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TRIALLING REALISTIC MATHEMATICS EDUCATION (RME) IN ENGLISH SECONDARY SCHOOLS

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This article provides an outline of the initial stages of the implementation of a project exploring the use of Realistic Mathematics Education (RME) in English secondary schools. The paper explores some differences in the approaches used by the RME pupils and those taught under the National Strategy. We conjecture that teaching under RME encourages pupils to refine and develop their informal strategies.

PREAMBLE

Since September 2003 we have been working on a project exploring the implementation of Realistic Mathematics Education at KS3 in some English schools. The project, funded from September 2004 by The Gatsby Foundation, is described in more detail an Appendix to this article. Our focus in the article itself is to compare Realistic Mathematics Education (RME) with our current approaches to teaching under the National Strategy.

It is hypothesised that under The Strategy the majority of teachers offer relatively closed activities and exercises to allow some degree of certainty that the outcomes for the lessons can be achieved. A minority of teachers, however, would attempt to utilise more open activities and hope to channel pupils' intuitions to support their mathematical development. In doing this, teachers are constantly battling with the pupils' diverse range of relatively primitive beliefs and strategies and the desire to achieve specific mathematically sophisticated outcomes outlined in the unit plans.

In preparation for the lecture, we asked some year eight pupils to try some fraction-comparison questions without using equivalence. We are not suggesting that the data collection and analysis would stand up to rigorous scrutiny but we have at least evidence to suggest that the pupils being taught under RME were more sophisticated and varied in their approaches to the questions. See Streefland (1991) for further discussion of these issues.

RME offers an alternative approach to teaching which attempts to support the development of pupils' strategies and provides an account of teaching and learning trajectories suggesting how pupils develop and how this development can be supported. From this initial exploration we consider that it is worth conjecturing that the emphasis on short-term content objectives and procedures actually inhibits the development of intuitive approaches. Evidence is starting to build that our objectives-led curriculum causes difficulties for pupils and teachers (Askew (2004)).

FRACTIONS UNDER THE STRATEGY

Initially, we will now look at fractions under the National Framework.

Each year, as part of the feedback from KS3 SATs, schools are informed of what areas of the curriculum need attention from teachers, and in which areas pupils have improved since the previous year (QCA (2005)). Under the heading "To help improve performance teachers should …", fractions have been mentioned each time for the past six years with a typical comment being "… develop children's strategies for ordering two or more fractions, moving them towards the use of a common denominator" (QCA (2003)). In the same time period, under the heading "Well done, some examples of progress …", fractions have not been mentioned once! It is evident from this, and from detailed feedback on individual questions, that fractions remain a problematic area for many pupils at KS3. Equally evident is the message that, to try and remedy this, more emphasis should be given to the notion of equivalence and the use of common denominators.

However, when we look at the current curriculum, we find that Unit Plans as early as Year 2 include "Recognise and find simple fractions and the equivalence between them", and that the issues of equivalence and common denominators are then worked on in each subsequent year until Year 9 when pupils should be able to "recognise and use the equivalence of fractions". So we have the strange situation where, on the one hand, pupils continue year on year to experience difficulty with fractions and the notion of equivalence, yet on the other we already teach this very notion from Year 2 all the way through to Year 9 (and for many pupils beyond)!

So what can teachers do about this? When we address teachers with this issue, some of the more sophisticated responses include suggestions such as "create and use activities which tap into pupils' informal, intuitive ideas" (of which the chocolate bar activity (Aspin *et al.* (1987)) would be a prime example), "Use a context to stimulate these ideas and to create interest and motivation", and "Use whole class discussion in order to share and refine these strategies". These same teachers, however, also talk of feeling "the need for closure and an end product" and of at some point "dropping the context and working in a more abstract setting". The frustration for many teachers remains the issue of how to move from pupils' informal, intuitive ideas to the more formal notions demanded at an early stage by our Curriculum. And for many, the solution to this is ultimately to teach the formal, and in doing so replace much of what the pupils initially brought with them to lessons. The continuum from informal to formal may well be there in the mind of the teacher, but rarely in that of the pupils.

FRACTIONS UNDER MATHEMATICS IN CONTEXT (MIC)

Over the last two years, however, while trialling MiC materials (Romberg (2003)) in local schools, we feel that we have started to see an alternative way of working to that described above. This difference can be seen theoretically by considering the RME philosophy which underpins MiC, and also practically by looking at pupils' work which is emerging from our classrooms. So what is different about the RME/MiC approach? We will focus on just two of these.

Firstly, when one looks at an MiC book, we see that everything appears to be embedded in a 'real-life' context. This is not, however, an applications course and it is important to note that 'realistic' is really a mistranslation from the original Dutch word, and that 'Realisable', (for pupils) or 'Meaningful' or 'Imaginable' may all be more appropriate. The contexts, researched over many years, are chosen initially not for 'social' reasons (i.e. to interest, motivate etc), but for mathematical ones. They serve as both a route into the maths, and also a route through it. Essentially the context is there to help the pupils to make sense of the work, and to keep it close to the pupils' reality. We see this as different to the UK, where context may be used initially (to interest and possibly to aid accessibility), or at the end of a topic (to add complexity in the form of 'word problems'), but is generally viewed as 'getting in the way' of mathematical; development rather as a means of enhancing it.

