

What happens when you divide by 5? Divide what by 5?

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This article developed through working with KS2 teachers from a primary school that had expressed a concern about mathematical attainment in KS2 classrooms. The article aims to explore the use of constructs, as labels, to identify and evaluate decisions, actions and events and the challenges involved in the process of developing powers of analysis and self-awareness to enhance teaching and learning. The wider perspective for this is discussion of the impact of research on practice and of practice on research (Watson 2010), (Hoyles, Joubert and Pope 2012), (Advisory Committee on Mathematics Education 2006). This article is essentially an enquiry into interplay between research and practice and informal action research, in the sense that some change in practice was considered desirable.

Keywords: Teacher as researcher, constructs, language of discourse, emancipation, investigative approaches to learning, generality.

The context

The primary school is a very successful three-form entry school in North London. OFSTED has recently graded the school as outstanding. However, the mathematics results do not match the improvements in English and science. This is despite considerable internal work, collaboration with the local authority and some visits from an external consultant. Hence the request for a more sustained form of CPD. A nine-day programme of support over five weeks for years 3, 4, 5 and 6 was agreed on. The pattern of support was to plan with year teams, team-teach some lessons, together with de-briefing. There were also several progress meetings with the senior leadership team (SLT) and year team leaders attended the year team planning sessions from the SLT. It was agreed that the support would have an action research approach to theory and practice. “Theory without practice is sterile; practice without theory is blind . . . nothing is so practical as a good theory” Lewin quoted in Mason and Johnston-Wilder (2004b, 32). The initial meeting with the SLT defined a focus on developing confidence and subject knowledge with the teachers. There was also a classroom focus on children developing their own understanding of mathematics and contributing to mathematical knowledge and understanding. This article is based on some of the work with the Year 4 team and reports on (accounts for) a few specific moments (accounts of), which seemed to offer particular significance.

The larger canvas

Initial impressions were that the teachers were very professional and hard working. In fact the head teacher said that she thought the teachers did too much work. There had previously been some discussion with the SLT about learning as distinct from teaching and the need to develop independence and confidence for children in

mathematics lessons. These points have some commonality with questions raised by Back and Beauchamp

From this small sample of lessons observed prior to the intervention of the CPD course we have gathered a picture of homogeneous practice across schools and classrooms which delivers the letter of the curriculum: plenty of interactive teaching, differentiated tasks, well structured lessons with clear objectives. However the observations raise serious questions about equity, mathematical challenge and the autonomy of pupils as learners. (2007, 18)

There also seems to be some resonance with the notion of emancipation, a prominent feature of The Humanities Project directed by Lawrence Stenhouse (1971, 154-162). This notion of emancipation has been characterised as having three levels – the level of the student, the teacher and the school. For the student this means to be independent of a teacher's authority, rather like the 'autonomy' referred to by Back and Beauchamp (2007, 18), that is, to "discover and own [knowledge] for oneself" (Hopkins, 2). For the teacher, the development of 'teacher as researcher' was intended to help provide a critical stance toward curriculum structures. Stenhouse went as far as to say that "curriculum research and development ought to belong to teachers" (1975, 142) and additionally "it is not enough that teachers' work should be studied: they need to study it themselves" (1975, 143). Hopkins suggests, "this applies as much to the National Curriculum and 'personalized learning' as it did to the HCP" (2002, 3). In other words, it has relevance today for the classroom and the wider political scene. Although action research requires individuals to look at their intentions, values and beliefs, some writers on action research claim that the most 'emancipatory' action research is collaborative, with personal and political benefits, and according to Hopkins, "Pride of place must go to John Elliott's (1991) Action Research for Educational Change" (2002, 6). Early on in the CPD program, it became apparent that teachers tended to teach generality rather than allow children the opportunity to explore generality. Prestage and Perks report on a similar situation, saying that "a curriculum that is focused on testing" gives rise to "a pedagogic style of 'give the rule, practise the rule' which focuses on practising the finished product for a test" (2006, 69). The teachers at the school relied on a published scheme of work and made regular use of practise SATs questions. Children's ability to answer these practise SATs questions was high on the teachers' agenda. Prestage and Perks refer to a Guardian interview (2006, 3), where Black states 'research from around the world shows that, when the stakes are high, teachers teach to the tests. This produces a short-term, three year uplift in results before they plateau" (2006, 65). This may partly account for the school's issue.

