Perceptions of symmetry: A window into how 13 year old students appear to understand symmetry.

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This study describes Year 8 students in England (aged 12-13) using Dynamic Geometry Software (DGS) to investigate triangles and quadrilaterals which can be generated by dragging two rigid perpendicular lines within a shape. The dialogue and the dragging and measuring strategies employed by the students seem to illustrate that they viewed the shapes through the lens of symmetry. On being questioned about the meaning of symmetry their notions of it were process based rather than coming from an esoteric understanding of the meaning of symmetry.

Keywords: Dynamic Geometry Software, dragging, measuring, symmetry

Dynamic Geometry Software and its affordances

Dynamic Geometry Software (DGS) has tools based on the Euclidean elements of points, lines and circles. The menus in the software allow geometric constructions to be performed on the basic elements. In this way the properties of a geometrical figure constructed in DGS can be programmed into the figure. The dynamic nature of the software stems from the affordance of the dragging mode (Holzl, 1996). Any figure which has been constructed on the screen can be dragged to demonstrate many examples of such a figure (Laborde, 1993, Olive, 2000). The facility of DGS to allow geometric properties to be programmed into a figure is an important affordance of the software.

Other important affordances of DGS are the drag mode, which keeps the designed-in properties of the geometric figure as constants while the figure is dragged, and the Measures menu. Measurements of lengths and angles of a geometric figure, for example, can be made and these update when the figure is dragged (Hollebrands, 2007).

A number of strategies for dragging and measuring have been identified. Students may use random dragging to investigate the properties of a figure on the screen or they may use dragging in order to maintain certain properties of a figure (Arzarello et al, 2002). There is also the dragging test which is used to check that a constructed figure keeps its properties when dragged and many studies have been reported which concentrate on how students learn to construct geometric figures which are resistant to dragging (for example, Holzl et al, 1994, Jones, 2000).

Using dragging and measuring strategies can help students to move between the spatio-graphical field of geometry (ie the experimental practical side of geometry) and the theoretical field (Olivero and Robutti, 2007). For example, students might make some measurements in an isosceles triangle constructed in DGS and observe what happens to the measures of the sides as the figure is dragged. This could help them to form a conjecture that two sides are equal in length and so help the students move from the spatio-graphical to the theoretical field. Another example could be when the students have used deductive reasoning to prove that the diagonals of a rhombus bisect each other at right angles. If they then construct the rhombus and use dragging and measuring to check the proof works in DGS then they are moving from the theoretical to the spatio-graphical field (Olivero and Robutti, 2007).

Moving between the two fields may help to support students as they develop their geometrical reasoning.

Hollebrands (2007) also noted different strategies students use when working with DGS: a reactive strategy is used when the student performs an action on the screen without being able to predict the result and then performs the next action depending on the result. A proactive strategy is used when the student predicts what will happen before they perform the action based on their knowledge of the software and the geometry. Although Hollebrands was writing about any kind of action performed in DGS it is obvious that she included dragging and measuring actions in this. In this study I have also observed students using a further dragging and measuring strategy which will be described later.

Research methods

In this study pairs of year 8 (12-13 years) students worked on a prepared task using one computer and the Geometers Sketchpad (Jackiw, 2001). The students were identified as having average achievement and were chosen by their class teacher as being confident to work with an adult they do not know (the researcher) and willing to talk about what they are doing. The on-screen activity and dialogue were recorded using image capture software.

The students worked in files set up by the researcher in the Geometers Sketchpad, The first file contained two rigid lines: a vertical 8 cm line and a horizontal 6 cm line which were constructed to maintain their orientation and length when dragged. These were referred to as bars to differentiate them from other line segments in the figure. The students were asked to drag one bar over another (which generated their intersection) and to join up the ends of the bars to complete the shape. They then constructed the interior of the shape thereby filling it with colour and helping with visualisation of the shape especially when it is concave. The students were then asked to drag the bars inside the shape and investigate which different shapes they could make. In essence the bars are the given constants in the task. The dragging mode is being used to generate different shapes whose diagonals are the bars (for quadrilaterals) or whose base and height are the bars (for triangles).

