Practitioner Use of Graphing Software to Teach about Algebraic Forms

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From analysis of teacher-nominated examples of successful technology-supported practice in secondary-school mathematics, the use of graphing software to teach about algebraic forms was identified as being an important archetype. Such practice was investigated through case study employing evidence from lesson observation and teacher interview. The practitioner model developed in earlier research (Ruthven and Hennessy, 2002; 2003) provided a framework for synthesising teacher thinking. Further analysis highlighted the crucial part played by teacher prestructuring and shaping of technology-and-task-mediated student activity. Moreover, this indicated how, in appropriating the technology, teachers modify their classroom practice and develop their craft knowledge.

Introduction

This paper develops a line of research on the incorporation of computer-based tools and resources into the mainstream practice of mathematics teaching in secondary schools. This line began by building a generic model of the terms in which teachers regard such incorporation as being successful (Ruthven and Hennessy, 2002; 2003). It has subsequently sought to identify and examine particular forms of professionally well-regarded practice; this paper reports on an archetypical use of graphing software.

Design and method for the study

Recommendations from professional leaders and evaluations from school inspections were triangulated in order to identify subject departments in state-maintained schools that were regarded as successful both in terms of the general quality of their practice and their use of computer-based tools and resources within it. To identify what practitioners themselves regarded as successful practice, focus group interviews were then conducted with each of these subject departments. In these interviews teachers were invited to nominate and describe examples of successful practice involving use of computer-based tools and resources. From the interviews with eleven mathematics departments, use of graphing technology was identified as a successful established practice, favourably mentioned in all the departments, and selected as a nominated example in seven, with graphing software generally preferred over graphic calculators. However, outside advanced courses, use of this technology still proved relatively infrequent, occurring on no more than a handful of occasions per school year with any particular class. The mathematical topics which were most frequently cited were linear equations (nominated in six departments, and mentioned in four more) and quadratic equations (nominated in only one department, but mentioned in four more, all of which nominated linear equations). The use of graphing technology to treat such equations also featured prominently in official curricular and pedagogical guidance, and lessons of this type were cited favourably, both in reports of individual school inspections, and in a national report on the impact on secondary mathematics of government ICT initiatives in schools.

Available project resources made it feasible to follow up only a limited number of cases. Given the relatively few differences in the forms of graphing practice that teachers described, and that graphing software (rather than graphic calculator) was clearly teachers' preferred technology, we followed up two cases of this type which provided some degree of contrast in approach. Other important considerations were that the teachers concerned had already provided quite full and thoughtful accounts during the focus group interviews, and that two lessons could be observed with each teacher (coded as teachers A and B), one on linear forms (coded as lesson 1) and the other on quadratic (lesson 2).

Detailed observation records were made of each lesson, incorporating a transcript of the main episodes, integrated with further observational material including copies of resources used and records of graphs displayed. Post-lesson interviews were conducted with teachers after each observed session. These were organised around a standard sequence of printed cards asking teachers about their thoughts, first while preparing the lesson (what they wanted pupils to learn; how they expected use of the technology to help pupil learning); then looking back on the lesson (how well pupils learned; how well the technology helped pupil learning; important things that they were giving attention to and doing). The resulting file for each lesson was first analysed to create a lesson summary outlining working environment, resource system, lesson agenda, and activity structure, followed by the main lines of pedagogical thinking reported by teachers. An analysis was then conducted across lessons and teachers, employing the broad themes from the compact version of the 'practitioner model' developed in earlier research (Ruthven and Hennessy, 2003).

A practitioner model of how graphing software helps teaching of algebraic forms

The themes from the earlier study provided a useful organising framework for synthesising the thinking reported by teachers in association with each lesson, making it possible to elaborate a practitioner model of the contribution of graphing software to the teaching of algebraic forms, grounded in the observation and interview data from this study.

