A-level mathematics reform: satisfying the requirements of university courses across the range of mathematical subjects.

Peter Osmon
Department of Education and Professional Studies, King's College London

Three groups of subjects have been identified that are mathematical to varying degrees, with requirements for three corresponding levels of foundational mathematical knowledge (Osmon 2009). But effectively only one post-16 mathematics qualification exists in England: GCE Advanced level. This matches the foundational requirement of the most mathematical group and serves as an entry requirement for their university courses but the numbers are far fewer than the total entering courses in mathematical subjects and consequently the other two subject groups' courses are populated with mathematically underprepared students. In principle, courses leading to the existing AS-level award and a new sub-AS award could meet the pure and applied mathematics needs of the other two groups, and means for specifying them are described. A progression scheme whereby these three courses form a single post-16 mathematics pathway, along which learners travel as far and as fast as their mathematical abilities and ambitions take them, and with the three awards as exit points, is described. It would use scarce mathematics teaching resources parsimoniously. But because of the widespread culture of mathematics aversion take-up would depend critically on universities using the awards as admission requirements.

Keywords: Foundational mathematics, mathematical subjects, post-16 mathematics pathway, progression scheme, mathematics aversion

Introduction

This paper brings to a conclusion a study begun about five years ago aimed at understanding the mathematical knowledge expected of students entering undergraduate courses in various subjects. The early stages were reported at BSRLM conferences during 2009-10 (Osmon 2009 et seq). The methods used were: informal interviews with heads of academic departments and admissions tutors across a range of universities and subjects; followed by analysis of UCAS admissions data to determine the mathematics requirements by subject and subject group across a sample of 10 universities, one per decile over The Times newspaper range of university rankings; followed up by comparisons of some first year undergraduate mathematics module specifications and textbooks.

The following recurring themes emerged from the informal interviews. (a) Some mathematics topics are important in our subject, but our students also need general mathematics competence. (b) Our students need more probability and statistics than they get at GCSE. (c) Students need to be able to use [apply] their mathematics- not "tell us the formula, sir". (d) They should keep their mathematics going between GCSE and university entrance- else they forget what they apparently knew. (e) A widespread culture of mathematics aversion, that has been documented elsewhere, by Brown, Brown and Bibby (2008) for example, has significant consequences: we don't label our subject as mathematical lest applicants are put-off and run to other courses/subjects. This aversion and its counterpart- mathematics regarded as an elitist subject- are impediments to improving mathematics knowledge under all the above headings.

From the UCAS data and follow-up clarification it became clear that there are three distinct groups of subjects with characteristic levels of foundational mathematics needed for their undergraduate courses: Light-mathematics subjects (L-m) (including Bio, Health, Management, and Social Sciences); Moderate-mathematics subjects (M-m) (including Business, Chemistry, Computing, Economics); Heavy-mathematics subjects (H-m) (including Engineering, Physics, Mathematics). The foundational level of mathematics knowledge required was characteristic of the subject, although more prestigious universities required higher mathematics grades at either GCSE or A-level for entry to their courses.

Thus mathematics to A-level standard was required for H-m subjects at all universities and no more than GCSE for L-m subjects. The M-m case was more complex: these subjects need foundational mathematics to about AS-level in size but Computing wants non-traditional content. However, A-level mathematics seems to be a monolithic award- the number of students taking mathematics to AS-level only is small- and, perhaps for this reason, AS-level is not a specific requirement for any university courses. *So, although three levels of foundational mathematics knowledge requirement were identified, there are effectively only two qualifications- GCSE and full A-level.* As shown in Figure 1 (a) what happens in practice is H-m subjects require full A-level for entry, M-m and L-m accept GCSE, and since applicants generally do not keep their mathematics going between GCSE and university entrance this is actually less mathematics knowledge than it seems and, presumably, these quantitative subjects have to be presented qualitatively.

Reform of post-16 mathematics

Curriculum structure

Currently, post-16, learners effectively study for the monolithic A-level award or else give up mathematics. Given the cultures of mathematics aversion/elitism it is unsurprising that only confident mathematicians choose the former. Those proceeding to M-m and L-m courses ought to continue with their mathematics- not necessarily to A-level standard- but there is no appropriate provision.

