Explicit and Implicit Pedagogy: variation theory as a case study

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Variation theory, promoted by Ference Marton and colleagues (Marton and Booth 1997, Marton and Trigwell 2000, Marton and Tsui 2004, Marton and Pang 2006) and augmented by Watson and Mason (2002, 2005) has roots going back at least as far as Isocrates (Papillion 1995). It proposes that learners must experience variation in the critical aspects of a concept, within limited space and time, in order for the concept to be learnable. But the presence of variation does not in itself guarantee that that variation will be experienced. As Kant implied, a sequence of experiences does not guarantee an experience of that sequence. *Implicit variation theory* assumes that the presentation of variation is sufficient in order for learners to learn what is intended, whereas *explicit variation theory* incorporates some degree of explicitness in the interaction between teacher and student.

The conjecture is proposed that tension between explicitness and implicitness is present in all attempts both to implement theories in practice and to justify or analyse pedagogical choices using theories, of whatever kind.

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Introduction

Much of mathematics education research into teaching and associated learning can be seen as an attempt to delineate conditions under which intended learning will take place. To improve learner experience and to improve the effectiveness of teacher-learner interaction is to implement practices that have been shown to be effective. Unfortunately, in almost every case, neither the practices themselves nor the milieu (Brousseau 1987) nor even indeed the didactic contract (Brousseau 1984) are described sufficiently precisely to enable someone else in some other situation to replicate the relevant actions and conditions.

My own position is that such a programme is unrealistic if not untenable, since cause-and-effect is an inappropriate mechanism either for analysing or for explaining how human beings behave creatively (as opposed to habitually where it may be an appropriate mechanism). My justification is that although human behaviour is largely mechanical, in the sense of the automatic activation of habits in reaction to situations, human beings are also capable of exercising will. As Jonathon Swift (1726) put it, “man is not a rational animal, but an animal capable of reason”. It is true that Skinner (1954), Pavlov (1924/1963) and colleagues showed conclusively that animal behaviour, and (much of) human behaviour is trainable through stimulus-response management, and Norretranders (1998) has argued convincingly that the assumption that human beings act through the activation of conscious intentions is (largely) an illusion. However the wilful component of the human psyche is ever present.

The issue of automaticity, of distinguishing between automatic, habitual reaction and intentional, considered response lies at the heart both of implementing theories and of using theories to justify pedagogic choices in mathematics education. A teacher adopting an *implicit* stance assumes that actions informed by and derived from a theory will have a predictable outcome, while a teacher adopting an *explicit* stance assumes that for many if not most learners it will be necessary to draw attention explicitly to what is available to be learned. Of course few teachers are at either extreme, but my contention is that this tension plays out in every classroom every day.

One manifestation of an implicit-explicit distinction was the strong suggestion (never actually statutory in the UK) that lesson objectives should be made explicit at the beginning of each lesson, and that these objectives should be returned to at the end. The assumption seems to be that human beings need to 'know where they are going' in order to get somewhere; that in order to learn from experience it is necessary to know what that experience is intended to be. It contrasts strongly with the underlying approach to pedagogy manifested in Eastern martial arts (Herrigel and Hull 1985) as manifested in the Karate Kid films (1984), where knowing what the experience is supposed to be would nullify or even negate that experience. The same holds for both Zen and Sufi teaching traditions (Harding 1961, Shah 1964, Cleary 1993).

Experience trying to specify learning objectives in the 1970s at the Open University suggested to me that expressing what was as yet unknown, in language the learner can appreciate before they have that experience, is highly problematic and that learners rarely engaged with the preliminary agenda setting, placing trust in the teacher-author to provide suitable tasks and resources. Further probing suggested that, at least for many Open University students, the question "why are we doing this?" arose predominantly when success seemed elusive or out of reach; when things are going well, there is no impetus to question the why's and wherefore's. Furthermore, and much more importantly, Dick Tahta (1980) distinguished between the *outer task* (what learners are invited to engage in) and possible *inner tasks* (what learners might experience or come up against). If someone knows or expects what they are 'supposed to' or are 'likely to' experience, they can either act so as to block that experience, or prematurely imagine that they have undergone it. This nullifies the intentions of the task, rendering it much less likely that suitable learning will be promoted. This raises the question of what constitutes learning.

