

Doing the same mathematics? Exploring changes over time in students' participation in mathematical discourse through responses to GCSE questions

Candia Morgan^a, Sarah Tang^a, Anna Sfard^b

^aInstitute of Education, University of London, UK; ^bUniversity of Haifa, Israel

The project “The Evolution of the Discourse of School Mathematics” uses the lens of GCSE examinations to investigate changes over the last three decades in what is expected of students in England. We have identified differences in the discursive features of examination questions through this period and now seek to investigate how these differences may have affected the nature of student participation in mathematics discourse. Students have been tested using questions varying in characteristics typical of different points in time. We discuss the design of the test, and present some preliminary results.

Keywords: assessment; examination; mathematical discourse

Introduction

During the past three decades or so in England there have been a number of changes in curriculum and assessment policy and government interventions in pedagogy and assessment practices. These changes form the background to our study, which seeks to investigate how school mathematics has changed over the period.¹ Rather than focusing on the documents and policies that seek to regulate the curriculum, we try to gain insight into the curriculum that students actually experience and the nature of the mathematical discourse in which they are expected to learn to participate. We take GCSE examinations as our window onto these expectations because of the well-documented relationship between high-stakes examinations, curriculum and pedagogy (e.g. Broadfoot 1996)

The study is framed by a theoretical assumption that understands doing mathematics as participating in mathematical forms of discourse (Sfard 2008). Hence we focus analytically on the discourse of examination texts and of student responses. Phase 1 of the project has involved the development of an analytic framework, described in (Tang, Morgan, and Sfard 2012), and analysis of a sample of examination papers. We have no space here for the full details, but present below some key findings that highlight differences found between examinations set at different dates. The main focus of the present paper is Phase 2 of the project, in which we investigate how students respond to examination questions that have differing discursive characteristics. We have constructed and administered two versions of a test, enabling us to compare student responses to ‘parallel’ questions. Below, we describe the design of the test and present some results, raising questions about the nature of the mathematical activity involved in examination success.

Phase 1: Analysis of examination questions

Our analysis of the changing discourse of examinations has made use of a sample of Higher Tier question papers from two of the three English examination boards. The

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sample includes the papers for summer examinations taken in each of 8 years. The years range from 1980 (pre-GCSE) to 2011, chosen to capture major changes in curriculum and examination policy and practice within this time frame. The analysis focuses on how mathematics and mathematical activity are construed and on the role of the student-examinee in this. Here we summarise some of the differences found in the analysis. Fuller details of some of these areas of the analysis have been and will be reported elsewhere (Morgan and Tang 2012; Morgan, Tang, and Sfard 2011; Tang, Morgan, and Sfard 2012).

Human agency in mathematical and non-mathematical processes

In considering the nature of mathematical activity construed in the examination texts, we ask to what extent mathematical processes are presented as being performed by human agents. Across the whole sample, agency in mathematical processes is overwhelmingly obscured. The means by which this is done has changed from passive voice “a tangent has been drawn” to use of relational statements “line AB is a tangent”. A reduction in the use of passive voice constructions has been a deliberate change made by the examination boards, following advice that passive voice lowers reading comprehension. The corresponding increase in relational statements, however, may further increase alienation, as the process itself (in this case “drawing”) is now absent. While human agents are thus largely absent from mathematical processes, they are to be found as actors in everyday practices in contextualised questions.

Contextualisation

The proportion of contextualised questions rose substantially in the first few years of GCSE, falling back in more recent years. Throughout the period, the majority of contextualized questions demand little engagement with the context itself. In the most recent years in our sample (2010 and 2011) we have coded approximately 40% of all contextualized questions as “ritual”, that is, of a standard form widely used as ‘exercises’ in the classroom (Nyabanyaba 2002). This compares to just 8% of contextualized questions coded as “ritual” in 1980.

Grammatical, logical and task complexity

The most recent examinations overwhelmingly use simple one-clause sentences. This is accompanied by a marked decrease in the use of conjunction “and” and implications (“hence”, “then”, “therefore”). Again, the reduction in grammatical complexity follows an explicit policy of attempting to avoid linguistic characteristics known to reduce reading comprehension. However, it is also relevant to ask whether, by avoiding the complexity of sentences with dependent clauses or clauses joined by conjunctions and implications, engagement with some important aspects of the logic of mathematics are also avoided.

We have also considered the complexity of the mathematical activity expected of students by considering the “grain size” (defined as the number of decisions required to achieve a solution) of tasks. The analysis of this factor has not yet been completed, though preliminary results suggest that, while the majority of tasks are of grain size one or two, the proportion of tasks with higher grain size has decreased. In 1987, 16% of tasks involved three or more decisions, while only 8% of tasks had this level of complexity in 2011.

