Gattegno's 'powers of the mind' in the primary mathematics curriculum: outcomes from a NCETM project in collaboration with "5x5x5=Creativity"

Alf Coles

University of Bristol Graduate School of Education

In this paper I report on the outcomes of a 'Mathematics Knowledge Network' (MKN) project, aimed at developing rich tasks in the primary curriculum. The work was funded by the National Council for Excellence in Teaching Mathematics (NCETM) and carried out in collaboration with the Arts based charity "5x5x5=Creativity". The approach to the project was informed by Gattegno's ideas on the 'powers of the mind'. I report on the answers to three questions: can KS2 students access rich tasks designed for KS3, using their 'powers of the mind'?; what is the potential for KS2 students' capacities to use and apply mathematics?; what is the potential for collaboration between mathematics and arts based education?

Keywords: Gattegno, 'powers of the mind', creativity

Background

In this paper I report on the outcomes of a 'Mathematics Knowledge Network' (MKN) project, aimed at developing rich tasks in the primary curriculum. The work was funded by the National Council for Excellence in Teaching Mathematics (NCETM) and carried out in collaboration with the Arts based charity "5x5x5=Creativity". The motivation for the project came from a headteacher who wanted support in developing a more creative approach to teaching mathematics in school C, where he worked.

The project comprised twelve weekly sessions focused on developing skills of using and applying mathematics, partcularly being organised and systematic, with two classes of year 3/4 students (aged 7/8). I taught eleven of these sessions which were jointly planned with the two class teachers, who helped out in the lessons; there were reflection meetings each week to discuss, evaluate and plan the following week. In the last two sessions, following input from "5x5x5=Creativity", we gave all students a 'Learning Journal' in which to document their work, organise what they did, and take forward any questions they still had.

Research questions

We began this project with three key questions, the first one linked to my previous experience of working with rich tasks at KS3, 4 and 5.

- could KS2 students access rich tasks designed for KS3 and beyond?
- what is the extent of year 3/4 students' capacities for Using and Applying mathematics?
- what is the potential for collaboration between mathematics and arts based education practices?

Before getting to these questions, I set out the background theoretical ideas, derived from Gattegno, that informed the project design and implementation.

Theoretical framework: Gattegno's 'powers of the mind'

Through studying the early language acquisition of children Gattegno (1971) analysed four common "powers of the mind" (1971, 9) possessed by everyone who is able to master their mother tongue.

- the power of extraction - finding "what is common among so large a range of variations" (1971, 10)
- the power to make transformations – e.g., my Dad ~ my Mum's partner ~ my sister's Dad ~ my uncle's brother
- handling abstractions – e.g., any noun is a label for a general set of objects
- stressing and ignoring – e.g., focusing on one aspect of perception to the exclusion of others.

The heart of Gattegno's approach to education is to offer activities and contexts that allow learners to continue to access these four powers. I provide some more detail below on how this framework translates into practical implications for the classroom.

'The power of extraction' and 'stressing and ignoring'

These two powers combine to mean that *all* students can respond to being offered more than one example, image or action, and being asked to find what is the same or what is different (see Brown & Waddingham 1982; from where I took both activity starting points used with the Y3/4 classes). Given a pair, or wider range of items to compare students can extract common features by stressing some aspects of what they see or do, and ignoring others. At the start of one of the activities I offered the year 3/4 class, I drew on an interactive whiteboard the following image.

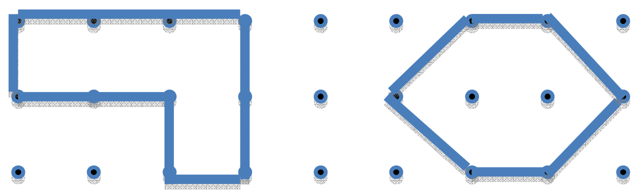

Figure 1: Introduction to working on Pick's Theorem

I began with the statement 'these are both four square shapes', and the invitation 'someone come and draw another, different four square shape'. I was aware that these students would not have been taught 'area' yet. I assumed that students would be able to use the power of extraction to see a common feature of the two shapes I drew, as was indeed the case in both classrooms. The starting statement 'these are both four square shapes' is deliberately not an explanation; the invitation is for students to begin *using* the concept of a 'four square shape'. I offer, as teacher, two examples of use; by attending to what is the same and what is different (stressing and ignoring) students will be able to continue such use. Only later, and perhaps never, will there be discussion of what it means to be a 'four square shape'. I am aware at KS3 that a slightly mysterious beginning, where there is patently some 'sense' of the situation to be made by the students can be engaging. I am also wanting to convey a message *about* our work together, that I am not going to be the one who provides explanations, and that there is work to do, which I assume everyone can do, using the powers of extraction and stressing and ignoring.

