view: that of the teacher and that of the student*

Green, S and Ollerton, M: *Mathematical anxiety amongst primary QTS students*

Ojeda, A-M: *Training and practice of teachers of probability: an epistemological stance*

Price, A: *Analysing data on the relationship between teaching and learning addition in a primary classroom*

Rowlands, S: *Two marks out of ten for constructivism*

Shayer, M, Johnson, D and Adhami, M: *Does 'CAME' work (2)? Report on Key Stage 3 results following the use of the Cognitive Acceleration in Mathematics Education (CAME) project in Year 7 and 8*

Smith, J: *Student teachers' responses to influences and beliefs*

November 1999

Brown, L and Coles, A: *The anatomy of a bid: from TTA to ESRC – looking at the developing algebraic activity in four Year 7 classrooms*

Daskalogiani, K and Simpson, A: *The formation and effects of attitudes towards mathematics in upper sixth form students*

Denvir, H, Rhodes, V, Brown, M, Askew, M, Wiliam, D, Ranson, E: *An investigation on the effect of homework on pupil gains in an assessment of numeracy in the first year of the Leverhulme numeracy research programme*

Houssart, J: *Seeing the pattern and seeing the point*

Huckstep, P and Rowland, T: *'Creative mathematics': real or rhetoric*

Pinel, A and Pinel, J: *Maths knowledge and understanding of primary student teachers: initial findings from a two year study*

	Prestage, S and Perks, P: *Towards a pedagogy of mathematics initial teacher education*
	Reyes, P: *Phenomenographic approach: a guide for action in education research*
Annual proceedings 2000	Askew, M, Denvir, H, Rhodes, V and Brown, M: *Numeracy practices in primary schools: towards a theoretical framework*
	Bills, C: *The influence of teachers' presentations on pupils' mental representations*
	Back, J: *Inducting pupils into mathematical discourse*
	Gardiner, J, Hudson, B and Povey, H: *Aspects of construction and proof in a hand-held dynamic geometry environment*
	Rogers, L: *The biogenetic law and its influence on theories of learning mathematics*
	Rowland, T, Martin, S, Barber, P and Heal, C: *Primary teacher trainees' mathematics subject knowledge and classroom performance*
	Stevenson, I: *Mechanics, modelling and pedagogy: reflections on the design of contexts for new technologies*
	Threlfall, J: *Mental calculation strategies*
February 2000	Brown, M: *Teaching and learning numeracy: policy, practice and effectiveness – BERA national event report*
	Winter, J: *Pupils doing algebra: interviews with Year 7 pupils in an ESRC project*

May 2000	Adhami, M: *Peer evaluation of whole class teaching*
	Anderson, J and Boylan, M: *National Numeracy Strategy: teacher questions and pupil anxiety*
	Barwell, R: *Underwriting action: discursive psychology and the discourse of a primary mathematics classroom*
	Caterall, R and Sangster, M: *The mathematics of money at Key Stage 1*
	Chae, S D and Tall, D: *Students' concept images for period doublings using computer experiments in chaos theory*
	Coltman, P, Petyaeva, D and Anghileri, J: *Revisiting the 'materials of play': effective learning in some aspects of shape and space*
	Houssart, J: *Nice and easy does it: pupils' responses to non-challenging tasks*
	Jaworski, B: ***r****esearch or* ***R****esearch and its relation to mathematics*
	Nardi, E: *The concept of supremum/infimum of a set: a problematic overture to the concept of limit?*
	Smith, C: *Using and applying mathematics: desirable teaching style or assessment construct*
	Threlfall, J: *Primary children's understanding of probability*
November 2000	Barwell, R: *How many burgers can a human being eat? Writing word problems when English is an additional language*
	Boylan, M and Lawson, P: *'I'd be more likely to speak in*

	class if…' How some Year 8 students experience teacher questioning and discussion strategies
	Georgiadou-Kabouridis, B: *Three way interaction: exploring the impact of a headteacher and a researcher on a novice primary teacher teaching geometry*
	Hoyles, C and Küchemann, D: *Year 8 students' proof responses: some preliminary findings from four groups*
	Nardi, E and Iannone, P: *Adjusting to the norms of mathematical writing: short stories on the journey from cipher to symbol*
	Olivero, F and Sutherland, R: *The Churchill Cabri project: background and review*
	Paine, J: *A Maths continuum: visualisation versus number agility*
	Pinel, A and Pinel, J: *Towards numeracy and progression: benchmarking within a pyramid project*
	Sharma, R: *Researching students' symbol sense*
Annual proceedings 2001	Coles, A and Brown, L: *Needing to use algebra*
	Jones, K and Morgan, C: *Research in mathematics education: some issues and some emerging influences*
	Olivero, F: *Conjecturing in open geometric situations using dynamic geometry: an exploratory classroom experiment*
	Prestage, S and Perks, P: *Models and super-models: ways of thinking about professional knowledge*
March 2001	Coles, A: *Listening: a case study of teacher change*

	Day, C: *Developing a Vygotskian practice in mathematics education*
	Denvir, H and Askew, M: *Pupil participation in 'interactive whole class teaching'*
	Hadjidemetriou, C and Williams, J: *Evaluation teachers' knowledge in relation to their children's learning*
	Mohammad, R: *Co-learning partnerships: a way to teacher development*
	Nardi, E: *Domain, co-domain and relationship: three equally important aspects of a concept image of function*
July 2001	Barwell, R: *Difficulties with mathematical word problems*
November 2001	Edwards, J-A and Jones, K: *Exploratory talk within collaborative small groups in mathematics*
	Foxman, D: *Mental calculation methods used by 11 year olds in different attainment bands: subtraction questions in the 1987 APU survey*
	Fujita, T: *The use of practical and experimental tasks in geometry teaching: a study of textbooks by Godfrey and Siddons*
	Georgiadou-Kabouridis, B: *Conceptualising the factors that influence the mathematics teaching of a group of newly qualified teachers in Greece*
	Hall, P: *Improving teaching methods: how listening to students can make a difference*
	Jones, K and Fujita, T: *Developing a new pedagogy for geometry*

Mason, J: *Mathematics as a constructive enterprise*

Pope, S and Sharma, R: *Symbol sense: teachers' and students' understanding*

Price, A: *Developing a framework for analysing children's learning of mathematics inside and outside the classroom*

February 2002

Afantiti-Lamprianou, T and Williams, J: *A scale for assessing probabilistic thinking and the representativeness tendency*

Brown, T: *Teacher identity in initial teacher training*

Goulding, M: *Do teachers understand the theory behind CAME?*

Pope, S: *Uisng small software on graphic calculators*

Williams, J and Ryan, J: *Argumentation and teachers' role: the general and the particular*

Parker, M: *Discussion and developing mental strategies when solving computer generated problems*

May 2002

Coles, A and Brown, L: *Constraints and freedoms in mathematics and mathematics teaching: descriptions of practice in Year 8 secondary mathematics classrooms*

Küchemann, D and Hoyles, C: *The quality of students' reasons for the steps in a geometric calculation*

Perks, P: *The interactive whiteboard: implications for software design and use*

Sutherland, R, Godwin, S, Olivero, F and Peel, P: *Design initiatives for learning: ICT and geometry in the primary*

	school
November 2002	Godwin, S and Beswetherick, R: *Reflections on the role of task structure in learning the properties of quadratic functions with ICT – a design initiative* Pope, S: *The use of origami in the teaching of geometry*