A functional taxonomy of multiple representations: A tool for analysing Technological Pedagogical Content Knowledge

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This study investigates the development of prospective mathematics teachers' use of multiple representations during teaching in technology-rich environments. Forty prospective teachers took part in a teacher preparation programme which aims to develop technological pedagogical content knowledge (TPCK). As part of this programme, prospective teachers participated in workshops during which the TPCK framework was introduced focusing on function and derivative concepts. Various components of TPCK were considered. This study investigates one particular component of TPCK: knowledge of using multiple representations of a particular topic with technology. The content we focus on in this paper is the "concept of radian measure". Two out of forty prospective teachers introduced the concept of radian measure as part of their micro-teaching activities. The data obtained from semi-structured interviews, videos of prospective teachers' lessons, their lessons plans and teaching notes was analysed to investigate prospective teachers' knowledge of representations and of connections established among representations using technological tools such as Cabri Geometry software. We use the framework of "functional taxonomy of multiple representations" which differentiates three main functions that multiple representations serve in learning situations: to complement, constrain and construct. We discuss the educational implications of the study in designing and conducting teacher preparation programmes related to the successful integration of technology in teaching mathematics.

Keywords: technological pedagogical content knowledge, multiple representations, concept of radian measure, prospective mathematics teachers.

Introduction

This study is part of a research project which aims to develop prospective mathematics teachers' Technological Pedagogical Content Knowledge (TPCK) (Mishra and Koehler 2006). TPCK has been a useful framework for exploring what teachers need to know or to develop for effective teaching of particular content. The components of TPCK have been examined by only a few researchers. Among those, Pierson (2001) and Niess (2005) used four components of PCK suggested by Grossman (1990) to define the components of TPCK. In our research project, four of these components were adopted from Grossman (1990). A component regarding multiple representations was added as the fifth component of TPCK:

- Knowledge of using multiple representations of a particular topic with technology
- Knowledge of students' difficulties with a particular topic and addressing them using technology

- Knowledge of instructional strategies and methods for a particular topic using technology
- Knowledge of curricular materials available for teaching a particular topic using technology
- Knowledge of assessment of a particular topic with technology

In the project, TPCK framework with its five components was used to design a course for prospective mathematics teachers. Prospective teachers participated in the activities concerning each component. This study focuses on a particular component of TPCK, namely the knowledge of using multiple representations (MRs) with technology. This paper aims to bring the content dimension into play focusing on the concept of radian measure and investigates how two prospective mathematics teachers integrate technology into their lessons to use multiple representations (MRs) of radians.

Theoretical framework

To investigate prospective teachers' development with regard to using MRs in technology-rich environments, we use the framework of "functional taxonomy of multiple representations" which differentiates three main functions that multiple representations serve in learning situations: to complement, constrain and construct (Ainsworth 1999). MRs might have complementary roles; that is, different representations involve distinct yet complementary information or may support different processes. MRs might also have constraining roles. Representations can confine inferences, allowing one to constrain potential (mis)understandings stemming from the use of another one. Finally, MRs might help students construct a deeper understanding by providing cognitive linking of representations which might eventually lead one to 'see' complex ideas in a new way and apply them more effectively (Kaput 1989).

Ainsworth (1999) describes pedagogical functions that MRs serve as mentioned above and proposes "systematic design principles". She suggests discouraging translation for complementary roles of MRs, to automate translation for constraining interpretation and to scaffold translation for constructing a deeper understanding. Although these principles are speculative, they provide a framework towards the pedagogy of using MRs. This study investigates how prospective teachers use MRs under this framework.

Methodology

Forty prospective mathematics teachers took part in the course. They were enrolled in a teacher preparation programme (which will award them a certificate for teaching mathematics in high school for students aged between 15 and 19) in a state university in Istanbul. As part of this course, prospective teachers participated in workshops during which the TPCK framework was introduced focusing on function and derivative concepts. With regard to multiple representations, the workshops focused on examples of MRs and how to make links between them with or without using technology. Students' preferences for different representations which might be used for different tasks were also discussed. The content we focus on in this paper is the "concept of radian measure". Prospective teachers prepared and conducted their own workshops and discussed the issues of representing radians. After these workshops, two out of forty prospective teachers (Gamze and Mutlu) introduced the concept of radian measure as part of their micro-teaching activities. Both prospective teachers were female and twenty-two years old. The data obtained from semi-structured interviews, videos of prospective teachers' micro-teaching lessons (where they teach to their peers in a computer lab), their lessons plans and teaching notes was analysed to investigate prospective teachers' knowledge of representations and of connections established among representations using technological tools such as Cabri software. To do that, representations either drawn on the board or constructed using the software were recorded. In addition to that, verbatim transcripts of micro-teaching lessons and interviews were coded to reveal how prospective teachers link different representations.