Teacher accounts of all the lessons made reference to various aspects of the theme of *Effecting working processes and improving production,* suggesting, for example, that the software made it possible to produce graphs "extremely accurately and extremely quickly" [B1], making "doing the activity an awful lot easier and quicker and more efficient" [B2], so that – in terms of time economy – students could "move through everything at a much quicker pace" [A2], allowing a topic to be addressed in only a single lesson [A1; B1]. In the lower- and average-attaining classes, this also helped make tasks accessible to students who would have found "organisation and presentation challenging" [B1] and would "have really struggled" [A1], echoing aspects of *Overcoming pupil difficulties and building assurance.*

These factors also underpinned some aspects of the theme of *Enhancing the variety and appeal of classroom activity,* in terms of the use of graphing software making lessons less "laborious" [A1; B1; B2] and less dependent on pencil-and-paper work [A2; B1], and increasing the immediacy and interactivity of tasks [A1]. For the higher-attaining classes, the teachers also talked of the potential of using technology to make tasks more "challenging" [A2] or "demanding" [B2] in mathematical terms.

With the lower-attaining class, technology was used to give students who "wanted to get the right thing in front of the class" [A1] the frisson of a very immediate and public test, by getting them to come out and check their proposal through using the software on the projected computer.

In relation to *Fostering pupil independence and peer exchange,* both teachers reported or were observed allotting short periods for playful exploration of the software by students, and for consequent sharing of discoveries [A1; A2; B1; B2]. Equally, for their higher-attaining classes, the teachers talked of using the technology to support "exploration" [A2] of more "open ended" tasks [B2], in which students "have their own control over the situation" [B2] and are "investigating, exploring, almost on their own" [A2]. Comparing her two lessons, Teacher A commented that she gave her higher-attaining class "more open-ended questions" to which they responded through "a text box on [their] graph to explain what differences [they had] seen", whilst her lower-attaining class were asked "much more particular questions" and "had a sheet to put very definite answers on… to focus them in". However, in the lower-attaining class [A1] as well as the higher-attaining classes [A2], when students came up with mathematical ideas going beyond the lesson agenda, she supported them in "go[ing] off at a tangent" [A2]. The contrast that Teacher B drew between the framing of tasks for her average- and higher-attaining classes was less strong: the former "had a very specific task", whereas the latter "had to do a slightly more open-ended task". She too was observed supporting students in going beyond the lesson agenda [B2] in a way which, as she pointed out, was only possible because of the availability of the graphing software.

Teachers' encouragement of informal exploration of the graphing software, and their assistance to students using it to engage in mathematical speculation and experimentation beyond the lesson agenda, also evidences how they saw this technology as a means of *Supporting processes of checking, trialling and refinement*. In both her lessons (suggesting that this had become part of her curriculum script), Teacher A posed the same speculative question about lines sloping in an opposite way, leading to a similar trialling episode being inserted into a conventional investigation [A1] and an introductory review [A2]. Likewise, the 'target practice' tasks in both of Teacher B's lessons (with the explicit linking of them [B2], indicating both a developing curriculum script, and an emerging activity format tailored to this type of topic) were conceived more broadly as examples of 'trial and improvement' [B1], dependent on feedback from the graphing software as to whether the graph of a 'trial' expression actually passed through the 'target' point which had been set. With the younger class, this required the teacher to renegotiate norms with students who were hesitant about the legitimacy of trialling [B1]; with the older class, the socio-mathematical agenda had moved on to developing habits of prediction and reflection to scaffold trialling processes [B2].

Finally, in terms of the theme of *Focusing on overarching issues and accentuating important features,* the teachers talked of how use of graphing software helped students to "get to grips with" [B2], "get an idea of" [B1], or "see straight away" [A1] the effect of altering a coefficient in the equation on the properties of its graph. Likewise, the teachers highlighted particular software devices which facilitated apprehension of equation/graph matching [A1; B1], comparison of gradients [A1], and examination of limiting trends [A2]. Nevertheless, teacher management and guidance also played an important part in helping students to gain such insights. In relation to one of the 'target practice' tasks, for example, key actions of Teacher B included constraining the type of expression to be graphed [B1], drawing attention to the equivalence of expressions [B1], and pressing students to seek further equations so as to generate graphs which were "steeper or shallower or sloping in the other direction" [B1]. Likewise, in both her lessons, Teacher A reported actively checking, and if necessary developing, students' understanding of the relationship between the equation of a graph and the coordinates of points lying on it [A1; A2], and prompting students to attend to the key mathematical properties which investigations aimed to establish [A1; A2].

