The investigation of the relationship between calculus students' cognitive process types and representation preferences in definite integral problems

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This study focuses on the students' cognitive process and preference of representation. We try to find an answer for that problem "How do students' preferences of the multiple representations change in definite integral problems according to type of cognitive process". The participants of the study are 26 undergraduate students who enrolled Calculus II course. The preferences of the student representation determined by the Representation Preferences Test and their type of cognitive process evaluated with Mathematical Process Instrument. Results show that the participants generally prefer algebraic representation. The visual type of participants' preference tendencies are influenced by input representations.

Keywords: multiple representations, cognitive process, definite integral problem

Introduction

A major problem for higher education mathematics research is understanding the difficulties emerging from the nature of abstract concepts. In several questions where the rules, formulas and operations were successfully applied, the fact that students could not comprehend the mathematical ideas behind this process and relate them to different contexts has been a common problem for researchers (Delice and Roper 2006). This problem might emerge from different problem-solving approaches of the students and the teachers (Schoenfeld 1992). The most primitive classification of these differences is related to whether visualisation is used. Krutetskii (1976) categorised cognitive processes into three and in addition to the analytic and visual preferences, drew attention to the existence of harmonic processes which use both preferences together. Analytic learners can easily work with abstract diagrams and tend to use verbal-logical components more than the visual-pictorial components in the problem-solving process (Presmeg 1985). Visual learners tend to present the problems using components they can understand visually. Contrary to analytic learners, visual learners use the visual-pictorial components more than the verbal-logical components (Aspinwall, Shaw and Unal 2005). Harmonic learners, on the other hand, are capable of using both the analytic and geometric approaches together in a well-balanced way. This study investigated students' different ways of thinking in the problem-solving process based on Krutetskii's (1976) concept of thinking structures which are called cognitive process types (Presmeg 2006).

Literature on cognitive process differences include various studies such as its role in the problem-solving process and success at problem-solving (Lowrie and Kay 2001), its relationship with spatial abilities (Kozhevnikov, Hegarty and Mayer 2002), and its effects on the representation transformation process (Haciomeroglu, Aspinwall and Presmeg 2010). While some studies stated that students' differences in the thinking process influenced their problem-solving performance, they avoided making generalisations (Lowrie and Kay 2001). Presmeg, in her studies on the visualisation process and learner difficulties (1985, 2006), used Krutetskii's (1976) classification and concluded that students struggled more in managing visual processes than in managing analytic processes. Some studies reported that analytic learners were more successful at problem-solving than visual learners (Lean and Clements 1981). However, this could be related to the difficulty level of the test and students' prior experiences (Lowrie and Kay 2001). Learners could have used visual solution strategies in order to understand data in the problems which were new and believed to be complex, and could have been inclined to use analytic strategies for easier problem types. In other words, the difficulty of the problem can affect the context and the representation types can affect the solution strategies (Sevimli and Delice 2011). Therefore, it is necessary to explore the relationship between preferred multiple representation types and differences in cognitive processes in the problem-solving process. Like cognitive process differences, multiple representation preferences are important individual qualities to be considered. Multiple representation theory can be used to relate concepts to different contexts and to assign different meanings to concepts and thus helps to deal with the abstract nature of mathematics and contributes to the comprehension process. Developing technologies allow easier access to multiple representations, and awareness of varied representations provides means to state a mathematical idea or relationship in different ways. The literature review reveals that the research is usually related to the effects of external factors (curriculum, technology etc.) on the use of multiple representations (Keller and Hirsch 1998, Kendal and Stacey 2003). The type of representation in the problem statement and the structure of the problem are also important in the identification of representation preference tendencies in the problem-solving process (Lowrie and Kay 2001). Thus, when investigating the relationship between cognitive differences and representation preference tendencies, the structure of the problem and students' preference flexibilities should be considered (Goldin and Kaput 1996).

This study was designed using a method that allowed students to choose different representations to solve definite integral problems. Therefore, regardless of students' problem-solving processes or performances, the relationship between preferred representation and differences in cognitive processes was investigated. This study is original in its investigation of the cognitive processes (internal effects) involved in the definite integral problem-solving process through multiple representations. The research question was "When choosing a representation, are cognitive processes affected by the representation types used in the problem statement?" Hence, the effect of cognitive process differences on representation preference tendencies was also investigated in terms of the representation used in the problem statement. This study is significant in that it was constructed with a consideration of cognitive differences in higher education classrooms and that it was focused on a research question which examined the relationship between the theoretical frameworks of cognitive processes and multiple representations.

Research methods

In this research, each cognitive process type was accepted as a case and each case, as well as the relationship among the cases, was explored as a whole (Yin 1994). The participants were 37 first year mathematics education students selected using purposeful sampling technique which is a non-probability sampling method. Data was collected during the spring term of the 2010-2011 academic year at a state university.

All of the participants had taken the course Calculus I in the previous term and at the time of data collection they were taking the course Calculus II.

Data collection tools

In this research, two different tests were used for two different purposes. The Mathematical Process Instrument (MPI) was developed by Presmeg (1985) based on Krutetskii's (1976) thinking structures theoretical framework. The participants were expected to identify their way of thinking during the mathematical problem-solving process. The instrument was used to categorise the participants as visual, analytic and harmonic according to their cognitive process types in the mathematical problem-solving process. The instrument also included a Questionnaire section with questions and a Solution preference key that presented different potential solutions to each question. The participants first took the Questionnaire section and their answers were analysed. Subsequently, the solution preference key was handed out and the participants were asked to choose the answer that was similar to their own solution method. If their answer was not given in the options, they were asked to choose the option given for original answers. The instrument had high validity and reliability scores and it had been frequently used in prior research (Presmeg 1985, Aspinwall, Shaw and Unal 2005).