Framing of student response

This part of the analysis concerns the degree of flexibility for students in producing and presenting their answer. In 1980 and 1987, students had a separate answer book, providing no guidance about the extent or shape of the expected response. Later examination papers provide space for answering on the same page as the question and, in many cases, gaps to fill in and lines to place the answer. There is variation between and within years in how the form of student answers is defined. Methods of framing include:

explicit statement of methods (e.g. “Use algebra to ...”). In recent years there is a tendency to ask students to “write down ...” or to “calculate ...” rather than simply to “find ...”. The use of imperatives (e.g. “Write down the amount ...”) rather than questions (e.g. “How many ...?”) also constrains possible approaches to finding an answer.

formatting answers. While in some cases a simple space or line is given for students to write their final answer, in others, the format of the answer is strongly determined. For example, the answer line for a question involving simultaneous equations might be given as “ $x = \dots, y = \dots$ ”. In recent years, the units of the answer are commonly included on the answer line (e.g. “..... kg”).

Phase 2: Testing students

In Phase 2 of the project, we ask what differences the discursive characteristics of an examination question make to the mathematics students engage in when answering. In order to investigate this, we have designed two versions of a test with ‘parallel’ questions involving characteristics typical of examinations set at different dates. In each case, an original question was included on one version of the test, while the other version of the test included a ‘contrived’ adaptation of the question, making use of discursive characteristics found in questions on a similar topic in another year. The questions were distributed to ensure that each test contained four original questions and four ‘contrived’ questions, four with ‘early’ discursive characteristics (1980 – 1995) and 4 with ‘late’ characteristics (1999-2011). This test has been administered to a sample of 158 Year 10 students from six classes in four London schools (all entered for Higher Tier GCSE). Half the students were assigned to each version of the test. In the next sections we present the design and results of two questions that gave rise to some striking differences in the responses to the two versions.

proportion – the ‘Election’ question

In table 1 we present the two versions of the question on proportion, summarising some of main differences structuring our design of the ‘contrived’ question. Table 2 then shows some of the differences in student responses to the two versions, focusing on the occurrence of some different correct strategies.

Table 1: Two versions of the proportion question

original 1987 question	'contrived' question, based on 1999
<p>In the 1983 General Election, 650 Members of Parliament were elected. Shortly before the election, an opinion poll indicated these voting intentions:</p> <p style="text-align: center;">Conservative 38%, Labour 32%.</p> <p>If Members of Parliament had been elected in the same proportions as the poll results, find how many M.P.'s of parties other than Conservative or Labour would have been elected.</p> <p style="text-align: right;">Answer [2]</p>	<p>Before the 2010 General Election, an opinion poll asked voters which party they intended to vote for.</p> <p>The results of the opinion poll were:</p> <p style="text-align: center;">Conservative 38% Labour 32%</p> <p>a) Write down what percentage of voters said they would vote for a party other than Conservative or Labour.</p> <p style="text-align: right;">..... [1]</p> <p>650 Members of Parliament were elected. The proportions of MPs elected for each party were the same as the poll results.</p> <p>b) Calculate how many MPs of parties other than Conservative or Labour were elected.</p> <p style="text-align: right;">..... [1]</p>
<ul style="list-style-type: none"> • <i>increased human presence in (non-mathematical) processes</i>: “voting intentions” vs. “voters said they would vote for ...” • <i>decreased grammatical complexity</i>: two temporal phrases (in reverse order of time!) vs. one; “in the same proportions” (qualifying phrase) vs. “the proportions were the same” (independent single clause sentence) • <i>decreased logical complexity</i>: “If MPs had been elected, [...] would have been elected” (conditional structure) vs. “The proportions were ...” (statement of fact) • <i>decreased grain size</i>: 1x3 vs. 1 + 1x2 • <i>increased explicitness of instructions</i>: “Find” vs. “Write down”; “Calculate” • <i>increased emphasis</i>: use of space to separate points; bold to highlight negation 	

Table 2: Some results for the proportion question

	1987	'new'
fully correct answers	52%	81%
strategy		
calculate 30% of 650	38%	72%
calculate 70% of 650 and subtract	15%	4%
calculate 32% and 38% and subtract both from 650	23%	9%

Unsurprisingly a high proportion of those doing the ‘new’ version of this question have calculated 30% of 650 directly. The structure of the question, divided into two explicit sub-tasks, suggests this approach. Although the overall success rate for the original version is substantially lower, the proportions choosing to use a correct strategy are relatively close (76% vs. 85%). It may be that those attempting the more complex strategies have made more errors in calculation; our analysis has not yet addressed this issue.

percentage change – the ‘Car’ question

In table 3, we present the two versions of the question on proportion. Table 4 then shows some of the differences in student responses to the two versions, focusing on the extent to which the context of the question is taken into account.