What then is learning?

Marton (in Marton and Booth 1997) proposed that something is available to be learned only when it has been experienced through being varied. Watson and Mason (2005) went further in the context of mathematics to propose that to learn concepts in mathematics is to become aware of aspects that can vary in examples while remaining as examples, and to become aware of over what range that change can take place. This gives rise to the constructs *dimensions of possible variation* and *range of permissible change*. The word 'possible' is intended as a reminder that what the teacher is ware of as a dimension of variation the students may not be, and different students may stress different aspects. Indeed, different teachers, and even the same teacher at different times, may stress different aspects. The word 'permissible' is intended s a reminder that students and teacher may vary considerably on the universe of discourse, on the objects being considered.

For example, shown a drawing depicting an 'angle', some students may attend to the length of the arms or the implied gestalt of the space between the arms in relation to measure. Through exposure to the same 'specific angle' with arm lengths varying, and acted upon by translation and rotation of the whole diagram, 'specific angle' is what remains invariant under all of these actions; exposure to one or both arms varying so that the 'specific angle' itself changes 'angle' is what remains invariant as the 'angle itself' changes magnitude. Such multi-level invariance occurs whenever the concept being considered is associated with a measure.

As examples of the range-of-permissible-change, when the word 'number' is used, students may find themselves with access to small whole numbers while the teacher is thinking of all integers, all rationals, all reals, or beyond. The word 'quadrilateral' may bring to mind a rectangle or parallelogram, or a convex figure, rather than stimulating access to the wide range of possibilities, including extreme examples (e.g. very very large edges or area, very small area, one angle close to 0 in value etc.).

The issue of degrees of explicitness of awareness of dimensions of possible change and ranges of permissible change remains subtle. Often it is the case that even without any explicit articulation or expression, learners incorporate (literally) these awarenesses. On the other hand it is well known that learners make natural but incorrect or incomplete assumptions, as evidenced for example in the construct of *figural concepts* (Fischbein 1993).

The way that variation is employed explicitly in sets of exercises is beautifully illustrated by the exercises in Krause (1975), but something similar can be found in many sources such as the first set of exercises in George Albert Wentworth's *First Steps in Algebra* (1894, 10). Before even introducing negative numbers, the task is to 'remove the parentheses and combine' (not simply get the arithmetic answer).

1. $9 + (3 + 2)$.	12. $7 - (3 - 2)$.
2. $9 + (3 - 2)$.	13. $9 - (4 + 3)$.
3. $7 + (5 + 1)$.	14. $9 - (4 - 3)$.
4. $7 + (5 - 1)$.	15. $7 - (5 - 2)$.
5. $6 + (4 - 3)$.	16. $7 - (7 - 3)$.
6. $6 + (4 - 3)$.	17. $(8 - 6) - 1$.
7. $3 + (8 - 2)$.	18. $(3 - 2) - (1 - 1)$.
8. $9 - (8 - 6)$.	19. $(7 - 3) - (3 - 2)$.
9. $10 - (9 - 5)$.	20. $(8 - 2) - (5 - 3)$.
10. $9 - (6 - 1)$.	21. $15 - (10 - 3 - 2)$.
11. $8 - (3 - 2)$.	

Note the variation between adjacent exercises, and adjacent sets of exercises. What is the student to make of this?

The fact that the arithmetic is simple means that students can check their answers (by doing the bracket first and then completing, or by using their parenthesis-free version. Note the way in which the shape or form varies from exercise to exercise.

There is no point in setting "odd numbered exercises for homework" since it is the relationship between exercises that really matters (Watson and Mason 2006). Might it be sufficient for students to work through these (one by one, with little or no attention to how they are related)? Or does learning-effectiveness depend on students spotting connections, or even on students articulating the 'rules' that they are experiencing being instantiated. In other words, who does the generalisation and how conscious does it need to be? The practice in Wentworth's books is to provide two 'examples' and then to state a general rule, followed by structured exercises like these.

