Developing an online coding manual for *The Knowledge Quartet*: An international project

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This paper provides a brief overview of the work to date of an international research team that has worked together since Fall 2011. The team members are mathematics educators and researchers who use the Knowledge Quartet (Rowland et al. 2009) in their work as researchers as a framework by which to observe, code, comment on and/or evaluate primary and secondary mathematics teaching across various countries, curricula, and approaches to mathematics teaching. The countries represented on the team include the UK, Norway, Ireland, Italy, Cyprus, Turkey and the United States. The team has developed a *Knowledge Quartet coding manual* for researchers which is freely available for other researchers to use. This is a collection of primary and secondary vignettes that exemplify each of the 21 Knowledge Quartet (KQ) codes, with classroom episodes and commentaries provided for each code. This work provides increased clarity on what each of the KQ dimensions 'look like' in a classroom setting, and is helpful to researchers interested in analysing classroom teaching using the KQ. This paper provides an overview of the Knowledge Quartet, describes the working methods of the team and offers examples of classroom vignettes that exemplify two of the codes as an indication of what can be found on the coding manual website (www.knowledgequartet.org).

Keywords: mathematical knowledge in teaching; classroom observations; data analysis; primary mathematics teaching; secondary mathematics teaching.

Background

Beginning in 2011 an international team of researchers began working collaboratively to develop a coding manual to support researchers interested in using the Knowledge Quartet (Rowland et al. 2009) in data analysis. The Knowledge Quartet (KQ) is an empirically grounded theory of knowledge for teaching in which the distinction between different *kinds* of mathematical knowledge is of lesser significance than the classification of the *situations* in which mathematical knowledge surfaces in teaching (see Rowland, 2008). It can be considered what Ball and Bass would call a "practice-based theory of knowledge for teaching" (2003, 5). Based on empirical grounded theory and an iterative process of grouping similar codes, four dimensions exist on the KQ framework which are depicted in Figure 1.

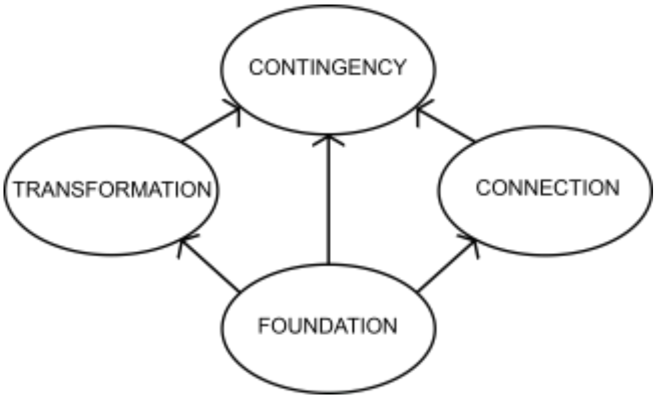

Figure 1: The relationships of the four dimensions that comprise *The Knowledge Quartet* (Rowland, Turner, Thwaites & Huckstep, 2009).

The KQ identifies three *categories of situations* in which teachers' mathematics-related knowledge is revealed in the classroom: transformation, connection, and contingency (Rowland, Huckstep and Thwaites 2005). *Foundation*, which comprises a teacher's mathematical content knowledge and theoretical knowledge of mathematics teaching and learning, supports each of these categories of situations. *Transformation* is the category most similar to Shulman's conceptualization of pedagogical content knowledge, that is, how a teacher takes his/her own content knowledge and transforms it into ways that are accessible and pedagogically powerful to pupils. This category pays special attention to the teacher's use of representations, examples, explanations, and analogies. A second dimension is *connection*, which is whether a teacher makes instructional decisions with an awareness of connections across the domain of mathematics (that mathematics is not, after all, a subject that contains discrete topics) and an ability to sequence experiences for pupils, anticipate what pupils will likely find 'hard' or 'easy' and understand typical misconceptions in a given topic. Since not all aspects of a lesson can be planned for ahead of time, *contingency* is the dimension that focuses on how a teacher must think on his/her feet in unplanned and unexpected moments, such as to respond to pupils' statements, answers, and questions. Within each of the four dimensions there exist four to eight codes which identify specific aspects of mathematics teaching to consider in planning, reflection, and evaluation.