Table 3: Two versions of the percentage change question

original 2010 question	'contrived' question, based on 1980
Arwen buys a car for £4000 The value of the car depreciates by 10% each year. Work out the value of the car after two years. £	The value of a car depreciates by 10% each year. If it cost £4000 originally, what would be its value after two years?
<ul style="list-style-type: none"> • <i>decreased human presence</i>: The introduction of “Arwen” in the original 2010 question suggests that the question is about an everyday practice, whereas the contrived question is in what Dowling (1998) calls the expressive domain: clearly school mathematics, not everyday, even though expressed in non-specialised vocabulary. • <i>increased grammatical and logical complexity</i>: simple sentences vs. conditional two-clause sentence demanding hypothetical reasoning • <i>decreased explicitness of instructions</i>: explicit “Work out” vs. question seeking information • <i>decreased specification of form of answer</i>: space for working delimited by line for answer; units given vs. open space 	

Table 4: some results for the percentage change question

	2010	'old'
fully correct answers	63%	48%
Contextualisation		
full sentence answer e.g. “The value after two years would be £3240”	0%	14%
£ sign used in answer	16%	62%
£ sign used in working but not consistently	20%	13%
£ sign used consistently throughout working	9%	18%

Again, the ‘new’ (original 2010) version of the question has a higher success rate. We have not yet investigated the strategies used but wish to draw attention to differences in how students located their responses in relation to the contextualisation of the question. In presenting their answer, 14% of those doing the ‘old’ version (11 students) wrote a full sentence, relating the numerical result to the value of the car. This was not done by any of those answering the 2010 version. We assume that the printed answer line with the £ sign frames students’ response so that there is no perceived need (or space) for other means of signalling the answer (although 16% still wrote their answer with a £ sign elsewhere on the page). Less easy to explain is the use of the £ sign in the working. While similar proportions used it at least once in their working, twice as many of those doing the ‘old’ style question used it consistently throughout.

Discussion

We have chosen to look at results for two questions differing substantially in their success rates between the ‘old’ and ‘new’ versions. However, our main interest is not in levels of difficulty but in the nature of the mathematical activity that students engage in when responding to questions with different discursive characteristics. Following Bezemer and Kress (2008), we ask what is gained and what is lost when the discourse changes.

The analyses offered here focus on students' choice of strategy and the contextualisation of their responses. In the 'Election' question, splitting the task into sub-tasks seems to have directed students towards using a more efficient strategy (and perhaps thereby achieving greater success). However, making a decision about strategy is in itself an important mathematical activity, involving students in exercising agency as mathematical thinkers. In the 'Car' question, in spite of the apparent attempt to make the context more 'relevant' by introducing human activity, the tight framing of the answer space seems to reduce the extent to which students engage with the context, not only in presenting their final answer but also throughout their working. In both cases, students' mathematical activity appears to be affected by subtle changes in the discursive characteristics of the questions. Examination boards have made some of these changes deliberately to increase student access and to prevent "language getting in the way of the mathematics". Our analysis suggests, however, that "the mathematics", which may appear the same, is itself changed for some students.

This analysis of student responses has allowed us to form conjectures about which features of the questions prompt particular types of response. In the next phase of the project we intend to interview students who took these tests to probe more deeply into the ways they participate in mathematical discourse as they read and respond to questions with different discursive characteristics.

References

- Bezemer, J. and G. Kress. 2008. Writing in multimodal texts: A social semiotic account of designs for learning. *Written Communication*, 25 2: 166-195.
- Broadfoot, P. M. 1996. *Education, Assessment and Society*. Buckingham: Open University Press.
- Dowling, P. 1998. *The Sociology of Mathematics Education*. London: Falmer
- Morgan, C. and S. Tang. 2012. Studying changes in school mathematics over time through the lens of examinations: The case of student positioning. In *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* ed. T. Y. Tso. (Vol. 3: 241-248). Taipei, Taiwan: PME.
- Morgan, C., S. Tang, and A. Sfard. 2011. Grammatical structure and mathematical activity: comparing examination questions. *Proceedings of the British Society for Research into the Learning of Mathematics*, 31 3. Retrieved from <http://www.bsrlm.org.uk/IPs/ip31-3/BSRLM-IP-31-3-20.pdf>
- Nyabanyaba, T. 2002. *Examining Examination: The ordinary level (O level) mathematics examination in Lesotho and the impact of recent trends on Basotho students' epistemological access*. Unpublished PhD dissertation. University of the Witwatersrand, Johannesburg, South Africa.
- Sfard, A. 2008. *Thinking as Communicating: Human development, the growth of discourses, and mathematizing*. Cambridge: Cambridge University Press.
- Tang, S., C. Morgan and A. Sfard. 2012. *Investigating the evolution of school mathematics through the lens of examinations: developing an analytical framework*. Paper presented at the 12th International Congress on Mathematical Education, Topic Study Group 28 on Language and Mathematics, Seoul, Korea.