Solving word problems algebraically in a spreadsheet environment in primary school.

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This paper presents some results from an investigation into the teaching of the algebraic solving of word problems in a spreadsheet environment in the sixth grade of primary school in Spain (11-12 year old pupils). The main aim of the study was to investigate whether the spreadsheet could be a mediator to the teaching of algebraic problem solving. Through the analysis of excerpts from a case study, the core of the paper is focused on two different types of the difficulties that students showed when solving problems algebraically in a spreadsheet environment.

Keywords: teaching and learning of algebra, prealgebra, solving problems, primary, software.

Introduction

Since the eighties there have been numerous studies that have tried to exploit the spreadsheet's capabilities into the teaching of mathematics. In this study we address the solving of arithmetic-algebraic word problems, which has played an important role in most mathematics curricula. Our study is based on the hypothesis that the use of the spreadsheet allows teachers to bring forward the solving of algebraic word problems to the last years of primary education. This hypothesis is supported by previous studies that have shown the potential of the spreadsheet in this sense (Dettori, Garuti and Lemut 2001; Friedlander 1996).

Among previous research using a spreadsheet, we highlight the *Spreadsheet Algebra Project*, developed by Rojano and Sutherland in the nineties. One phase of this project was conducted with primary students (10-11 years old) with no prior algebraic instruction, i.e. initial features very similar to those of our work. As a conclusion of their work, the authors pointed out that "a spreadsheet helps pupils express, explore and formalize their informal ideas" (Sutherland and Rojano 1993, 380). Despite stressing the spreadsheet's potential, the authors recognised some limitations of the tool. In this sense, they affirmed "most pupils aged between 10 and 11 do not spontaneously think in terms of a general formula when first presented with a spreadsheet environment" (1993, 379). This result seems to underline the need to teach both the basic rudiments of the spreadsheet and the algebraic solving of word problems in that environment. The inclusion of a teaching sequence in our work is the main difference between our research and the *Spreadsheet Algebra Project*.

On the other hand, Arnau (2010) analysed how the teaching of algebraic problem solving in the spreadsheet environment affects students' performance when they come back to the pencil-and-paper environment. The study was conducted with second-year secondary students (13-14 years old), just after they were instructed in algebraic solving of word problems with pencil and paper. The students were taught to solve word problems algebraically in a spreadsheet environment by the Spreadsheet method (hereinafter, SM). This method is an adaptation of the Cartesian method (hereinafter, CM) to the spreadsheet environment considering the restrictions imposed. The CM is the way in which solving problems is usually introduced in algebra texts by the use of algebraic language (Filloy, Rojano and Puig 2008). We can describe this method as a sequence of ordered steps: 1) the analytic reading of the statements of the problem to transform it to a list of quantities and relations among quantities; 2) choosing a quantity (or several quantities) which one designates with a letter (or several different letters); 3) writing algebraic expressions to designate the other quantities using the letter (or letters) introduced in the second step and the relations found in the analytic reading made in the first step; 4) writing an equation (or as many independent equations as the number of letters introduced in the second step) based on the observation that two (non-equivalent) algebraic expressions written in the third step designate the same quantity; 5) transforming the equation into a canonical form; 6) the application of the formula or the algorithm of solution to the equation in canonical form; and 7) the interpretation of the result in terms of the statement of the problem.

In contrast, the SM can be synthesized as: 1) the analytical reading of the problem; 2) assign a cell for each unknown quantity and choose one unknown quantity on which the remaining unknown quantities depend, this is known as *the reference quantity*; 3) represent in previous cells (except for the reference quantity) formulas that describe their relationship with other unknown quantities; 4) set up an equation by assigning a second cell for one quantity in order to have two expressions that represent the same quantity; 5) iterate the reference quantity until both cells of the equation have the same value; and 6) interpret this value in terms of the problem.

Arnau aimed to evaluate whether instruction in the SM produced an increase in students' competence in the CM when they returned to the paper-and-pencil. In contrast, it was possible for negative cognitive tendencies to appear after the return to arithmetic solving. In fact, his results showed some limitations of the instruction in the SM when students came back to the paper-and-pencil. Specifically, there was an increase in the number of algebraic resolutions, but with a decrease in the use of the language of algebra. In addition, for a subfamily of problems (age problems), there was a significant decrease in student competence to address problems algebraically. This result is attributed to the appearance of spontaneous strategies which are enabled by the spreadsheet's features. Arnau's results raise doubts about the possibility of using the spreadsheet in secondary school when students have already received instruction in the language of algebra.