The use of constructs

Constructs were used to help teachers develop the notion of noticing (Mason, 1999 and 2002) and to develop a language of discourse which would help teachers to research their own practice. "Research is when you don't only listen but also hear; don't only look but also see" (ATM 1987, 2). In other words, combining research and practice to 'hear' and 'see' how children responded to investigative approaches, opportunities to reason and generalise, exploration of patterns, relationships and connections within mathematics referred to previously. Some research papers were provided for the teachers to inform the planning process. For example in thinking about the pedagogic purpose or aims of activities, e.g. knowing and understanding and the distinction between *instrumental understanding* and *relational understanding*

(Skemp 1976, 20-26); and in developing pedagogic activity, e.g. the notion of *teaching without telling* (Smith 1986, 47). Other constructs that were readily assimilated by the teachers included:

- *account of* and *accounting for* - used to identify significant moments
- *Do Talk and Record* - teachers were aware that talk needed developing,
- *specialising, generalising, conjecturing and verifying* –the teachers were not comfortable with the word *specialising* and preferred ‘*particular*’

Mason and De Geest found that instantiations from 292 interviews with past MA students offered a range of descriptions of how constructs continued to be used, both for planning activities and for analysing occurrences in the classroom, thus supporting the notion that constructs offer a vocabulary and thinking framework for reflection. (2010). In common with this study, there were changes in teachers’ practice. A difference is that the teachers involved in this research had not followed a masters module and the use and understanding of constructs was developed as teachers worked on activities, to develop a vocabulary and thinking framework.

Working with teachers

Working with the teachers involved working together on activities that provided alternatives to ‘teaching the finished product’ (Prestage and Perks 2006). The activity for this article arose from planning for decimal division with the year four team. I posed the question “what happens when you divide by 5?” There was some initial discussion of “divide what by 5?” The teachers then developed the activity and found links to fractions, remainders, families of division, patterns and generality. At one point, teacher A said “I’m not getting this”. This may have been because it challenged the way she normally worked and challenged her subject knowledge, or at least the ‘connectionist’ aspect (Askew et al. 1997). Brissenden states “a teacher who challenges their pupils usually finds their own understanding is under constant challenge” (1980, 12). There were other examples of such challenges during the time spent working with teachers at the school. It was important that teachers were prepared to recognise and express such challenges. The best resolutions were when the teachers helped each other. This has parallels with aspects of using and refining natural powers, as stated by Mason and Johnston-Wilder “whatever the topic, there is likely to be more actual learning when teachers provoke learners into using these powers than when they closely structure the work for the learners” (2004a, 34). Teacher B remarked “So you are working with us in the same way we could with our children”. Working together on tasks provided opportunities to strengthen subject knowledge, to be explicit about pedagogic strategies and to model ways of working in the classroom. Returning to teacher A, it was the peer support that helped her to adjust her ‘schema’, (Skemp 1979, 219). There was a shift to ‘relational understanding’ – knowing both what to do and why and building conceptual structures, (Skemp 1976, 20-26). The collaborative approach also helped with ‘logical understanding’ - the ability to communicate something and to be able to convince other people, (Skemp 1976, 20-26). Teacher A agreed that the experience was both intrinsically satisfying and created a feeling of confidence. Changes to practice and development of subject knowledge can contain challenges in the affective domain and it may be that collaborative emancipatory action research (Elliott, ibid), is one way that teachers can feel confident in expressing and working with the “I’m not getting this” moments.

Working with children

The same activity that the teachers worked on was then tried with a mixed ability Year 4 class. The objectives for the lesson were given as *look for patterns, explain patterns and use a calculator to help*. The task was stated as ‘what happens when you divide by 5?’ The children were then asked in pairs to respond to the question. This was a pedagogic strategy familiar to the teachers. What was different for them was the nature of the task, in that it was more open and required the children to make decisions and generate examples – specialising and looking for patterns and underlying generalities. Also different for the teachers was a brief plenary session after about five minutes of pair work, to share and discuss the children’s responses to the question. The lesson then developed through the use of ideas generated by the children – using their natural powers (Mason and Johnston-Wilder 2004a, 34). This approach represented a change in practice for the teachers. Teacher C who said, prior to this lesson “I find that I am telling myself to shut up in the classroom” was expressing a change in practice. This was in response to discussion of one of the articles provided for teachers, Questioning Questioning, which suggests that teachers try teaching without telling, Smith (1986, 47). So ‘shut up’ became a metaphor for teaching without telling. Black and Wiliam say that questioning by the teacher can be seen as sharing power and control, and is indeed what counts as acceptable knowledge, with the pupils, (1998). A while after the plenary one child said, with confidence, “My story is in two parts, I do some dividing and then I organise my answers”. This resonates with learners as authors, which is a development of the notion of ‘author/ity’, Povey (1995):

