Surface area to volume ratio and metabolism: Analysing small group-task as Vygotskian activity

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Three students Dan, Levi and Thor, attempt a group-task containing worksheets A and B. While worksheet A asks students to calculate and compare surface area to volume ratio of a sphere for six successive units, worksheet B asks them to consider the metabolism of living cells and the bearing the ratio has on their functioning and size. While Levi and Thor own the group-task, follow its instructions and deliberate on its questions, Dan declares his intention of observing Levi and Thor and takes a free ride. Based on students' inscriptions and transcript of audio-recordings, I show how Levi and Thor work through calculations required in worksheet A with ease, even coming up with conjectures. In attempting worksheet B they are able to correlate better metabolism in cells with a smaller radius, yet question if that model is indeed borne out in reality. Three constructs from cultural historical activity theory and/or CHAT namely leading activity, germ cell of activity and learning activity are utilised to shed light on attempts by Dan, Levi and Thor at their group-task.

Leading activity; germ cell of activity; learning activity; surface area to volume ratio; metabolism in living cells

Introduction

This paper analyses the attempts of three adolescent students Dan, Levi and Thor at a small group-task, conducted as part of my doctoral and classroom study at a Gymnasium in Norway (Gade, 2006). Three constructs of cultural historical activity theory and/or CHAT are deployed. In what manner do leading activity (Karpov, 2005), germ cell of activity (Stetsenko, 1999) and learning activity (Davydov, 1999) shed light on students attempts at the group-task, is the question pursued.

The small group-task

Dan, Levi and Thor, formed a group of three students whom I observed while sitting beside them, as they participated in a collaborative classroom practice established by their teachers Olaf and Knut (Gade, 2006). Over the academic year I observed seven such groups as they took part in a practice geared pedagogically towards students' dialogue and meaning making, by way of cooperative learning in small groups (Gade, 2011). My data set includes field notes, quantitative responses by all students to group-tasks specially designed by Olaf and Knut and audio-recordings of the small group-tasks I set for each of the seven student groups. Partitioned into seven cycles, with each cycle corresponding to a section of the textbook, the table below summarises progression of instruction during the first three cycles. The four grounded themes in which such progression was understood makes up the four columns in the table. Dan, Thor and Levi's attempt is indicated **in bold**.

Theme → Chapter ↓	The collaborative practice	The consolidation of meaning	Problem solving know-how	Cooperation in problem solving
Number understanding	Establishment by the teacher of his intentions	Building of meaning in teacher-driven practice	Discussion by turning rules into questions	Cooperation established, consolidated
Equations and proportionality	Participation by students in their and other's intentions	Building upon of students meaning making	Building up solutions for application	Students conjecture reality with given graphs
Scale factor in similar figures	Participation by students with independent intention	Building upon of students intuitive knowing	Questions become problems that students can solve	**Students question reality with given model**

Titled *What can we find about surface area and volume?* the group-task I dwell upon was made up of two worksheets A and B. Conducted during students' recess time and rewarded with a bag of chocolates, I audio-recorded Dan, Levi and Thor's attempts besides also making field notes. I also addressed students' expectations of their task with appropriate instructions. Worksheet A asked students to fill in successive values for surface area, volume and their ratio for spheres of increasing units of radii. Students were then asked to express various ratios algebraically. Worksheet B consisting of Exercises 1 and 2, read as follows:

Exercise 1

The surface area of any body is an important measure in science.

What do you think the cat gains by stretching out as shown below?
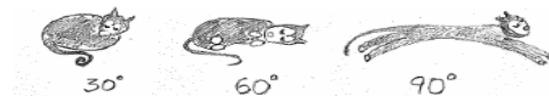
Can you offer an explanation in terms of its surface area and volume?

Do we humans behave similarly to the cat? How and why?

Cats and humans are warm blooded. Would the above observation change if we were discussing cold blooded animals?

Exercise 2

If we assume that living cells in our bodies are spherical what happens to the surface-area-to-volume (SA/ V) ratio as the cells get larger and larger? Why?

For any living cell, metabolism is the rate of chemical activity in the cell. Metabolism maintains life. For metabolism to take place materials like oxygen and water need to be absorbed. Of the two values Surface Area and Volume

Which value will you think determines the cell's metabolism _____?

Which value controls how much material gets in and out of the cell _____

For a spherical cell how would a larger surface-area-to-volume affect its metabolism?

Do you think it is advisable for organisms to have large cells or small ones? Why?

With the aims of my paper being methodological, I now turn to examine Dan, Levi and Thor's attempts with analytical constructs of CHAT, setting aside engagement with research relating to purpose and utility in task design for later (Ainley, 2011).