The works discussed above do not restrict the potential of spreadsheets in teaching of algebraic solving to learners before their instruction in its formal language. Based on the studies discussed above, we designed an experiment which aimed to analyse the viability by using a spreadsheet to teach algebraic solving of word problems at an educational level where the lack of algebraic language seems to make it impossible. In this paper we present briefly results which allow us to answer the main aim of this research, and we focus on some of the difficulties that primary students showed when they solved problems in an algebraic way.

Methodology

Twenty-one pupils took part in the experiment. They were enrolled in sixth grade (10-11 years old) and all of them were part of the same natural group. Basically the study consisted of two phases: the first aimed at teaching the SM while the second aimed at collecting data about how primary students solved word problems after being taught in the SM. Below we describe synthetically the main features of each stage.

In the first phase the pupils followed a sequence of lessons oriented to teach how to solve word problems in an algebraic way by using the SM. The teaching sequence was developed in the computer classroom of the school to which the students belonged. The teaching sequence lasted ten sessions, five sessions of 45 minutes and five of an hour. The sessions took place at the usual time of mathematics classes. Students were freely grouped in pairs to favour the communication process between them.

The teaching sequence was divided in two different stages of five sessions each. Since students had no prior knowledge about the spreadsheet's operation, the first stage of the teaching sequence was an introduction to the basic principles and features of the spreadsheet environment and its language. With this aim, we designed a set of mathematical activities which allowed the student to familiarize the following basic techniques: the identification of the spreadsheet's elements (cell, row, columns...), the introduction of formulas, the need of the use of parenthesis, etc. The second stage of the teaching sequence was devoted to the instruction of the SM. In the first session of this stage, the teacher solved a word problem using the SM. After that, each pair of students received a collection of problems to be solved independently.

After the teaching sequence, we carried out a task-based case study involving all students who participated in all ten lessons of the teaching sequence. Thus, the case study was completed by eight pairs, the same pairs formed for the teaching sequence. In the case study the pairs should solve four arithmetic-algebraic word problems in a spreadsheet environment. The case study was videotaped for a later transcription to a written protocol. The four selected problems were characterized by: 1) having a similar difficulty to those from the teaching sequence and 2) its most direct analytical reading was algebraic.

Results and findings

Before discussing some of the difficulties that emerged when pairs of primary students solved problems in an algebraic way in a spreadsheet environment, we present a quantitative summary of the case study. This summary allows a global view of the experiment's results regardless that the rest of the paper is intended to discuss the difficulties. Table 1 shows for each problem how many pairs addressed each problem, how many of them used the SM as solving strategy and, finally, how many pairs solved correctly each problem by using the SM.

Problem name	**Addressed**	**Addressed by using SM**	**Solved correctly**
1. Sport activities	8 of 8	8 of 8	7 of 8
2. Three friends	8 of 8	8 of 8	4 of 8
3. Jaime and David	7 of 8	7 of 7	3 of 7
4. Chairs and tables	6 of 8	6 of 6	3 of 6

Table 1. Results by problem.

The analysis of the transcripts from the case studies allows us to build a catalogue of performances which registers how primary students solve word problems algebraically in a spreadsheet environment. In this catalogue, errors and difficulties in applying the SM are highlighted. This fact might generate a negative view of the consequences of using this method. However, we refer to the results from Table 1 to reject that idea and to confirm that the students were able to solve problems algebraically.

Next we proceed to describe briefly some of the difficulties that were visible during the case study. We will use excerpts from the transcripts to illustrate how these difficulties are reflected in the spreadsheet environment.

Difficulties in operating with the unknown

The SM involves inexorably operating with the unknown and seven of the eight pairs demonstrated an ability to do it during the case study. However, in some situations individual performances emerged which revealed reluctance to operate with the unknown.

To illustrate this difficulty we use an excerpt from a pair during the resolution of the problem *Three friends*, which reads as follows: *Three boys won 960 euros. Luis won twenty-four euros less than Juan and a tenth part of what Roberto won. How much did each one win?* Before the analysis of the excerpt, we present an analytical reading of the problem.