The second data collection tool was a Representation Preferences Test (RPT) which was developed by the researchers. RPT was designed in order to determine participants' tendencies to use different representations for the definite integral and was used in earlier studies (Sevimli and Delice 2011). By representation preference, the participants were expected to identify the representation type which they believed would facilitate the process of solving a given definite integral problem. The test consisted of nine items each of which represented a different objective of the course. There were input and output representations in each of these questions. Input representations were given as part of the problem and output representations were the ones which the solution of the problem would include.

Data analysis

Participants' cognitive process types were determined by a standard deviation value added to and subtracted from the average. The participants were thus grouped into three categories of visual, harmonic and analytic according to their cognitive process types. On the other hand, each participant's representation preference for each question was analysed separately within categories of numerical, graphical, algebraic or mixed. A mixed representation is said to exist when more than one representation is used in relation to the same question. The effect of differences in cognitive processes on representation preferences was analysed by coding the cognitive process of each participant and representation type preferred for each problem. The data was analysed digitally using SPSS and descriptive statistics.

Results

MPI results

The results of MPI showed that the maximum score was 24; the minimum score was 6 and the average score was around 14. Given that the maximum score in the instrument was 36, the average score of the participants was low. MPI scores were used in order to determine the participants' cognitive process types. Consequently, the results revealed that 27% of the participants were Analytic, 54% were Harmonic and 19% were Visual.

The case of input representation

The results indicated that analytic participants mostly preferred algebraic representations when the input representations were algebraic (57%). Analytic participants, who used other representation types with similar percentages when input representations were numerical, predominantly preferred algebraic representations again when the definite integral problems were presented with a graphical representation (54%). The participants mostly preferred algebraic representations for this type of problems and thus the representation type in the problem statement was observed not to influence the analytic type of participants' preference tendencies much (Table 1).

	%	**Representation Preferences**			
		Algebraic	**Numerical**	**Graphical**	**Mixed**
Input Representation	**Algebraic**	57	9	20	14
	Numerical	35	21	20	24
	Graphical	54	-	31	15

Table 1: Analytic participants' representation preferences according to input representations

Preference tendencies of harmonic type participants according to input representations showed high percentages of algebraic representations (Table 2). When a problem was stated in algebraic or graphical representations, harmonic participants predominantly preferred algebraic representations. When input representations were numerical, the preferences changed to numerical (40%) and graphical (31%) representations. This type of participants preferred mixed representations, where two representations were jointly used, to a lesser extent. The results presented in Table 2 in relation to harmonic type participants' behaviour according to input representations indicated that input representations did not directly affect preference tendencies, algebraic representations were preferred the most and this type of participants used numerical representations more.

	%	**Representation Preferences**			
		Algebraic	**Numerical**	**Graphical**	**Mixed**
Input Representation	**Algebraic**	53	2	34	11
	Numerical	13	40	31	16
	Graphical	48	14	21	17

Table 2: Harmonic participants' representation preferences according to input representations

According to MPI scores, representation preferences of visual participants who got high scores were shaped by the input representation in the problem. When the problem was stated with an algebraic representation, algebraic (66%); when stated numerically, numerical (42%); and when stated graphically, graphical (55%) representations were preferred more (Table 3). It is difficult to suggest that a single representation type was predominantly preferred in this group in general. Participant preferences changed according to the representation type in the problem and thus input representations affected preference tendencies. Moreover, given the general answers to RPT, visual participants were observed to prefer algebraic representations less than the other participants.

	%	Representation Preferences			
		Algebraic	**Numerical**	**Graphical**	**Mixed**
Input Representation	**Algebraic**	66	4	20	10
	Numerical	16	42	26	16
	Graphical	16	4	55	25

Table 3: Visual participants' representation preferences according to input representations

Discussion

The main research question in this study was related to the effect of input representations in the problem statement on learner preferences. Harmonic participants preferred numerical representations when input representation in the problem was numeric. Otherwise, harmonic and analytic participants were observed to have similar preference tendencies in relation to the input representation and to prefer algebraic representations regardless of the representation used to express the problem. On the other hand, visual participants' preferences changed according to the input representations. Visual participants believed that if the problem was presented algebraically, algebraic representation; if presented graphically, graphical representation; and if presented numerically, numerical representation would facilitate the process more. Furthermore, preferences parallel to the input representations in the problems were mostly observed with highest percentages for visual participants. Kendal and Stacey (2003) also stated that input representations in a problem statement could affect preferences. The fact that the effect of input representations were marked for visual participants in this study revealed that visual perception of the problem was important for these participants. Visual learners believed that during the mental manipulation of the visual input, the solution could also be visual and their visual skills influenced their representation preferences.

Analytic and harmonic participants predominantly preferred algebraic representations regardless of the input representations. This indicated that for visual participants visual perception of the problem was more important in the solution process. Moreover, for the three types of cognitive processes, the most preferred representation type was algebraic representation. The findings revealed that the methods and approaches used in the traditional teaching process affected students' problem-solving behaviour. This study is significant in the identification of students' ongoing solution tendencies. Teachers and programme developers should design course content and teaching approaches with a consideration for the differences in the learning environment.

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