'The power to make transformations' and 'handling abstractions'

The use of notation can never be far from the doing of mathematics. We know from students' facility with their first language that there is no issue in the use of arbitrary labels (such as 'dog') to stand for a potentially infinite variety within a set of similar things. The key to accessing this power to handle abstractions is again that they are *used* rather than defined. Having set up a notation, students' use of the power to make transformations can be accessed in creating or provoking questions linked to the notation. For example, in starting the first activity I worked with students to find the number of ways they could arrange three red and two blue books on a shelf. I wrote one combination and invited students to contribute the rest. When it seemed as though there were no more different possibilities I introduced the (standard) notation $\binom{5}{2} = 10$ to mean that with 5 books in all, of which 2 were blue, the total number of combinations found was 10. With this notation we were able to talk about students trying (5,1), (5,3) or different numbers of books (4,0), (4,1), etc. Students wrote their answers on a board at the front of the class and were able to see patterns or make predictions (e.g., how many combinations if the bottom number is zero or one). I am not wanting to discuss other possible notations, as the choice is essentially arbitrary (Hewitt 1999). I want students to use the notation I offer to help organise their work and notice patterns in their answers in relation to this notation.

Could KS2 students access rich tasks designed for KS3 and beyond?

The answer to this question was an emphatic 'yes'. There seemed evidence for the universality of Gattegno's claims for his 'powers of the mind'. Students assessed as operating at level 1 were able to access both tasks we offered. A Teaching Assistant who supported one of the groups wrote (in a personal communication to me):

> The recordings that the children did, both on the paper an in the journals, highlighted the importance of allowing children to convey their own thoughts and understandings of the subject. I was able to witness the way that they explored their ideas on paper, after an introduction to the 'problem', and discover answers and patterns for themselves.

This teaching assistant worked with a table of students who had the lowest prior attainment in the group; I take her statement as evidence that these students were able to access the tasks through the 'introductions' – as is confirmed by the range of different routes students took in working on the problems. In other words the tasks, developed for students in year 7 and beyond, were accessible to the full range of a mixed ability year 3/4 class.

What is the extent of year 3/4 students' capacities for Using and Applying Mathematics?

Changes in students' organisational skills were particularly evident on the red/blue book task. By the second session almost all students had moved from writing down solutions without discernible pattern, to using some kind of system. In Fig 2, there are four cases worked on by the student, and it can be seen the student has used slightly different systems. In three cases the total is correct, but some combinations have been missed in the (6,3) case. However it is possible to observe in all cases a common strategy of keeping some aspects the same and moving others to generate successive lines. This was a typical feature of students' work.

Despite my convictions about the relevance of Gattegno's ideas and the untapped potential often claimed existed in children, I was struck by the quality and extent of the 7/8 year old students' capacity to organise their work, make and test predictions, use algebra and reason about the answers they found – all skills found at high levels with the National Curriculum strand of Using and Applying Mathematics.

