Conceptions of 'Understanding mathematics in depth': What do teachers need to know and how do they need to know it?

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In recent years there has been much debate about the preparation and supply of mathematics teachers, e.g. Williams (2008), Smith (2004). There has been a corresponding growth of interest in what constitutes subject knowledge for mathematics teaching, and how this is developed. Much research has focused upon primary teachers, whereas the nature of subject knowledge required by secondary mathematics teachers has been relatively under-researched. In this paper I report on work in progress in an investigation into what characterises 'deep understanding of mathematics' as understood by two specific groups of secondary pre-service and serving mathematics teachers. Additionally I comment upon data collected on degree classification and outcomes of postgraduate initial teacher education.

Keywords: subject knowledge, Mathematics Enhancement Course, deep understanding, secondary PGCE

National context, government initiatives

There seems to be an endemic problem with the supply of appropriately trained teachers of secondary mathematics in Britain. Various writers have analysed the situation and proposed remedies (eg Smith 2004, Tickly and Wolf 2000). Teacher shortages are linked to pupil underachievement in the subject (HMI report, cited by Smith, op. cit. p 21). The UK government has responded to the challenge of teacher supply by funding various schemes such as extra bursaries for those training to teach in 'shortage subjects', financial incentives in the form of the 'golden hello' for teachers of some shortage subjects, and a widening of routes to achieve Qualified Teacher Status such as the Graduate Teacher and 'Teach First' programmes. The Mathematics Enhancement Course and the Mathematics Development Programme for Teachers have developed in this context.

The Mathematics Enhancement Course (MEC) sits within a wider framework of Subject Knowledge Enhancement (SKE) courses. The MEC is aimed at graduates who wish to train as secondary mathematics teachers, whose mathematics background is insufficient for entry to PGCE or other routes to Qualified Teacher Status, but who otherwise are suitable candidates for initial teacher education programmes. It has a strong focus upon the development of subject knowledge (Teacher Training Agency, 2003). Universities have considerable freedom to interpret this, and perhaps unsurprisingly there is in many courses a focus on pedagogical subject knowledge as well as pure subject knowledge.

The Mathematics Development Programme for Teachers (MDPT) is a part-time course for serving teachers. It is aimed at teachers who are already teaching mathematics at secondary level, but who did not originally qualify in the subject. It is primarily a subject knowledge enhancement course, but like the MEC, there is an inevitable overlap with pedagogical subject knowledge; indeed, course participants are very keen to pick up new ideas for approaches to school mathematics topics.

Models of teacher knowledge

This study is informed by the work of a number of writers. Much contemporary research on teacher knowledge refers to Shulman's (1986) work, and the breakthrough idea of the existence of pedagogical subject knowledge. In particular, Ball and colleagues have refined Shulman's model (e.g. Ball, Thames and Phelps 2008) and developed survey items designed to measure teachers' pedagogical content knowledge (Rowan, Ball, et al. 2001). The Knowledge Quartet developed by Rowland et al (2009) provides another useful model, and links can be made between this and the Shulman-Ball view.

Understanding mathematics in depth

In her articulation of the concept of 'profound understanding of fundamental mathematics', (PUFM) Ma (1999) made a significant contribution to the development of theory, which builds upon earlier work. Her theory has resonance with ideas of procedural and relational understanding as expounded by Skemp (1976). Ma delves into this area to explore rich examples of exactly what this profound understanding can mean for teachers, and the ways in which teachers gain it. Within the original specification for MECs there was an emphasis on the nature of subject knowledge that should be developed by the courses, which drew explicitly on the work of Ma (1999), in particular the idea of

> "profound understanding of fundamental mathematics, emphasising deep and broad understanding of concepts, as against surface procedural knowledge" (TTA, 2003, p 3).

Adler and Davis (2006) report on a study of events and episodes in mathematics teacher education courses in South Africa. They claim that there is an emerging discourse about, and growing support for the idea that

> 'there is specificity to the way that teachers need to hold and use mathematics in order to teach [it] and that this way…differs from the way mathematicians hold and use mathematics'(p272).