Leading activity

Laying emphasis on sociocultural activities that a child takes part in during his or her development, the construct of *leading activity* (Karpov, 2005) refers to those activities which produce major developmental changes in students. Premised on the fact that the social environment provides the building blocks for development, the activity leading development in infancy is emotional communication, followed by object-centered activity and play in early stages of childhood. While taking part in educational activity is the leading activity for middle childhood, interactions with peers and adults is the leading activity for adolescents. Mediated by theoretical interactions with scientific concepts, participation in such activity brings about formal-logical thinking and allows also for self-analysis and independent thinking. I argue the group-task pursued by Dan, Levi and Thor to be their leading activity, which they were asked to attempt together. While Levi and Thor took responsibility for pursuing different parts of the task, Dan chose the easy way out of observing them and avoided any manner of engagement with objectives of the task. In various extracts of their attempts below I use common transcription symbols and refer to myself as RES. I begin with Levi and Thor's completion of the table in Worksheet A as given below:

- Fill in the table below for spheres of different radii:

SN	Radius (r_x)	Surface Area (SA_x) in 2 decimals	Volume (V_x) in 2 decimals	$\frac{SA_x}{V_x}$
1	1 unit	12.57	4.19	3
2	2 units	50.27	33.51	1.5
3	3 units	113.09	113.1	1
4	4 units	201.06	268.08	0.75
5	5 units	314.16	523.6	0.6
6	6 units	452.39	904.78	0.5

The utterances that accompanied such efforts included Levi's initial conjecture

      05:19 Levi   67   That's pretty wrong ... it's hmm in the power of three

In a short while Levi poses a conjecture and asks

      06:51 Levi   99   Is there any connection here?

While filling the SA/V ratio for a radius of 6 units, Levi guesses as below

      11:30 Levi   164   That should be five

In line with CHAT, the above extracts show that scientific concepts like radius, surface area and volume of a sphere mediate Levi and Thor's thinking as they filled the table, alongside concepts like formula, ratio and rounding off of decimals. Formal-logical thinking, self-analysis and independent thinking that such activity is said to promote, is evidenced in these extracts. For example in calculating volume of spheres for increasing units of radii, Levi observes a pattern in terms of the power of three (67). This in turn leads to self-analysis when he searches for a connection in the patterns he observes (99). While calculating surface area to volume ratio Levi is then able to build on the logical thinking he finds emerging in his efforts and conjectures that the last ratio he is to calculate *should* be five (164). This aspect is demonstrative of his independent thinking. With such thinking evidenced in students' attempts throughout the group-task, I now turn to the next construct.

Germ cell of activity

Germ cell of activity is a CHAT construct based on the premise that human learning leads development. Stetsenko (1999) explains learning to lead development because it is through learning that children master, through and within interactions with an adult, new cultural tools, which together constitute the cornerstone of mental functioning and human development. Stetsenko identifies any germ cell to have three components – social interaction as the main source of development, cultural tools which mediate psychological functioning and the zone of proximal development (zpd) as the main portal through which development occurs. While I detail an instance of a zpd in my doctoral study in Gade (2010), I presently utilise germ cell of activity to shed light on Levi and Thor's attempts in the three extracts I offer. Firstly, Levi and Thor discuss how surface area to volume ratio was related to different radii of the sphere.

Time	Person	Turn	Utterance
14:49	Thor	248	it lowers , it lowers [down]
	Levi	249	[half]
14:50	Thor	250	[Half]
	Levi	251	[There] it is half ((Ratio of 1.5 compared to 3))
	RES	252	hm hm
14:55	Levi	253	Its hm..one third ((Ratio of 1 compared to 3))
	RES	254	hm hm
14:58	Levi	255	One fourth ((Ratio of 0.75 to 3))

In a sequence of turns (248 to 255) Thor and Levi then articulate how the surface area to volume ratio decreases in magnitude. I argue the sequence of these utterances to evidence how Levi and Thor learnt or mastered the new cultural tool of surface area to volume ratio, which in turn was based on their mastery of volume and surface area of spheres of different radii. It was mastery of these cultural tools that mediated their mental or psychological functioning. Evidencing the role of social interaction in such activity, I now turn to Levi and Thor correlating the metabolism of living cells to their volume and radius. When I ask how spherical volume would affect metabolism (472) Levi first correlates lesser energy in cells with a smaller surface area to volume ratio (473). Upon such mediation Thor makes his observation *Oh what you mean is if the bigger the cells are ... the worse you have it*. (477, 479)