The students' use of dragging and measures

At first the students dragged fairly randomly to see what shapes they could make. This may be akin to the random dragging described by Arzarello et al (2002) or the reactive dragging described by Hollebrands (2007). After they had spent some time investigating shapes the students were asked to drag the bars in order to make a shape of their choice. Usually at this point, the students dragged the bars purposefully, whilst attending to the measures displayed on the screen so that the shape looked fairly accurate by eye. This may be akin to the proactive strategy mentioned by Hollebrands (2007).

When the students used the software to measures lines and angles in the figure they usually found that measurements which they expected to be equal, for example adjacent sides in a kite, were not exactly equal (see figure 1). The students were then observed to make fine adjustments to the position of the bars whilst they attended to the measures in order to get them as close as possible. I have named this dragging strategy 'dragging to adjust measures' as this describes what the students were trying to do.

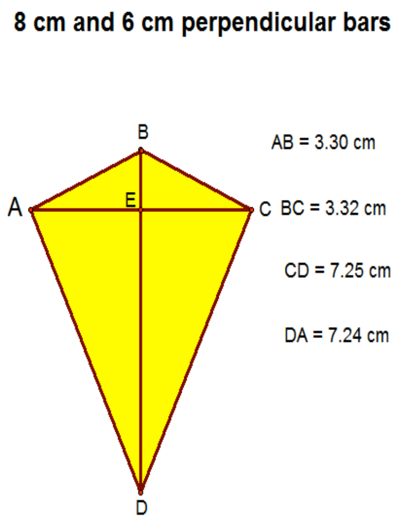


figure 1

The students were rarely able to drag the bars so that the measures were exactly equal but they seemed to accept this and understand that they could not get the figure to be perfect. The reasons for not being able to make the measures read in such a way as to suggest the figure was perfect may stem from issues with the software itself. The computer image of the figure is a screen representation of the theoretical figure so, whilst it may embody certain properties (in this case perpendicular diagonals of 8 cm and 6 cm), it still has imperfections which may be the result of the size of the pixels and the curvature of the screen.

The measures are calculated according to an algorithm in the program which is based on the co-ordinates of the end points of the line segments and thus may have small inaccuracies built in (Olivero and Robutti, 2007). Whilst we understand that figures we draw on paper are likely to be inaccurate, we tend to think of the computer image as being accurate and this is not necessarily the case.

The students' understanding of symmetry

The students had an understanding of symmetry which was processed based and seemed to stem from the way they had been taught to recognise symmetrical shapes in school. The following conversation took place after two girls had generated a concave kite and then stated that it had symmetry. They were asked what this means.

Tilly: It's got a line of symmetry
Res: So if that's the line of symmetry what must be true? Can you tell me anything true about that shape
Tilly: Erm, not sure
Res: What does a line of symmetry mean?
Alice: It's the same on the other side
Res: What do you mean by 'you have the same either side'. Can you tell me some things which might be the same?
Tilly: Well the shape of it. If you look where BD is, it's got the same like points and everything on the same, like point.
Alice: I've done it before but, I'm trying to think. Did we trace it and then checked if it there were lines of symmetry?
Res: When you traced it and you were checking lines of symmetry what did you do?
Tilly: Traced half of it and folded it over.
Alice: and then the lines were like you could see. whether the lines were the same

Dragging to maintain symmetry

Arzarello et al (2002) have described how students sometimes drag to maintain certain properties of a figure and symmetry may be such a property. Students in this study were observed to use dragging to maintain line symmetry. For example, Tilly and Alice were observed to drag the horizontal bar along the vertical bar such that it was bisected by the vertical bar. They did this whilst trying to generate an accurate kite. They moved the horizontal bar AC so that it touched the end of the vertical bar BD (figure 2), then moved it down to generate a concave kite (figure 3) and then up, thus generating a kite (figure 4).