They find that in the context studied, compression or abbreviation of mathematical ideas (in contrast to unpacking) is dominant practice.

> 'There is a limited presence of interesting instances of unpacking or decompression of mathematical ideas as valued mathematical practice' (p271).

Adler's recent work includes a study (the QUANTUM project) based partly in South Africa and partly in the UK, in which the focus is on 'understanding mathematics in depth'. The UK strand of the project focuses particularly on Mathematics Enhancement courses in England, and course tutors' and students' conceptions of 'understanding mathematics in depth' (UMID). This context provides an interesting site for investigation, since MECs have been devised specifically as subject knowledge courses for intending mathematics teachers (Adler, Hossain et al, 2009).

Quantitative data: analysis of entry and exit grades

Tennant (2006) studied students on a secondary mathematics PGCE. He investigated the relationship between formal academic qualifications, as measured by classification of first degree, and effectiveness in initial teacher training as measured by level of performance in the Standards for Qualified Teacher Status. Tennant found there was no correlation between his students' degree classification and their level of success in initial teacher training. Following

Tennant, and notwithstanding the limitations of the method, I analysed data from three consecutive PGCE mathematics groups studying between 2006 and 2009 at one university. Degree classifications on entry to PGCE were scored thus: a first class or higher degree as 1, a 2:1 as 2, a 2:2 as 3, a 3rd as 4 and a pass degree as 5. In England, student teachers' performance across aspects of professional knowledge, teaching, planning and assessment which constitute the Standards for Qualified Teacher Status (QTS), are graded on a five point scale consisting of three pass grades (1,2,3) where 1 is the highest score, and two fail grades (4,5). These provided the exit scores.

I found no relationship whatsoever between entry and exit grades. I then went on to compare outcomes of former MEC students with those of non-MEC students. The data revealed small but statistically insignificant differences in the means (Z-test, 1-tailed test, 5% level).

Overall QTS grades	Mean	SD
MEC (n=43)	1.8	0.5
Non-MEC (n=63)	1.7	0.5
Subject knowledge QTS grades		
MEC (n=43)	1.7	0.6
Non-MEC (n=63)	1.6	0.5

Comparison of QTS grades on exit from PGCE: MEC and non-MEC students

Tennant (op. cit.) suggests that students with a degree in mathematics may actually have gained a very narrow understanding of some areas of the subject, with little sense of the overview and connections between areas. Interestingly, this is where today's bespoke subject knowledge enhancement courses may become relevant. Tennant contends that in mathematics, degree results cannot reliably be used as indicators of subject knowledge for teaching. This begs the question of what indicators could or should be used, and how admissions tutors might best make decisions about entry to PGCE. My results support Tennant, and further they indicate no difference between MEC students and others in terms of their outcomes upon completion of PGCE. In other words, MEC stands up to scrutiny when compared with traditional degree pathways, and is doing its job as a successful alternative route to PGCE.

Qualitative data: semi-structured interviews

This part of the investigation is being carried out by means of semi-structured interviews with a sample of recent PGCE and MDPT course members from one university. Of the PGCE students, roughly half are ex-MEC and half not. This is an interesting sample as it is made up of a group of people who are at an early stage in developing their expertise as teachers of mathematics. I suggest that they are all, for various reasons, at an early stage in their encounter with the importance of understanding mathematics in depth for teaching. All have recently completed training and/or subject knowledge enhancement courses, and this experience is still fairly fresh with them.

The development of interview items has been informed by my involvement in ongoing work by Adler, Hossain et al (2009) into understanding mathematics in depth. I am adopting a hermeneutic approach to interpretation of participants' responses. At the time of writing, 14 interviews have been carried out, each taking about 30 minutes. It is intended to carry out a further 10 interviews. Questions probe, *inter alia*:

- participants' experience of mathematics during their training/course and how this prepared them for their own teaching,
- topics which the respondent understands well, and where this understanding came from,
- what 'understanding maths in depth' means to the participant.