Time	Person	Turn	Utterance
	RES	472	volume, how would that affect metabolism
23:32	Levi	473	it will be lesser … energy
23:36	RES	474	lesser energy if, if the ratio is less [or if] the ratio is more
23:41	Thor	475	[ya]
	Levi	476	[ya]
	Thor	477	Oh what you mean is if the bigger the cells are
	RES	478	Hm mm
23:50	Thor	479	the worse you have it
	RES	480	Right why do say so
23:55	Thor	481	Because you just said that the bigger the radius is..
	RES	482	Hm hm
	Thor	483	the lower the metabolism effect is

This last utterance from Thor is indicative of psychological functions maturing in him in relation to my question and his being led in his zpd, in line with Stetsenko (1999). Thor also relates his new found reasoning by pointing to *This ... mathematics* (511).

Time	Person	Turn	Utterance
24:28	Levi	497	I agree
	RES	498	you agree.. why?
24:30	Levi	499	because metabolism is good and the metabolism is big with while when the radius is low
24:32	RES	500	Ok and why is that
	Levi	501	I don't know @@ I have no clue
24:40	RES	502	@ No no you have it
24:41	Thor	503	@ it is from you said you said SA / V
	RES	504	yes the ratio
	RES	505	So can you explain in one line what's the relationship between the ratio and the rate of metabolism
24:51	Levi	506	It increases when the radius ... goes … down
	RES	507	It goes down and you have a reason for that
	Levi	508	This ((Levi points to the worksheets))
25:08	Thor	509	[@@]
	RES	510	[What's the reason]
25:10	Thor	511	This … mathematics
25:13	RES	512	Yes, the table
	Thor	513	the table it gives us a solution and answer for it

Far from working with simpler concepts known prior to the group-task, in the above extracts Levi and Thor evidence how they are able to work with concepts derived in the course of ongoing activity. Qualifying formal-logical thinking that Karpov (2005) drew attention to, the three extracts in this section show how the mastery of concepts called into question the thinking that mediated Levi and Thor's development.

Learning activity

Davydov (1999) explains real *learning activity* as one in which surrounding reality is transformed by one's efforts. Such activity has object-related content differing from other kinds of human activity and necessarily involves creative or reforming elements. Learning in such activity relates to transformation, where any contradictions that are come across are handled in an objective manner. The construct of learning activity sheds light on the discussion Levi and Thor had towards the end of their attempts, as evidenced by two extracts I now offer. In line with Davydov, Levi and Thor's pursuit of surface area to volume ratio in relation to metabolism of living cells was an object which distinguished this group-task from others. The notion of reality relevant to their attempts is highlighted by the manner in which Thor considers their calculations like a theory upon which to base his assumptions (554). Levi however does not agree with Thor (549, 551) and asks *But is it like that* (555).

Time	Person	Turn	Utterance
26:36	Levi	543	but, ya, ya, are they
	RES	544	Well what is the word here we say
	RES	545	we assume
	Thor	546	[we assume]
26:39	RES	547	[we ass=ume]
	Thor	548	[We don't know] but
26:41	Levi	549	[We don't know]if we [assume]
	RES	550	[it seems correct]
	Thor	551	[we don't know]
26:43	Levi	552	Ya, if we make the assumption then this table says what we have … … [concluded]
	RES	553	[Ya]
	Thor	554	Its like a theory, ya
26:49	Levi	555	But is it like that

The above extract is followed by another turn (562) wherein Levi questions reality by reiterating *I don't think so* illustrating how Levi contradicted Thor, yet in an objective manner. Illustrative of handling the dialectic between emerging theory and perceived reality, Levi and Thor transformed the way they conceived living cells in terms of their stating that smaller radii were needed for better metabolism. Thor's summary of his attempts at the group-task via *The bigger the animal is ... the smaller the cells are* (578) brings my analytical attempts of the same to a close.

Time	Person	Turn	Utterance
27:10	Thor	569	they have smaller cells
	RES	570	Just because the animal is larger do you think the cells would be larger
	Levi	571	No, it should be
	RES	572	what does this tell us
27:16	Levi	573	Since the metabolism is got to higher
	RES	574	yes, yes
	Thor	575	If that has to be higher then
	Levi	576	then the cells got to be low … smaller... ya
	Thor	577	Hmm hmm
	Thor	578	The bigger the animal …. the smaller the cells are

In conclusion

In line with non-dualistic perspectives of CHAT, it was possible to discern how the social environment was not extrinsic to but provided the very building blocks for Levi and Thor's psychological development as they attempted a group-task which qualified simultaneously as learning activity, germ cell of activity and leading activity.

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