8 cm and 6 cm perpendicular bars

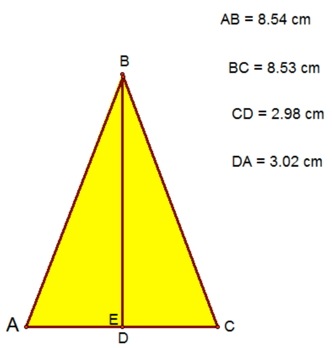


Figure 2

8 cm and 6 cm perpendicular bars

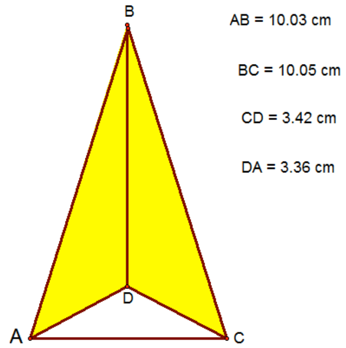
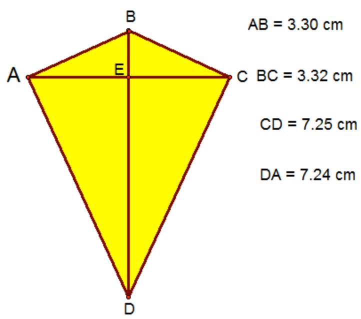


Figure 3

8 cm and 6 cm perpendicular bars



When the girls were asked what they were trying to do they said

"we're trying to get BD in the middle of the shape"
"one er B and D vertically in the middle of the shape"

Figure4

Clearly the girls were attending to the vertical axis of symmetry. In fact very few of the students in the study have used the horizontal bar as a line of symmetry although some have mentioned it, usually as a second line of symmetry. This could be a natural result of the fact that, as Pinker (1997) suggests, human beings are reasonably symmetrical about the vertical axis and we tend to use the vertical axis as our frame of reference.

Half a rectangle?

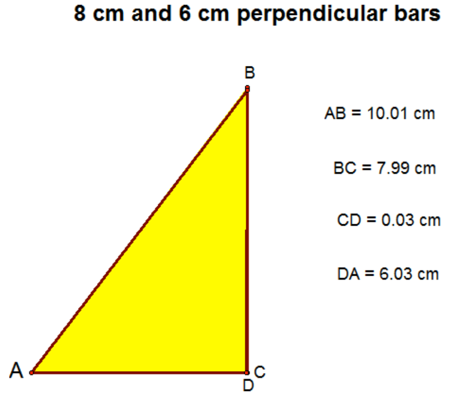


Figure 5

At one point the girls dragged the bars to make a right angled triangle (Figure 5). A discussion followed as to whether this shape is half a rectangle or half a square. This was interesting because the girls clearly visualised that if the right angled triangle was copied and then the copy was transformed the result would be a rectangle (or square, until they had argued it through).

Tilly: Cos if you get another one of them, and turn it round and make a

Alice: It would make a rectangle

Res: So you think it would make a rectangle?

Tilly: And if you put A er D at the top, join it with the B point then put BC on the other side then it would be a same I think.

Res: What do you think Alice?

Alice: er, er, wait, I need to

Tilly: no actually it would be a rectangle

Alice: It would because, if you think about it, if you did, if you flipped that over the other side so it was like symmetry, you would get the same, and if you had both of them, then it would be a rectangle

When Alice talked of flipping the shape it would suggest she thought of reflecting it and that would have resulted in a kite rather than a rectangle. However Tilly talked of turning it round which would result in a rectangle. Nevertheless this suggests that the girls were able to visualise the right angled triangle and its transformation making a complete rectangle.

Discussion

The nature of the task requires the drag mode to be used to generate different geometric shapes. This has led to a newly observed dragging strategy where the students ‘drag to adjust measures’.

It appears that the students viewed the shapes through a lens of symmetry. Clearly the task, with a longer vertical bar and a shorter horizontal bar is likely to influence the way that the students viewed the shapes.

Symmetry and transformations may be involved in the way that humans visualise objects. Battista (2008) has surmised that unconscious visual transformations including rotations might be used as a mechanism for visualising spatial structures. Pinker (1997) claims that, as a way of limiting cognitive load, humans memorise figures in their normal upright orientation and use mental rotation in order to recognise figures in other orientations. He even suggests that reflections are visualised as 180 degree rotations in the plane perpendicular to the frame of reference.

There could be implications for the way in which geometry is taught. Many children currently learn about the properties of right angles, congruent sides and angles, and parallelism before the properties of symmetry. However it may be more intuitive for children to learn about symmetry first and to develop other properties from it.

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