To date, the majority of writing about the Knowledge Quartet has been focused on describing the framework (Rowland et al. 2009) and its origin (Rowland 2008) and has been written to support teacher development of mathematical knowledge in teaching (MKiT). In recent years team members have been using the KQ as a tool to support focused reflection on the application of teacher knowledge of mathematics subject-matter and didactics in mathematics teaching (Corcoran 2007; Kleve 2009; Rowland and Turner 2009; Turner 2009) and working with early-career teachers, pre-service teachers and their school-based mentors, and with university-based mathematics teacher educators, in applying the KQ to the development of mathematics teaching. Through these interactions we have seen that participants often conceptualise one or more of the dimensions of the KQ *in ways that differ from the understandings shared within the research team* which conducted the classroom-based research leading to its development and conceptualisation. Therefore, we have seen that the framework is open to interpretive risks and mis-appropriation. Furthermore, the majority of the writings have focused on explaining the essence of each of the four dimensions rather than identifying definitions for each of the underlying codes. These considerations are supported by Ruthven (2011):

> Essentially, the Knowledge Quartet provides a repertoire of ideal types that provide a heuristic to guide attention to, and analysis of, mathematical knowledge-in-use within teaching. However, whereas the basic codes of the taxonomy are clearly grounded in *prototypical teaching actions*, their grouping to form a more discursive set of superordinate categories – Foundation, Transformation, Connection and Contingency – appears to risk introducing too great an interpretative flexibility *unless these categories remain firmly anchored in grounded exemplars of the subordinate codes*" (85, emphasis added).

Beyond his categorization of generic and content specific aspects of teacher knowledge, Shulman (1986) also identified ataxonomy for the *forms* in which knowledge might be represented, including propositional knowledge, case knowledge, and strategic knowledge. Case knowledge contains salient instances of theoretical constructs in order to illuminate them, and a subcategory of this domain is the use of prototypes. It is within case knowledge that we situate the project at hand.

Project aim

Compared to previous work, this project focused on researchers (not teachers) and expanded KQ use into secondary grades and across countries and curriculum. The aim of the project was to assist researchers interested in analysing classroom teaching using the Knowledge Quartet by providing comprehensive coverage to 'grounded exemplars' of the 21 contributory codes from primary and secondary classrooms. An international team of 15 researchers was assembled. All team members were familiar with the KQ and used it in their own research as a framework by which to observe, code, comment on and/or evaluate primary and secondary mathematics teaching across various countries, curricula, and approaches to teaching. The team includes representatives from the UK, Norway, Ireland, Italy, Cyprus, Turkey and the United States.[4]

Project methods

In Autumn 2011 team members individually examined their data and identified available codes that they could contribute to the project. A template was developed in which the *scenario* of how the episode unfolded was captured. Often this included excerpts of transcripts and/or photographs from the lesson. Then a *commentary* was written, which analyzed the excerpt and explained why it is representative of the particular code and why it is a strong example. In January 2012 each team member submitted scenarios and commentary for at least three codes from his/her data to offer as especially strong, paradigmatic exemplars. Drafts of each scenario were written by individual team members remotely and shared via Dropbox. In February 2012, scenarios were assigned to each team member to review for agreement of the code with the scenario and improvement of the commentary. In March 2012, 12 team

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members met for the *Knowledge Quartet Coding Manual Conference* at the University of Cambridge. Groups of three team members evaluated and revised each scenario and commentary. Throughout the spring and summer, individuals again worked remotely to revise scenarios based on conference feedback.

To date, 55 total scenarios and commentary have been written. These scenarios and commentary combine to form a 'KQ coding manual' for researchers to use. This is a collection of primary and secondary vignettes that exemplify each of the 21 KQ codes, with classroom episodes and commentaries provided for each code. The collection of codes and commentary is freely available online at *www.knowledgequartet.org*. The website provides an overview of the Knowledge Quartet and its four dimensions as well as the work to-date of the international team's scenarios and commentaries describing mathematics teaching across multiple countries, topics, and pupil ages. Additional scenarios and commentaries continue to be added to the website.

Sample scenarios

In order to exemplify our work we will present two scenarios which illustrate two of the codes. First we present an example of the code *Responding to students' ideas* (RSI), a code within the *Contingency* dimension. Second, we present an example of the code *Decisions about sequencing* within the *Connection* dimension.

Responding to students' ideas

The following scenario from a lesson that took place in 2002 (Rowland 2010) is offered here as a kind of prototype of the RSI code. Jason was reviewing elementary fraction concepts with a Year 3 (pupil age 7–8) class. The pupils each had a small oblong whiteboard and a dry-wipe pen. Jason asked them to 'split' their individual whiteboards into two. Most of the children predictably drew a line through the centre of the oblong, parallel to one of the sides, but one boy, Elliot, drew a diagonal line. Jason praised him for his originality, and then asked the class to split their boards 'into four'. Again, most children drew two lines parallel to the sides, but Elliot drew the two diagonals. Jason's response was to bring Elliot's solution to the attention of the class, but to leave them to decide whether it is correct.