Findings

In this section, findings obtained from the data analysis will be presented in two sub-sections. Each sub-section is devoted to each prospective teacher's lesson and how they use MRs and links between them to teach concept of radian measure.

Findings regarding Gamze's lesson

Gamze started her lesson by giving a brief history of angle. She then defined angle, positive arc and negative arc. She assessed prior knowledge of unit circle by giving various points and asking her peers to find whether they were on the unit circle or not. After defining the angle of 1 degree verbally she explained it graphically on the board. In other words, she used graphical representation for a complementary purpose. She explained 1 radian in a similar way. She first explained it verbally as follows:

> 1 radian is the angle which faces an arc equivalent to the length of a radius (Gamze).

She then drew a graphical representation of any angle other than 1 radian and asked her peers the following question:

> How many radiuses are there in this arc? (Gamze).

She then asked her peers to find out the measure of the central angle facing the whole circle using the Cabri Geometry software. Together with the class, she constructed a circle and found that the measure of the central angle is 6.28 (which is nearly 2π) radians (See Figure 1).

As can be observed from Gamze's approach, she used MRs for constructing a deeper understanding.

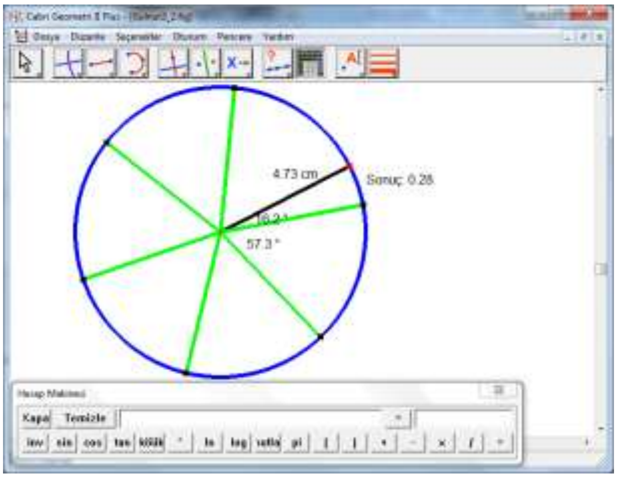

Figure 1 Graphical representation of 6.28 radians by Gamze using Cabri Geometry software

Another way she used such a translation is concerned with the algebraic representation of radian which is arc-length divided by radius, L/R. She used Cabri Geometry software to translate the algebraic representation of radian measure (L/R) to graphical representation. She asked her peers to construct three circles and find the measure of the angle as shown in Figure 2.

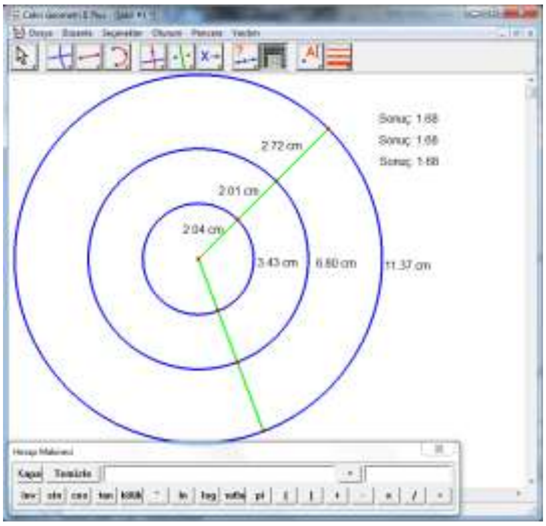

Figure 2 Graphical representation of L/R by Gamze using Cabri Geometry software

She used scaffolding to translate between MRs by asking questions as follows:

> Measure the arcs on these three circles and radiuses of these circles. Are they the same?...What I wonder is whether the ratio of arc and radius is the same?...Ratios are the same. So it's not dependent on the length of the arc. Radian is the ratio of the length of an arc over the length of the radius. So it's L/R (Gamze).

