Working group on trigonometry: meeting 3

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These notes record the discussion at the third meeting of this working group. The focus was on feedback from tasks undertaken since the last meeting.

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First lessons on trigonometry

Anne Watson had surveyed local teachers asking them about their first trigonometry lessons. She had analysed the replies and found several distinct approaches: similarity (students are offered a collection of triangles with a fixed angle and divide measured sides); functional (angle changes and one side changes); multiplier (angle changes with unit radius); combinations (discover from a mix of angles and ratios); technical (here is how to find missing values). She found that asking what was on the board and what language was emphasised indicated (a) the underlying belief about what is important in trig and (b) whether the teacher wanted a sense of closure to the lesson or was happy to leave things a bit loose. Some reported that the first lesson is a kind of hurdle to help learners feel; comfortable with the ideas, while others wanted to end with formulae, or SOHCAHTOA, or a labelled image of an appropriate triangle.

Colin Foster then reported his experiment with a first lesson. He had taken a gentle pace leaving ‘lots of air’. He had obtained Jeremy Burke’s outline plans, which start by establishing angle as a key object, but was unable to use them because of lack of computer access. Instead, students cut out a collection of differently-oriented similar right-angled triangles. Cutting gave them a sense of the same angles. They were then asked to sort them. Some tried to fit them together to make a shape; others tried to lay them out in a line in order of size; others stacked them in the same orientation. Deciding that they could turn triangles over and they were still ‘the same’ was non-trivial for some. When they saw, excitedly, that they were ‘all the same’ he asked them what that meant. Some measured lengths, or differences in lengths, but interestingly stacking and lining up did not necessarily mean they had internalised the sameness of the angles, since some students had to measure them to find this out. Students also tried adding angles and adding side lengths. It was hard to talk about ‘sameness’ and Colin introduced the notion of ratio himself since dividing is not a natural thing to do. Students then wondered why 0.7 appeared frequently.

We discussed the importance of intuitive understandings of ‘parallel’. This manifests an awareness that even very young children have: extending straight lines maintains direction. This appears in some of the ways they sorted their triangles.

Division is a non-obvious way to articulate the sense of sameness. Colin was happy to focus on moving towards this, but some students asked him ‘are we going to do SOHCAHTOA?’, having heard this idea elsewhere. In discussion we agreed that an imposed need to have a single lesson objective is unhelpful when the teacher wants to focus on developing a new sense of relations. Half a term could usefully be devoted to the ideas in trigonometry. We recalled a participant saying that is his/her experience trigonometry is not taught as a separate topic, but arises as a set of tools in various mathematical and other contexts, focusing on ratio and informal awareness of angle: doors opening, stairs rising, etc. The problem is how students can move from the application of ratio in particular cases to the formal appropriation of trigonometric methods as tools for other situations.

One fundamental question about teaching trigonometry is whether measuring to get lengths to calculate with is misleading. In this lesson, the invariance of tan 35 is not measurable. Do we give students the impression that mathematical invariants can be found empirically? The aim is to draw students' attention to ratio, not to lines and angles as separate elements. There is a choice between what is pedagogical (measuring and dividing) and what is mathematical (comparison; invariant ratio).

Sorting questions

A set of 12 typical questions that might be used in teaching trigonometry was then examined. We took a critical look at each question, wondering whether we would use them at all. Rather than repeat the details here we merely list the issues that arose:

- The questions over-emphasised the kinds of things that might be asked in an examination, for example, one that was about giving answers correct to a certain number of figures and another about mis-orientating the triangle – we favoured incorporating these issues into tasks that were more about concepts.
- Learners have to read diagrams: e.g. locate right angles, assume straight lines.
- We preferred tasks that could be extended towards more challenging aspects of trigonometry and non-routine examples.
- One task asked students to show that $4x + 3y = 12$ from a right-angled triangle and its ratios. We liked the combination of concepts, but wondered whether the need to manipulate algebraic expressions might be a distraction. To prove the result involves setting up ratios and equalities so there would be some engagement with fundamental trig concepts.
- Can we construct pivotal tasks affording engagement with key ideas in trig?

Programme of work

- Historical roots of trigonometry.
- Awareness of similarity: In our second meeting we suggested that this could arise from the way we look at objects which are at various distances from us, and also from a dynamic sense of growth.
- Hypothetical learning/teaching trajectory to articulate the difficulties learners may have moving from perimeter/diagonal considerations in polygons to circumference/diameter relation for circles.

We shall review some of this work at the next BSRLM meeting on 14th November. This is an open group and all are welcome to join. If you would like copies of earlier readings please contact anne.watson@education.ox.ac.uk.