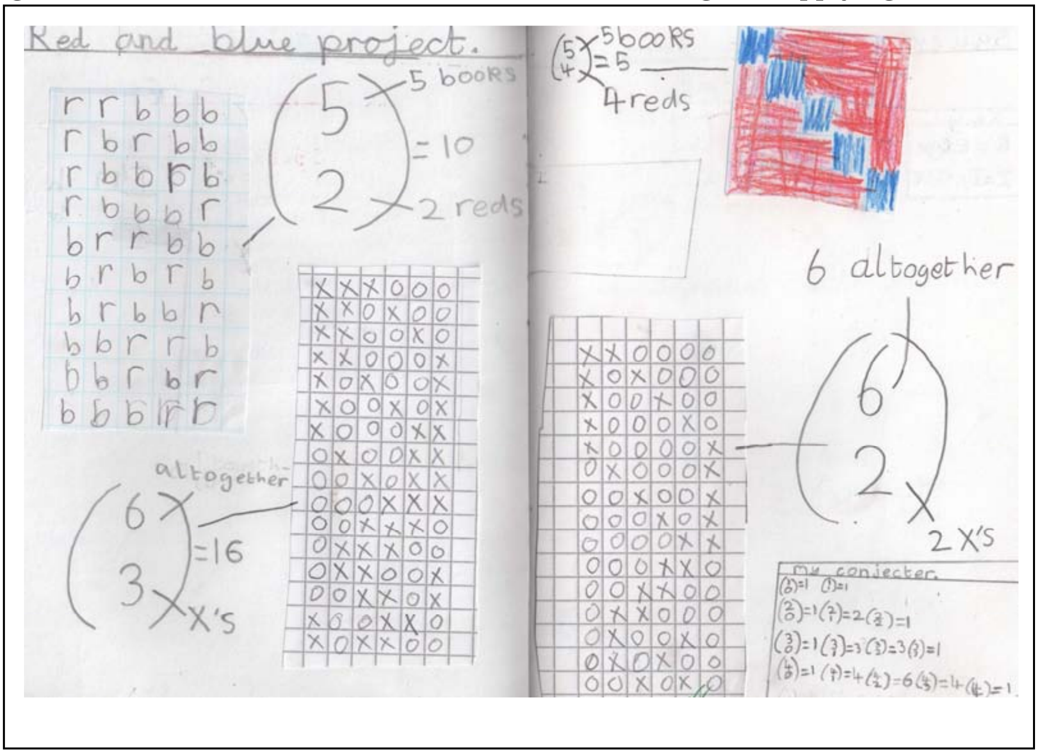

Figure 2: A student's work on the red/blue problem

There was evidence of pattern spotting in almost every student's journal. There were many examples of students making and testing predictions. The most striking (due to the sophistication of the prediction) was the following (see Fig 3).

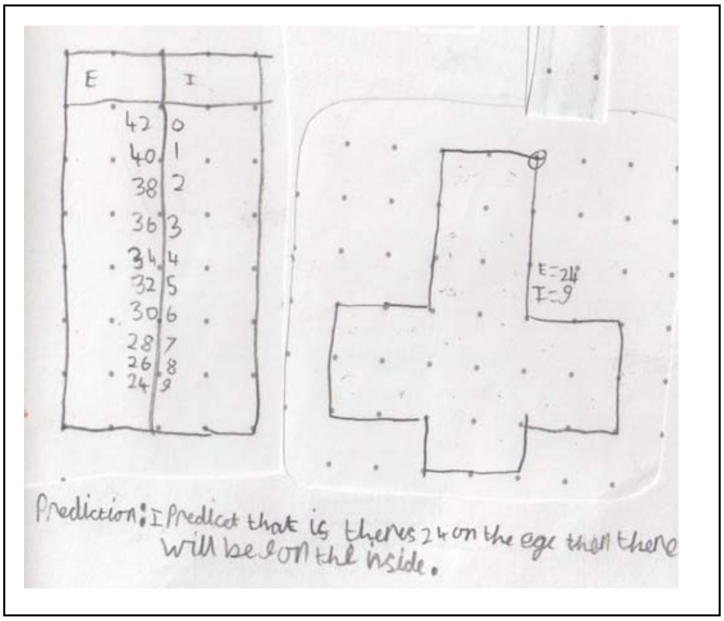

Figure 3: Student prediction and test

The student here followed a pattern they noticed that for any area shape, to get E when I=0, you need to double the area and add 2. He also noticed (as did many students) that as I increased by 1, E decreases by 2. The prediction above (which is tested and found to be correct) follows from the student choosing area 20, predicting that when I=0, E=42 (20x2 +2) and then following down the table until he got to I=9.

One student (who his teacher reported was not achieving highly in relation to the class in mathematics) articulated in discussion a prediction about the number of possible combinations for (5,5), (6,6). I encouraged him to predict for (100,100) and other high numbers and then offered him the notation (n,n) to express his idea and the word ‘conjecture’. His learning journal includes the following section (see Fig 4).

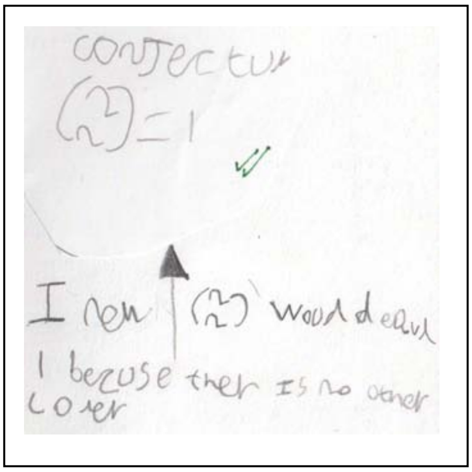

Figure 4: A student conjecture with a justification

I interpret his writing as follows: “I knew (n,n) would equal one because there is no other colour”. The reasoning was written on the journal page, the conjecture cut out from work done in the original session. There is evidence here of the students’ capacity to work with abstraction in expressing this generality with a reason behind it (with no other colour you cannot generate a new arrangement). I was left to wonder the extent to which the relative literacy difficulty this student seems to have might be contributing to his perceived lack of success in the standard maths curriculum.