Evidently the three subject groups (L-m, M-m, H-m) need progressively more foundational mathematics and it seems clear the structure of the curriculum and associated awards should be reformed to encourage learners to develop their mathematics as far as possible thereby widening their opportunities: an ideal mathematics curriculum structure would provide *a pathway along which learners can travel as far and as fast as their abilities and ambitions take them.* This pathway should have exits corresponding to the minimal mathematics needs of the three subject groups. Is such a structure feasible?

First, consider the situation described by Figure 1(b). A-level mathematics is shown as an entry requirement for H-m courses and AS-level for entry to M-m. For this to work, content leading to AS (called here A1- so as to distinguish content from the award) would have to be modified to meet at least some of the needs of Computing (most obviously some set theory and logic) with knock-on consequences for A2 content perhaps. L-m's needs remain unmet.

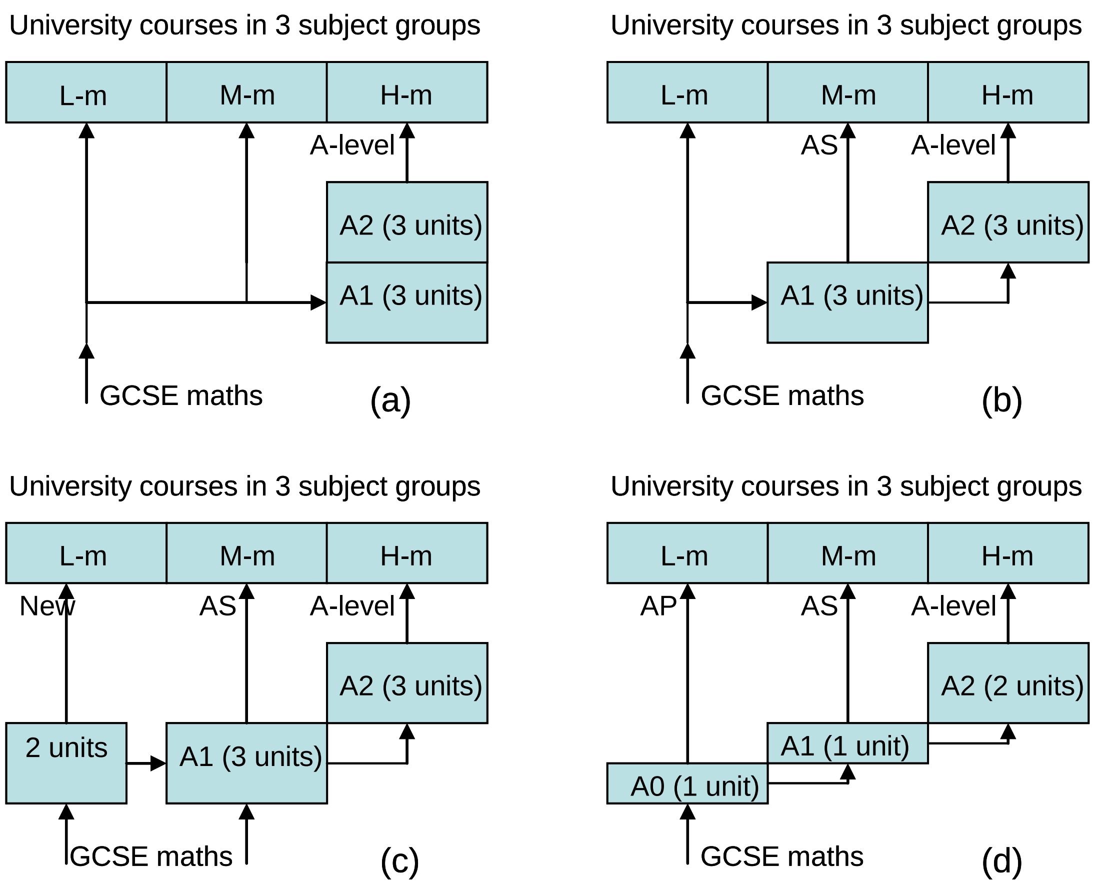

Figure 1. Mathematics requirements possibilities for entry to university courses in the three subject groups: (a) currently- GCSE required for groups L-m and M-m, and A-level for H-m, (b) modified so that AS-level is needed for entry to M-m group subjects, (c) showing ACME's proposed new sub-AS award as entry requirement for L-m subjects and AS for M-m subjects, (d) with the author's proposed AP award for entry to L-m subjects, AS for entry to M-m, and full A-level for H-m.