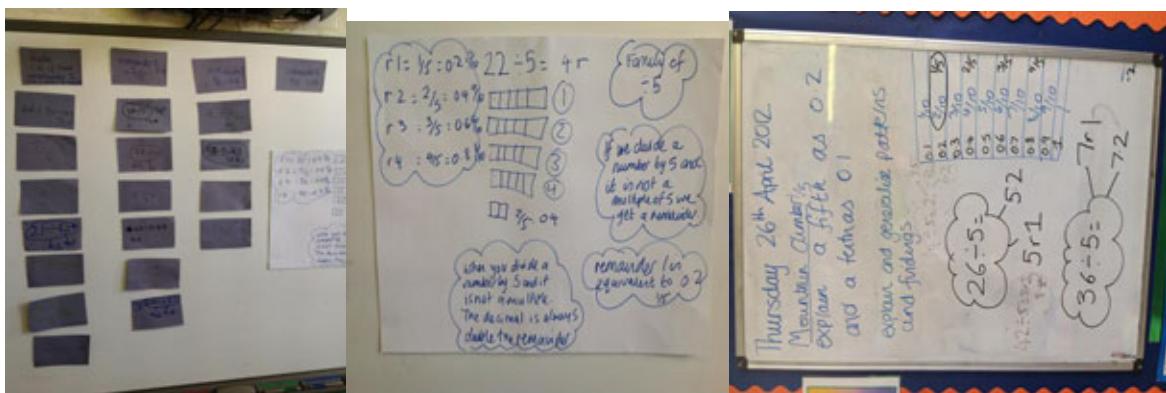
Mathematics can be appropriately construed as narrative because it is ‘an essentially interpretive activity (Brown, 1994, p 141), mathematical expression being thus understood not as objects with internal inherent meaning but as hermeneutic acts uttered within a social space that is contingent upon context, culture and coherence. (Povey et al. 1999)

It would seem that a shift from the “omniscient teacher . . . telling”, to “mutual learners does not reduce the teacher’s role nor his or her authority. Rather, the teacher takes on the additional role of encouraging others to share it” Bruner (1996, 21-22). So the action research with the teachers has perhaps changed practice and an effect, quoting Bruner again, has been to “help the child understand better, more powerfully, less one-sidedly” (1996, 56).

Finished products created by the children

The children initially focussed on dividing by 5. The focus on patterns and the use of the calculator provided motivation and interest. Patterns began to emerge and children posted post-it notes of their results on the white board. The first picture below shows this collection and the second picture shows some detail of their generalisations. The children were able to discuss what was the same and what was different about the collection of post-its, which served to strengthen their understanding of generality. This investigative approach also subordinated the procedural skills required. “. Subordination not only offers practice but also situates a skill within relevant changing contexts ... this is quite different from students being told to follow a procedure and repeat it with no clear understanding” Hewitt (1996, 83). Hewitt also says that that this is different from an investigation, which may only

contain a low level of mathematics content and adds, significantly, that “what is subordinated ... is retained by children over a long period of time (1996, 83).



The third picture shows the teacher's *mountain climber* approach to collecting the children's generalisations. Climbing mountains is generally a worthwhile challenge and specific views can be quite breath taking. The teachers were confident with generalisations in the plural, as they had similar experiences when working on the activity themselves. They were also able to focus on seeing and hearing what the children were doing, talking about and recording, and on the variety of ways the children specialised and generalised. This use of constructs was providing a language of discourse, which made the de-briefing sessions more precise, e.g. in terms of pedagogic aims and pedagogic activities. Hopkins asserts, “pedagogy should be the heartland of classroom research” (2002, 5).

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