Emergent themes

Analysis of this qualitative data is still at an early stage, but some interesting responses have been identified. The majority of respondents conceived 'understanding mathematics in depth' (UMID) as 'knowing why' and also in terms of being able to communicate ideas to others. This emerged both through an open question about their conceptions of UMID, and via a ranking exercise later in the interview in which participants were offered five different interpretations of UMID and invited to rank them in order of importance.

Most teachers responded by relating the question about UMID to themselves, e.g.

> "you have got to kind of get in there and explain why… and I suppose it's just going that little bit deeper. If you have got a deeper knowledge then they will trust you as well because they can sense that you do know what you're talking about"
>
> "Explaining it to others, I just think that's probably one of the harder things to do. You understand it in your head but if you can actually articulate it, explain it, demonstrate it, lead someone through a process, however it works, the depth is really there".

but some teachers responded by discussing how they identify UMID in their students, e.g.

> "So rather than just regurgitating things that they have been... tools and tricks that they have been taught, they actually deeply understand every question and where it is coming from.
> And they can also transfer skills. They don't pigeonhole this is an algebra question, this is a coordinates question, you know, they have got the ability to transfer the skills to right across all the subject areas".

and one identified how she knows if students do <u>not</u> exhibit UMID:

> "I have taught a few of those groups… and I feel like I've got them through and they will get their C's by just learning things, algorithms you do step one, you do step two, then you do step three and there's your answer. And they know how to do it but they actually don't really understand what they're doing".

Some other themes are emerging at this stage as a result of teachers commenting on their experience of mathematics during their training/subject knowledge course. These include:

Identity, confidence:

> "I am a much more confident teacher from being here and doing all the different things we have done, taking them back with me and I actually feel like part of the mathematics department…and I am comfortable with it, whereas two years ago if someone had said you will be teaching statistics in year 11, I would have looked at them and laughed.." (MDPT, 09/10)

Transformation of teaching approach:

> "…gave me a clear understanding of why things work… and I know how to think about those things now, and I know how to draw it out of pupils rather than going in and saying, 'this is how it works because it does'." (MDPT, 09/10)

Identity, change of approach:

"I have started to see things perhaps deeper and try to anticipate questions of why really - because I was never the why person" (PGCE 09/10, non-MEC)

Understanding maths in depth

"With the MEC, I think that was really delving deep… spending two or three weeks on a certain topic and really delving deep and grabbing it by its whatever…" (PGCE 07/08, MEC)

Conclusion

The lack of a relationship between degree classification on entry and success on a postgraduate Mathematics ITE programme is no doubt well known to those working in the field, and so my findings here serve merely to underscore this point. However, I suggest that the data comparing PGCE outcomes of former MEC and non-MEC students does add something new to the picture. This is despite the obvious limitations regarding conversion of degree classifications to an interval scale, and reliability and validity of the QTS grading system. Overall, discussion of quantitative data serves mainly to provide a context and background for my qualitative study.

If one wishes to explore teachers' conceptions of deep understanding of mathematics, then it is necessary to hear their voices, and an appropriate way to achieve this is through interviews and subsequent analysis of the data thus gathered. The role of the researcher is complex. I am aware that one cannot 'stand outside' the domain of enquiry and look within; one is a part of that domain. The researcher interacts with the subject matter through choice of questions to be asked, through the way responses are interpreted, etc. In the interactive context of interviews, the researcher is in relationship with the interviewee and this cannot be neutral – there are shared understandings and interests, there is an underlying context and in some cases, a history to that specific interaction. In the case of this study, all interviewees were ex-students of mine. This itself provides a context which must be acknowledged.

The teachers interviewed are giving their espoused views of what understanding mathematics in depth means, and I have not attempted to relate this to, for example, any external judgement or my judgement of their depth of understanding. I seek merely to hear their voices. I hope that as I become more skilled in attending to and interpreting what is said, those voices will reveal some thoughts and ideas of 'new' mathematics teachers which will be important for us all to hear.

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