The Advisory Committee on Mathematics Education (ACME 2012) has proposed a new two year post-16 course leading to a 2-unit sub-AS award which could also offer a gentler entry route into AS for less confident learners. (However this route comprises up to 5 units altogether and hence requires an investment of time comparable with full A-level and so might prove unattractive.) Figure 1 (c) shows this 2 units sub-AS award (labeled New in the figure) in the role of entry requirement for L-m, and shows AS as prerequisite for M-m but- since ACME wants AS largely undisturbed- for content reasons this would not work. However a sub-AS award which would enable less-confident mathematicians to keep their mathematics going after GCSE and develop it somewhat before entering L-m courses is an attractive concept.

The author therefore proposes the scheme in Figure 1 (d) in which the content leading to AS is split into two halves (labeled here A0 and A1) where A0 is the content for a new sub-AS award (which might be called Preliminary A-level- abbreviated here to AP) to meet the needs of L-m and A1 is the content to complete AS so as to meet the needs of M-m. A2, as currently, tops up A1 leading to the award of full A-level to meet the needs of H-m. This gives a progression of three courses- A0, A1, A2 leading to 3 corresponding levels of award- AP, AS, full A-level. There is no apparent rationale for the current packaging of AS and A2 mathematics courses as 3-units each- rather than 2-units like arts subjects- and this may indeed help to brand mathematics as elitist and hence encourage aversion. 4-units for the mathematics A-level course implies 2-units for the AS course and 1-unit for the AP course. A 1-unit course is novel- but are there any real difficulties? These arrangements would offer *staircase progression in post-16 mathematics: learners climb the staircase as far and as fast as their abilities and ambitions take them.*

Implementation

We now consider how a carefully thought through implementation scheme might encourage access. Three categories of post-16 mathematics learners are envisaged: least, middling, and most confident who may be expected to aim for the corresponding three awards: AP, AS, A-level. We may hope however that some at least will underestimate their mathematics learning potential so that, after initially aiming for AP might come to realize they could actually be capable of AS and likewise some AS-aimers could achieve full A. (Conversely, of course some may decide they have been over-ambitious.) Ideally this implies provision for some learners to commit late to A1 or A2 (and conversely for some to drop back to less ambitious goals). Can a course scheme provide this degree of flexibility while also using scarce teaching resources parsimoniously?

To see what would be involved, assume two years of post-16 mathematics, a teaching year in two halves with A0, A1, and A2 all taught and assessed in two halves: A0a, A0b; A1a, A1b; A2a, A2b. *And assume learners review their goals after each half year*. Evidently the scheme should accommodate the following cases:
AP: direct to AP in 2 years; *AS*: direct to AS in 2 years; *A*: direct to Full-A in 2 years.
But also: *APtoAS*: AP initially, switching up to AS; *AStoAP*: AS initially, switching down to AP; *AStoA*: AS initially, switching up to Full-A; *AtoAS*: Full-A initially, switching down to AS.

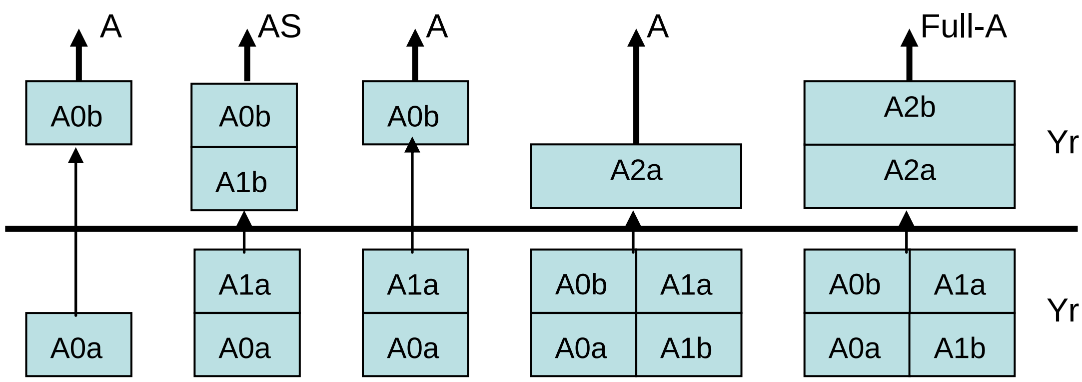

Figure 2. How all these cases can be accommodated- provided the two halves of A1 are independent- so that they may be studied in either order (A1a then A1b) or (A1b then A1a)- then A1a and A1b need only be taught once in each year (parsimony). The reader is invited to verify that all cases are covered.