As can be seen from the excerpts and Figure 2 above, Gamze promoted an understanding of the meaning of radian angle. In other words, she used graphical and algebraic representations for constructing a deeper understanding.

Gamze's reflections on her lesson also indicate that she used MRs for constructing a deeper understanding:

> Different representations are important for conceptual relationships. I tried to use multiple representations to promote understanding and translations… I think Cabri Geometry is very appropriate software to show the relationship between

> radian and length of arc. I used Cabri Geometry to emphasise the arc. I wanted to show the relations in a dynamic way (Gamze).

Findings regarding Mutlu's lesson

Mutlu started her lesson by explaining the concepts of angle, directed angle and directed arc on the board. Drawing a circle and a central angle on the circle, she asked her peers the relationship between an angle and the length of the corresponding arc.

> Why do we need radian as an angle measurement when we already have degree? (Mutlu).

She first explained graphically what 1 radian is then expressed it verbally. In other words, she used verbal and graphical representation for a complementary purpose (See Figure 3).

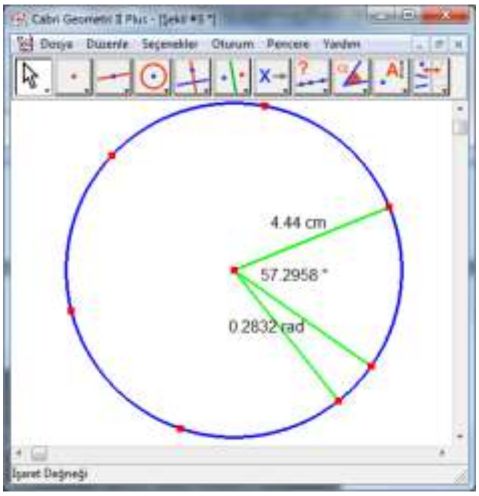

Figure 3 Graphical representation of 6.28 radians by Mutlu using Cabri Geometry software

She then asked her peers to find out how many radians a round angle is:

> If this is 1 radian, then how many radians is the whole circle? Let's look at this together… Yes, 6 radians and there is some left here. It is nearly 0,28… Is this number familiar to you? Nearly 2π (Mutlu)

As can be seen from the excerpts and Figure 3 above, Mutlu, together with her peers, constructed a graphical representation and discovered that there are 6 radiuses and approximately 0.28 radiuses on a circumference of the circle. In that sense, she used Cabri Geometry to construct a deeper understanding of the concept of radian measure. In other words, she translated verbal representation to graphical representation using Cabri Geometry software to construct a deeper understanding of radian.

Mutlu's reflections on her lesson indicate that she also used MRs for constraining interpretation as well as the other two purposes of MRs. She mentioned that radian is generally understood in terms of π (not as an arbitrary real number such as 2). To constrain this interpretation, she said that she asked her peers to construct a graphical representation using Cabri Geometry and found that a round angle is approximately 6.28 radians.

Discussion and conclusion

The analysis of data indicated that both prospective teachers used MRs aiming at a conceptual understanding of radian, i.e. for constructing a deeper understanding. They also used MRs for complementary purposes while only Mutlu used MRs for constraining purposes.

For complementary purpose, both prospective teachers defined concept of radian measure verbally and explained it using graphical representation by drawing a circle and an arc on it. For constructing a deeper understanding of the concept of radian measure, both prospective teachers asked their peers to find out the measure of the central angle facing the whole circle using the Cabri Geometry software (See Figure 1). This requires a translation from verbal to graphical representation which is not automatic but rather should be constructed using the software. The case for using MRs for constraining purpose was observed only in Mutlu's lesson. She mentioned that radian is generally understood in terms of π (not as a real number such as 2). To constrain this interpretation, she asked her peers to construct a graphical representation using Cabri Geometry and showed that a round angle is approximately 6.28 radians.

These observations let us draw two main conclusions. First any successful preparation program for technology integration should provide opportunities for participants to appreciate the contribution of MRs for an effective use of technology aiming at a conceptual understanding. Several studies (such as that of Juersvich et al. 2009) suggest that the links among the MRs are not often established by teachers during instruction. Second, functional taxonomy of MRs provides a theoretical lens to analyse (prospective) teachers' practice of technology integration regarding how MRs can be effectively used in technology-rich environments.

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