An even more structured approach can be found in Tuckey (1904). The extract here is adapted from the original:

> Multiply each of the terms in the top row by each of the terms in the bottom row, in pairs:
>
> $x + 1 \quad x + 1 \quad x + 2 \quad x + 3$
>
> $x - 1 \quad x - 1 \quad x - 2 \quad x - 3$

The variation available in the form of the answer (two terms or three) and relationships between the constants in the factors and the coefficients in the product seem likely to provoke conjectured generalisations. But might it be possible for some learners simply to record the answers (however obtained) without being alert to patterns in the answers? Might it be possible for a teacher to be unaware of the benefits of learners looking for quick ways to get all the answers and so to encounter, perhaps even to articulate, relationships? How explicit might a teacher need to be? And might this explicitness vary from situation to situation?

The notion of variation has deep roots. Papillion (1995) finds resonances in Isocrates (436-338 BCE) and coins the term *hypodeictic rhetoric*:

> … rhetoric that uses praise and blame -- mostly praise -- and a strong sense of comparison to set out situations as examples for those around to learn and from which those around could create policy for the future (1995, 158)
>
> … joining narration with argument through praise by comparison (ibid, 159).

Confucian culture education often makes explicit use (Liu Yizhu 2004) in a manner similar to Wentworth and Kraus, and it can also be found in Russian textbooks. Davidov (1972/1990, 6) proposed that

> “the completeness and adequacy of the generalization depend on the breadth of the variations of the attributes that are combined, on the presence in the raw material of highly “unexpected” and “unusual” combinations of the common quality with the concomitant attributes or form of expression."

Thus variation seems to be considered to be effective when it generates disturbance of some sort for the learner, which fits with the theory of *cognitive dissonance* of Festinger (1975).

Central Issue

Using the word *suffer* in its original sense of ‘to experience’, is it sufficient to ‘suffer’ variation, for learning to take place? More generally, for any theoretical construct used to inform or analyse pedagogical choices, is it sufficient to ‘suffer’ a sequence of actions in a succession of activities, for learning to take place?

Alf Coles (personal communication) pointed out that there are potentially levels or degrees of explicitness. For example, the Wentworth task on parentheses might be expected to have the effect of constantly bringing the learner ‘up short’ due to continued variation in at least one aspect of the exercises, in contrast to repetitive but unstructured exercises. The Tuckey exercise could be seen as repetitive, but it could be that the strength and value of the exercise lies precisely in the (assumed natural desire on the part of the learner) to reduce effort by finding patterns to predict answers.

This matches my own analysis of interactions between teacher, learner and mathematics (Mason 1979) in which, for example, rather than denigrating exposition as an outmoded mode of interaction, arranging that learners are in a position to hear/see what is being said/displayed can release the power available in effective exposition. This takes effort on the part of teachers as well as learners: in order to experience the clichés “I see what you are saying” and “I hear what you are saying”, it is necessary for the teacher to be clearly ‘saying and displaying or pointing out what they see/hear’, and for the learner to have had recent experience that prepares them. For example, Bob Burn (2008) presented a collection of pre-lecture tasks which have been used to prepare learners to make sense of and to appreciate a subsequent lecture, as well as tasks for following up such experience so that learners encounter significant dimensions of possible variation and associated ranges of permissible change.

Exposition is but one of six different modes arising from an analysis of the initiating, responding and mediating roles played by three impulses (teacher, student and mathematics) within a milieu (Mason 1979). Although these interactions can take place spontaneously, one of the components must take the initiative, one must respond, and one must mediate in order for anything to happen. In other words, appropriate conditions are essential. These act as catalysts and contexts which hold and direct energies so that the action can take place, rather than exerting a cause-and-effect structure on the activity.

Many authors have drawn attention to the issue of explicitness and implicitness. Chazan and Ball (1999) point to the importance of teachers using explicit prompts. Goos (2004) also argues that such prompts are important in order to connect with students' thinking and to point to how that thinking needs to be extended—especially when students appear locked into one solution strategy, or when whole-class discussion appears to be making no progress.

It seems therefore that no matter how detailed the constructs used to inform specific pedagogic strategies and particular didactic tactics, the complexity of human learning calls for artful sensitivity on the part of the teacher in relation to learners, as to when and in what ways to shift attention from the doing of tasks to becoming aware of what was effective and what ineffective in the doing of those tasks, what mathematical themes and concepts have been involved, and what more could be explored about them. In other words, it is part of the art of teaching to make choices about appropriate degrees of explicitness about what it might be useful for students to have 'come to mind' in the future in similar situations.

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