Quantities	*Relations*
Total money (T = 960)	$T = L + J + R$
Money won by Luis. (L)	
Money won by Juan. (J)	$J = L + Mlj$
Money won by Roberto. (R)	$R = L \cdot Vlr$
Number by which the money won by Luis has to be multiplied to get the money won by Roberto. (Vlr = 10)	
Extra money that Juan wins over the money won by Luis. (Mlj = 24)	

The pair A-M in the problem Three friends

At the moment of the next dialogue the spreadsheet showed the aspect of Figure 1.

	A	B	C
1	total	960	
2	luis	2	
3	juan		
4	roberto		
5			

Figure 1. Content of spreadsheet before the dialogue between A and M

The following interchange reveals difficulties in operating with the unknown. At the moment in which the dialogue takes place M and A are trying to construct a formula to represent the money won by Roberto (cell B4).

M: Luis… Luis' money divided by ten…

A: M...

M: But we don't know how much Luis has.

A: That is the problem.

In this short excerpt, M proposes to write the wrong relationship $R = L/Vlr$. This proposal is originated because of their inability to reverse the original relationship. However, what we highlight here is that they are not able to construct the formula =B2/10, which would represent what M has verbalised. They reject this possibility because L (cell B2) is an unknown quantity.

Difficulties in representing the relationships obtained in the analytical reading

Reversing the relationships included in the analytical reading of a problem could be a source of difficulties in the resolution process. In fact, the problem *Three friends* was posed to evaluate this difficulty because the resolution of this problem implies the need of reverse at least one of the three relations exposed above. The solver can choose to reverse the additive relation $T=L+J+R$ in which the total is known and the parts are unknowns, or reverse one of the other relations: one additive and another multiplicative. During the case study, all the pairs opted for this second approach, although some of them could not correctly finish the reverse process. Furthermore, the spreadsheet environment restricts the solver's flexibility and forces him to only pose one equation. This is an important difference between the SM and the CM. In CM it is possible to use as many letters and equations as the user wants. By contrast, as we pointed out previously, SM does not allow the students to set a system of equations. In the problem *Three friends*, no pairs tried to reverse any relationship initially, but as result of the failure in solving of the problem, they decided to modify relations or formulas from the analytical reading.

Next we present an excerpt from the pair I-D in which it is shown how the inability to reverse a relationship lead them to spontaneous strategy as the concatenation of relationships.

<u>*The pair I-D in the problem Three friends*</u>

	A	B	C
1	tres muchachos	960	
2	Luis	=B3-24	
3	Juan		
4	Roberto		
5	Total	=B2+B3+B4	
6			

Figure 2. Content of spreadsheet before the dialogue between I and D

D: Can we start other problem? And we think it meanwhile…

Int: Think it a bit more.

D: Ok. Three boys won… won nine hundred and sixty Euros, Luis won twenty-four less than Juan.

I: B4 entre ten. (He modifies the cell B2, writing (B3-24)B4/10 in B2. The spreadsheet reports a syntax error).

D: Something is wrong. Don't put brackets.

I: (I modifies the cell and writes B3-24B4/10. The spreadsheet reports a syntax error).

D: What does it put down in Roberto?

I: Plus. (I modifies the cell B2 by writing B3-24+B4/10. The cell takes the value -24).

At the beginning, D asks to change the problem because they seemed not to find a way to represent the relationships. I verbalized the correct relation L = R/Vlr and tried to write down it in B2. Since this cell is not empty, he decided to concatenate the new formula with the previous one, producing an incorrect formula. The spreadsheet reports a syntax error because no operator separates the quantities 24 and B4. This message leads them to modify the formula without reasoning about the meaning of the represented formula, which concludes with a wrong formula.

Conclusion

We have shown two examples of different types of difficulties when solving word problems in an algebraic way in the spreadsheet environments. The operation with the unknown is a difficulty previously documented in the paper and pencil environment (see, for instance, Filloy and Rojano 1989). This case study offers evidence that this difficulty is not exclusive to the paper-and-pencil environment, and is observed too in the spreadsheet environment. However, the second difficulty addressed here represents a different case. Difficulty in representing relationships is clearly favoured by the features of the SM because the solver has no options to avoid the reverse of one relation. In this context, the solver can try to avoid this process, what lead him to face wrong strategies as the concatenation of relations.

In relation with the principal aim of this research, the results seem to indicate that it is possible by using the SM to bring forward the algebraic solving of word problems before the teaching of the CM.

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