The National Curriculum places at level 5 (Using and Applying) the skill of deriving a rule, expressing it algebraically, and giving a reason, which I see in Fig 4. The work in Fig 3 is, without the notation, an example of a student working with three variables (Area, E and I) – this appears at level 7 of the same strand. It seems therefore that students in Year 3/4 (and not just those achieving highly) are able to operate at a level of sophistication not normally expected until well into Secondary School, if they are given the opportunity to use the ‘powers’ of their minds.

What is the potential for collaboration between mathematics and arts based education practices?

This project was (and continues to be) a collaboration with “5x5x5=Creativity”. This educational charity places artists in schools to work with students and research the learning that takes place (all Arts based until this project). “5x5x5=Creativity” run professional development sessions for artists and teachers involved in their projects, and at one of these days that I attended with one of the year 3/4 teachers there was an input about the use of ‘Learning Journals’. Following discussion between the teachers at school C we decided to ask for a learning journal for each of our students and to offer them as a way of reflecting on their work and taking forward outstanding questions or issues. At the training day there were examples of journals kept by children and adults, which mostly took the form of collage. I decided I should start a journal about my work on the project, partly to be able to offer the students an image of what could be in a journal. We introduced the journals to students in the second to last session, and all the staff in the room were so impressed by the work the students did that we decided they would be the focus of the final session as well.

The two teachers I worked with commented to me on the value they saw in terms of getting students to reflect on their work. One teacher reported finding it hard previously to get students to be creative or inventive when asked to reflect on a piece of work in any subject. With the learning journals however the students seemed to enjoy going back over what they had done (Reflection meeting 12/11/10). One aspect the teachers spoke of was the chance for students to look at the work they had done and re-organise it. This is illustrated in Figure 5, with one student who cut up her work on 'Pick's Theorem' and organised it according to area (which it had not been on paper) and then added a table of results.

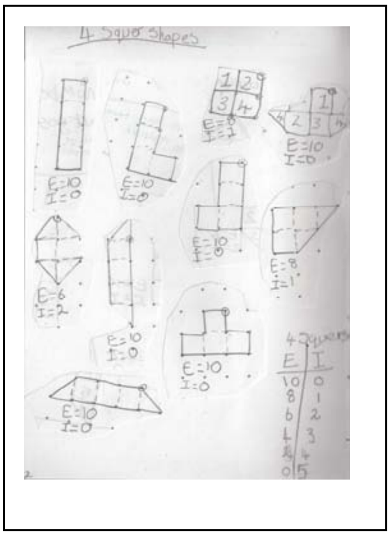

Figure 6: A student re-organising her work by area

The table of results for E and I is written on the paper of the journal and was not part of the student's original work. There is evidence here for the journal allowing the student to collect common shapes together, focus on the values E and I in creating a table – and then extend the table to shapes she has not found. In this example through reflecting on her work via a journal the student is accessing higher levels of skill in terms of Using and Applying Mathematics, that she had previously done on the same topic. The use of journals is just one technique common in Arts education that has been shown to have applicability to mathematics. The project has convinced me there is potential for further fruitful collaboration here.

Conclusion

In presenting this report at the day conference of BSRLM in London in March, several participants at the session were interested in any effect the project had in terms of the school or teachers' views of student 'ability' in mathematics. There were certainly students (e.g. see Fig 4) who surprised their teachers in terms of the quality of work they were able to do (compared to their prior attainment). The students were arranged in tables according to 'ability' in both mixed attainment classes. There was at least one case of a student moving 'up' a table following work produced in a project session; but no evidence of problematising the concept itself.

I am mindful, following a question at the conference session, not to downplay the role of the teacher (which was generally me). I tried, in the section on the powers of the mind, to offer more insight into some of the awarenesses behind the choice of starting points and initial questions, than I did in London, but am aware this remains brief and inadequate. It is of course not simple to work with students so that they access their powers no matter what the starting point. In some sense this report can be seen as an existence proof confirming e.g., the work of Cohen (1989) and Gattegno (1971), in terms of what is possible for 7/8 year olds to achieve mathematically. I hope to continue the work with the school and report findings in a future meeting.

References

Brown, L., and J. Waddingham. 1982. *An Addendum to Cockcroft. A Response by Avon Teachers to the Cockroft report "Mathematics Counts"*. Bristol, Resources for Learning and Development Unit (RLDU).

Cohen, D. 1989. *Calculus by and for Young People (Ages 7, Yes 7 and Up)*. Champaign Illinois, Don Cohen - The Mathman.

Gattegno, C. 1971. *What we owe children: the subordination of teaching to learning*. London, Routledge Kegan Paul.

Hewitt, D. 1999. Arbitrary and necessary Part 1: a way of viewing the mathematics curriculum. *For the Learning of Mathematics* 19: 3.