Working group on trigonometry: meeting 5

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These notes record the discussion at the fifth meeting of this working group. The focus was on reading two research reports and considering the future of this group.

Keywords: trigonometry, curriculum

Introduction

The two papers we discussed were:

Singh, S. (1986) Teaching trigonometry in the (micro)computer age. *International Journal of Mathematical Education in Science and Technology*,17:3,315 — 323.

Kendal, M. & Stacey, K. 'Teaching trigonometry'. Downloadable from http://staff.edfac.unimelb.edu.au/~kayecs/publications/1997/KendalStacey-Trig.pdf

Most of the first paper consisted of programmes for teaching trigonometry, but the first part offered a classification for school trig: computational and analytical. The first category includes solving right angled triangles and using trig identities to solve equations or to substitute; the second a functional approach. It was stated without justification that the first should be taught first and understood before the second. The content of the first roughly corresponded to what is tested at GCSE, and the second to what is tested at A-level. There was no mention of similarity in this article, nor if the uses of trigonometry with vectors, such as in applied mathematics.

The second paper – of which we have also read a version published in a refereed journal – compares two methods of teaching: ratio and unit circle, but tests their effectiveness with a test that relates more closely to the ratio understanding, since it depends on solving right angled triangles, including transforming ratio equations. It is inevitable that different teaching methods would lead to different understandings and this research had not taken that fully into account. For example, the approach to solving right angled triangles for those who had been taught using the unit circle included transforming the triangle so that it sat in a virtual first quadrant, and then scaling to make the hypotenuse be unit. We were not surprised that this was a harder way to solve right angled triangles than methods based on identifying suitable ratios.

Challenging the curriculum

One view put forward was that there was no longer a need for trigonometrical understanding, since many of its traditional mathematical functions can now be done by other means: computer algebra systems; use of standard integrals; robots in the workplace; and so on. One of its main functions was to provide substitutions for calculus and expansions of functions, but so long as engineers and physicists and others who use these can accept the outcomes of CAS as tools knowledge of their

structures is not necessary. It was felt that its traditional role as a hurdle to higher GCSE grades was based on an outmoded set of mathematical needs.

A contrary view is that, as we have shown in previous meetings, trigonometry provides a mathematical context for coordinating conceptual understandings of:

Triangles Right angles Different orientations Compound shapes Angles Measurement of angle & length Ratio Proportion Similarity Expressing ratio as a number Understanding new notation	Wave functions Transformation of functions Inverse functions Rearranging formulae when a function is involved Levels of accuracy and rounding Use of letters to mean labels, unknowns or variables Components of vectors Surveying Polar coordinates and parametric curves

It is therefore a suitable mathematical arena for exploring connections and relations within mathematics, and joining different strands meaningfully. It also still has practical applications and interesting historical roots. In countries where mathematics is taught in different strands (algebra; geometry etc.) it provides a coherence to the subject as a whole.

The future

The problem remains that there is no good quality research about trigonometric understanding. A recent proposal associated with this group was highly-rated but not funded. It is likely that various people will keep trying to get funding. Meanwhile, we felt that the working group has run its course and there is little more to do, but we are in a position to prepare a curriculum statement, and to encourage teachers and others to work further on developing curricular approaches which develop trigonometric understanding.

Members of the group will also read the research carried out by Craig Pournara of University of Witswatersrand, which seems to be the only credible research in this area, and consider further what might be achieved in the future.

Past discussions can be found in previous BSRLM proceedings and copies of the papers discussed by the group, and anything else, can be obtained from anne.watson@education.ox.ac.uk.