Pure mathematics

It may be possible to deduce desirable pure mathematics content for each of the proposed components (A0, A1, A2) by asking subjects and subject groups their needs. Thus, consider the following *procedure*. First, assemble subject advisory bodies representing each subject's professoriat and its professional body (for computing this would be the CPHCD and the BCS); ask each advisory body to specify their subject's foundational mathematics need; then assemble an overarching body for each subject group to merge these subject needs. Next map these requirements onto AP, AS and A-level as follows. The L-m subject group go first to map their needs onto A0. (In the informal interviews "Our students need more probability and statistics than they get at GCSE" was a theme across all subject groups and it seems likely the L-m group would say this is their primary content requirement.) The M-m subjects go next to specify further material for A1, then the H-m group should specify what else they need for A-level. Iteration may be necessary.

"Some topics are important in our subject, but our students also need general mathematics competence" suggests subjects and subject groups might be sufficiently flexible in their requirements that this procedure can converge on an agreed specification for content. Note that in this reformed curriculum there are no options or choices- *parsimony of content* in ACME's terminology- just a single mathematics road- so that learners are not required to specialise prematurely.

Applied mathematics

"Students need to be able to use [apply] their mathematics". Making or exploring mathematical models can be a rewarding experience and regarding applications as instances of modelling, has the potential to increase the attractiveness of post-16 mathematics- see Maasz (2011) for example. But the learners need to be familiar with the application domains of the models they work with- generally these are their everyday world and also their other subjects: physics, geography, or whatever- and, of course, the appropriate pure mathematics. Modelling problems requiring probability and statistics knowledge occur in all application domains, and emphasis on this topic in the pure part of the curriculum will give learners access to them. The New Zealand experience seems relevant: Hodgen (2013) have remarked that an applied probability and statistics pathway with a wide range of applications has attracted uniquely large numbers into post-16 mathematics learning.

Summary and conclusions

Currently A-level mathematics meets the foundational mathematics needs of H-m subjects. But the needs of M-m and L-m subjects (far more students) are not addressed, and most students entering these courses have no mathematics beyond GCSE: consequently M-m, L-m courses must teach their subjects relatively qualitatively. We are forced to conclude that post-GCSE mathematics is currently unfit for the purpose of preparing the majority of those entering HE courses in mathematical subjects. This is pretty disgraceful! Clearly, awards meeting the needs of L-m and M-m should be devised and introduced urgently. But the cultures of mathematics aversion/elitism would limit take-up. To counter them (a) school children need to appreciate that knowledge of mathematics affects their life chances and (b) mathematics should take care not to present itself as an elitist subject and should instead provide for mathematically less confident learners to access and progress post-GCSE.

Reform of structure and content of A-level mathematics

Accordingly, a reformed structure for post-GCSE mathematics has been proposed offering access and progression such that: *learners may progress as far and as fast as their abilities and ambitions take them.* The structure comprises three courses and three awards, matched in size to the needs of L-m, M-m, and H-m subject groups, and organised progressively with steps of increasing size (1, 2, and 4 units) so as to lower the access barrier to post-GCSE mathematics without diminishing the standard of full

A-level while accommodating learners travelling at different speeds. This scheme has the same number of assessment points as current A-level. As to content, a procedure has been proposed whereby the universities would collectively determine the scope of these post-GCSE mathematics awards so that they progressively meet the needs of their courses in the three subject groups.

Practicalities

More students taking post-GCSE mathematics- in order to prepare for L-m, M-m courses- would stress the finite national mathematics teaching capacity. However these reforms would use the limited capacity economically: more students but working within a single unified post-16 curriculum. In ACME's terminology the reforms' use of teaching resources is *parsimonious*.

These structure and content reforms to the curriculum would hopefully make the study of post-GCSE mathematics more attractive and enable learners to develop the appropriate foundations for the whole range of undergraduate courses in mathematical subjects. But realistically they are unlikely by themselves to counter the culture of mathematics-aversion/elitism and *take-up is likely to be disappointing unless the universities require these awards* for entry to all their courses in mathematical subjects.

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