This scenario is interesting mathematically, and not so 'elementary' in the context of the Year 3 curriculum. Responding to Elliot's solution, either by teacher exposition, or in interaction with the class, makes demands on Jason's content knowledge, both Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK), in three significant respects. Jason has to decide whether the non-congruent parts of Elliot's board are equal, but also what notions of 'equal' will be meaningful to his 7–8 year-old students, and what kinds of legitimate mathematical arguments about area will be accessible to them.

Decisions about sequencing

Connection as a dimension in the Knowledge Quartet is "concerned with the decisions about sequencing and connectivity" (Rowland, Turner, Thwaites, & Huckstep, 2009, 36). One of the codes within this dimension is *Decisions about Sequencing* (DS), which is concerned with "introduc(ing) ideas and strategies in an appropriately progressive order" (37). We suggest that in the scenario described below, the sequence of the exercises was consciously done by the teachers. The teacher had prepared four different exercises to be done in whole class before the pupils were told to work individually with tasks from the textbook. The lesson objective was to learn to calculate with improper fractions. Placing this lesson in the *Connection* dimension of the KQ and coding it as *Decision about Sequencing* is based on the progression of the exercises discussed below.

The first exercise 6/8+5/8 was presented with both illustrations and numbers. To work out this exercise pupils calculated with numbers, converting improper fractions to mixed number which was illustrated on the figure by pulling shaded pieces from one rectangle to fill up the other on the smart board. In this exercise it was possible to get the correct answer 1 3/8 by counting shaded pieces on the illustration. The second exercise was presented without numbers. The teacher had shaded 5/8 of one circle and 4/8 of another circle and pupils were asked how large a part of the first was shaded and then of the second before they worked out the answer. The answer, 9/8, was converted to 1 1/8 which was illustrated on the figure. This time, it was not possible to pull the pieces. The teacher erased from one circle and filled up the other. When starting the third exercise 3/5+3/5 and 7/10+5/10, the teacher said, "let us try without illustrations". This suggests that he consciously wanted the pupils to calculate the sum of two fractions which added up to an improper fraction, without having illustrations as mediating tool.

The fourth exercise was different from the first three in several ways. It was illustrated with two circles, each divided in quarters. All quarters were shaded and the calculation 2-1/4 was written below. This time the illustrations were not on two sides of an equal sign, the calculation was subtraction, and it started with a whole number. The exercise for the pupils was both to illustrate how much to erase from the figure and also to work it out with numbers.

Hans' choices of illustrations and numbers / only illustrations / only numbers reflected a progression in the lesson. However, the fourth exercise required a subtraction and thus introduced an added complexity. In this example subtraction was thought of as "take away", but could also be comparison. Thus there was a leap, or a missing link. It might have been preferable here to have an exercise that was adding, with one of the numbers as a mixed number and the other as a fraction. Also, whether the exercises chosen were appropriate for developing a solid concept of improper fractions may be discussed. In all exercises the fractions were presented as part of a whole. According to research, fractions as part of a whole is inconsistent with the existence of improper fractions and possibilities for obtaining a well-developed concept of fractions are limited if one focuses on fractions as part of a whole (Kleve, 2009).

Discussion

Both of the proceeding classroom vignettes are offered as exemplars of a given KQ code (RSI and DS, respectively). We readily acknowledge there may be 'room for improvement' and indeed have identified some possible instructional decisions to this end. It was not uncommon in our work for scenarios to seem strong exemplars of one KQ code, and simultaneously lacking in another. Other scenarios were considered strong examples of multiple KQ codes, and in these instances the team worked toward a consensus of which KQ code seemed 'best' exemplified by the scenario.

An underlying question during this project was whether any adjustments would need to be made to the Knowledge Quartet when applied to secondary grades.

Although the content involved is different in upper grades, it was not necessary to add or remove any codes to capture effective mathematics teaching to pupils beyond the primary grades. The majority of the scenarios on the website are from primary grades (which is helpful in that the mathematics do not get to be so difficult as to burden the reader trying to sort out the mathematics instead of thinking about the code), and approximately one-quarter of the codes are from secondary grades and will be helpful to researchers interested in using the KQ to analyze secondary teaching.

The team continues to collect and write scenarios, with the near-term goal of having at least three scenarios per each of the 21 KQ codes. We encourage the use and sharing of the *www.knowledgequartet.org* website as this work provides increased clarity on what each of the KQ codes 'look like' in a classroom setting and is helpful to researchers interested in analyzing classroom teaching using the KQ across a wide range of countries, contexts, and pupil ages.

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