The significance of instrumental induction, task design, and teacher mediation

Sensitised by literature review and wider theory in the field to aspects of teaching practice which remained largely tacit in this model, further key issues were examined. Analysis of 'institutional' and 'instrumental' aspects of tool use (Artigue, 2002) has developed in response to difficulties encountered with the educational use of sophisticated technologies designed for use by professional mathematicians. It provides a conceptual framework for analysing the process through which students (and indeed teachers) progressively appropriate a material artefact to create a mathematical instrument. Graphing software, however, has been explicitly designed for educational use. The teachers described the packages they were using as "instinctive" [B] and "user-friendly" [A&B]. They identified several aspects of the user interface which made the software readily accessible and interpretable by students (contrasting the software favourably with graphic calculators in many of these respects): the clearly labelled scales [A&B] and the gridlines in the background to assist comparison of gradients [A]; the 'hand' tool for dragging the image to view sections of the graph outside the original display [A]; the colour coding which associated particular equations with their graphs when several were displayed simultaneously [A&B]; the acceptability of expressions defined in the form x= as well as y=, and defined implicitly as well as explicitly [A]. Nevertheless, however much these graphing packages had been "designed to do things easily" [B], the teachers still played an important role in inducting and supporting students into use of the software for mathematical purposes.

Both teachers followed a dual approach to establishing a collective repertoire of instrumented graphing techniques. Prior to undertaking a classroom task involving graphing, unless the teachers had confidence that the core techniques required were already familiar to the class [B2], they introduced [A1; B1] or reviewed [A2] them. More serendipitously, they also allotted short periods to playful exploration of the software by students, and to subsequent sharing of new possibilities [A1; A2; B1; B2]. Supporting and developing use of the software was also an important dimension of teacher interaction with students while they were working on tasks. In the observed lessons, teachers guided basic operation of the software, prompted strategic action with it, and supported mathematical interpretation of its results. Teacher actions included: explaining how to enlarge a target point to make it more visible [B1], and how to enter x^2 in the equation editor [B2]; helping students to understand why the software had produced a horizontal line rather than the expected sloping one (as a result of entering y=5+4 rather than y=5x+4) [B1], or a straight line rather than the expected curve (as a result of entering $y=x+2^2$ rather than $y=(x+2)^2$) [B2]; prompting students to drag the displayed image to expose more of a particular graph [A1], or to pursue the limiting trend of a graph [A2]; and prompting students to zoom out on the displayed image of $0.00000009x^2+x+1$ to test whether it was a straight line, and introducing the comparison with $0x^2+x+1$ [B2].

Likewise, the structuring of lesson tasks through prepared materials and teacher intervention was crucial in realising many of the benefits attributed to using the graphing software. Essentially, whatever task format was adopted, the learning goal of these lessons was to induct students into an accepted mathematical organisation of the multimodal systems constituted by equations of the types $y = mx + c$ or $y = ax^2 + bx + c$ and their graphs. Achieving such an organisation depends on managing the double semiotic of the system through coordinating algebraic and geometric registers, while also managing its multi-dimensionality through isolating phenomena and controlling variables.

Within the lessons observed in this study, the investigation tasks (and indeed the lesson expositions) employed by teachers were largely successful – through sequencing and patterning example types and sets – in providing a logical decomposition of the multimodal mathematical system under consideration. For example, one quadratic investigation specified four groups of quadratic functions (such as $y = x^2$, $y = x^2 + 2$, $y = x^2 - 2$, $y = (x + 2)^2$, $y = (x - 2)^2$) and suggested examining cuts on the axes, extreme values, and lines of symmetry, to explore what the graphs in each group had in common [A2]. It is this didactical organisation of the topic which underpinned the use of graphing software to help students grasp the effect of altering a coefficient in the equation on the appearance of its graph. Nevertheless, a breakdown in such organisation emerged in the other quadratic investigation when students failed to formulate the intended property of the coefficient b from the family of forms selected to exemplify it [B2]. Lacking the same level of didactical organisation, however, the 'target practice' tasks called for much higher levels of teacher mediation. This emphasises that graphing software and lesson tasks form a resource system, in which the technology's contribution to supporting learning is powerfully conditioned by task structuring.

The adaptation of teaching practices and the development of craft knowledge

Developing a coherent resource system incorporating the use of graphing software required teachers to extend their practice in several ways. They developed strategies both to familiarise students with (and later to review) core techniques for using the software, and to allow students to explore (and then to share their discoveries of) a wider range of technical possibilities. They devised or appropriated suitable tasks and supporting textual materials to underpin classroom activity that employed computer graphing to investigate the topic of algebraic forms. As illustrated earlier, they were also developing a repertoire of strategies to support students in tackling these tasks, concerned not just with guiding software operation, but with prompting strategic action, and supporting mathematical interpretation.

In terms of activity structure, teachers suggested that use of graphing software made investigative lessons more viable. Moreover, the availability of projection facilities permitted all the investigative lessons observed in this study to be organised within an activity structure in which episodes of individual or paired student activity at workstations were interleaved with whole-class activity, concluding with plenary review. Moreover, the emergent type of 'target practice' task was associated with a rather different activity format for individual or paired student work, capitalising on the interactivity of the software to centre activity around a process of trial and improvement. Likewise, both teachers had adapted the whole-class exposition and questioning format to exploit the opportunity to use the software to provide immediate feedback on student predictions, for example by students 'taking the stage' to use the projected computer to test their suggestions.

These preceding elements of adaptation were interwoven in the development of teachers' curriculum scripts for the topic of algebraic forms, as evidenced in the lesson agendas they formulated, and in the detail of their classroom action (including interaction) during the observed lessons. On the basis of explicit comment by teachers or recurrent patterns of teacher action some of these examples clearly represented mature developments in teachers' curriculum scripts for the topic. Other examples provided evidence of teachers extending their repertoire of approaches to supporting students and (re)directing them towards desired states, intended responses and resultant learning. This included teachers' capacity for reactive teaching in response to new types of student initiative made possible by the graphing software.

Conclusion

This study has shown how the general themes of the practitioner model of successful classroom practice (Ruthven and Hennessy, 2002; 2003) provide a useful organising framework for synthesising teacher thinking about the contribution of graphing software to the teaching of algebraic forms. Further analysis has highlighted the crucial part played by teacher prestructuring and shaping of technology-and-task-mediated student activity in realising the ideals of the practitioner model. Although teachers consider graphing software very accessible, successful classroom use still depends on their inducting students into using computer graphing for mathematical purposes, providing suitably prestructured lesson tasks, prompting strategic use of the software by students, and supporting mathematical interpretation of the results. Accordingly, this study has illustrated how, in the course of appropriating the technology, teachers adapt their classroom practice and develop their craft knowledge: establishing a coherent resource system incorporating the software, adapting activity formats to exploit interactive possibilities, and extending curriculum scripts to encompass classroom graphing by machine; in particular to prestructure students' technology-and-task-mediated activity and then to shape it reactively through prompting strategic action and supporting mathematical interpretation.

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References

Artigue, M. 2002. Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning* 7(3): 245-274.

Ruthven, K., and S. Hennessy. 2002. A practitioner model of the use of computer-based tools and resources to support mathematics teaching and learning. *Educational Studies in Mathematics* 49(1): 47-88.

Ruthven, K., and S. Hennessy. 2003. Successful ICT use in secondary mathematics – a teacher perspective. *Micromath* 19(2): 20-24.