A second significant difference is in the use of what are termed 'models' to support pupils as they develop mathematically (van den Heuvel-Panhuizen (2003)). These models initially emerge from a context and in many cases may at the early stage be little more than a representation (a picture say) of a contextual problem. Importantly, however they then become a tool for solving problems and developing strategies. In doing this they bridge the gap between the informal and the formal and hence teachers feel less need to replace pupils' informal knowledge in the way discussed earlier. Models also allow pupils to work at differing levels of abstraction, so that those who have difficulty with more formal notions can still make progress and will still have strategies for solving problems (we shall see examples of this later). Within the context of fractions models would include pie charts, ratio tables, fraction bars, and the double number line. We will exemplify the important characteristics discussed above by focussing on just one of these, namely the fraction bar. Figure 1 shows a number of contexts used during Year 7 work on fractions.

Very briefly, at an early stage pupils work with submarine sandwiches and are encouraged to draw in order to show what they are doing. Being conveniently rectangular, this serves as an introduction to the bar as a possible model for working with fractions. A little later, pupils are working with cans of coconut milk and addressing issues such as whether we can pour Can a into Can b if, say, one is $^{1}/_{3}$ full and the other $^{3}/_{4}$ full. Pupils regularly draw rectangles instead of cans at this stage, a representation encouraged by also looking at cutting up (conveniently rectangular) food, and then shading a (rectangular) meter to represent the occupancy of a car park. The model by now, though still closely connected to the context, is becoming a tool for comparing different fractions. This is extended further (Figure 2) when addition and subtraction of fractions is first met (one of the cartons is $^{3}/_{10}$ apple juice, and the other $^{1}/_{4}$, and a bar of length 40 is used to compare the amounts).

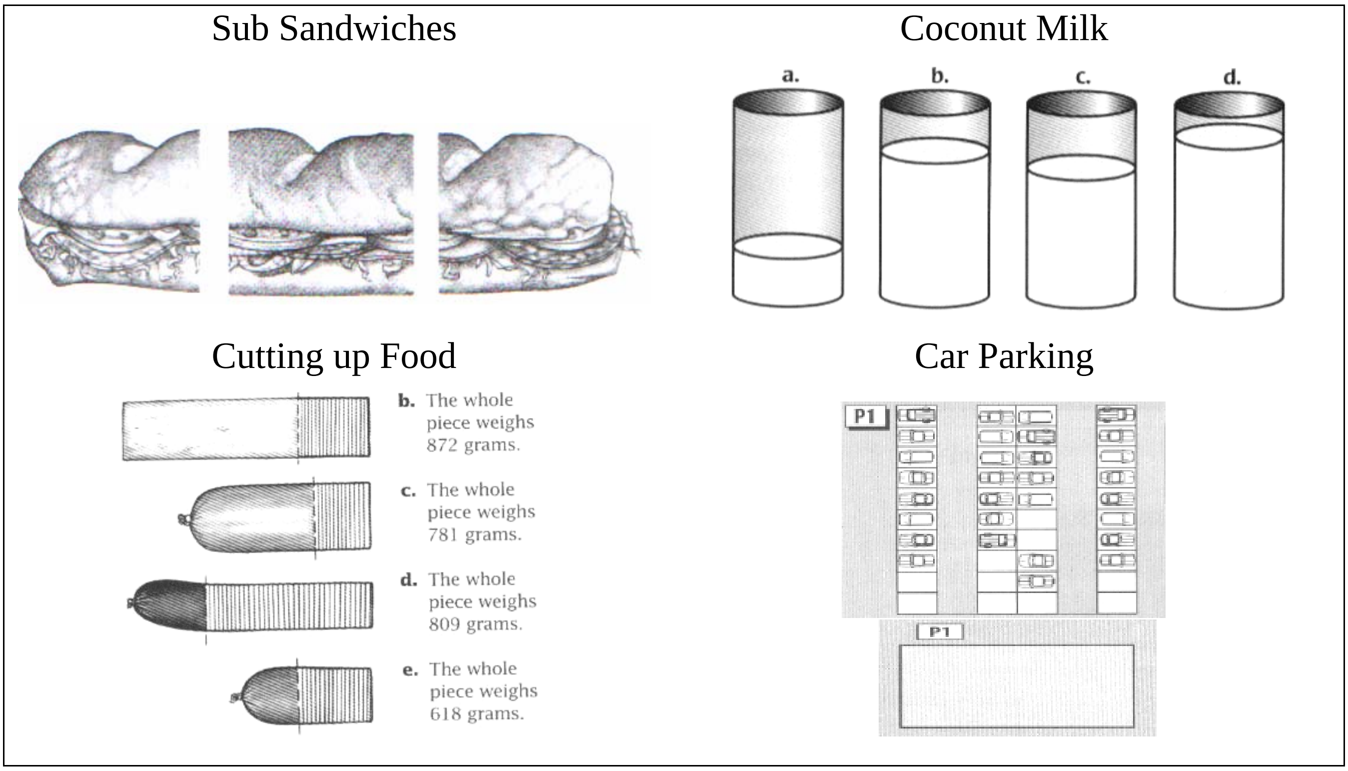


Figure 1

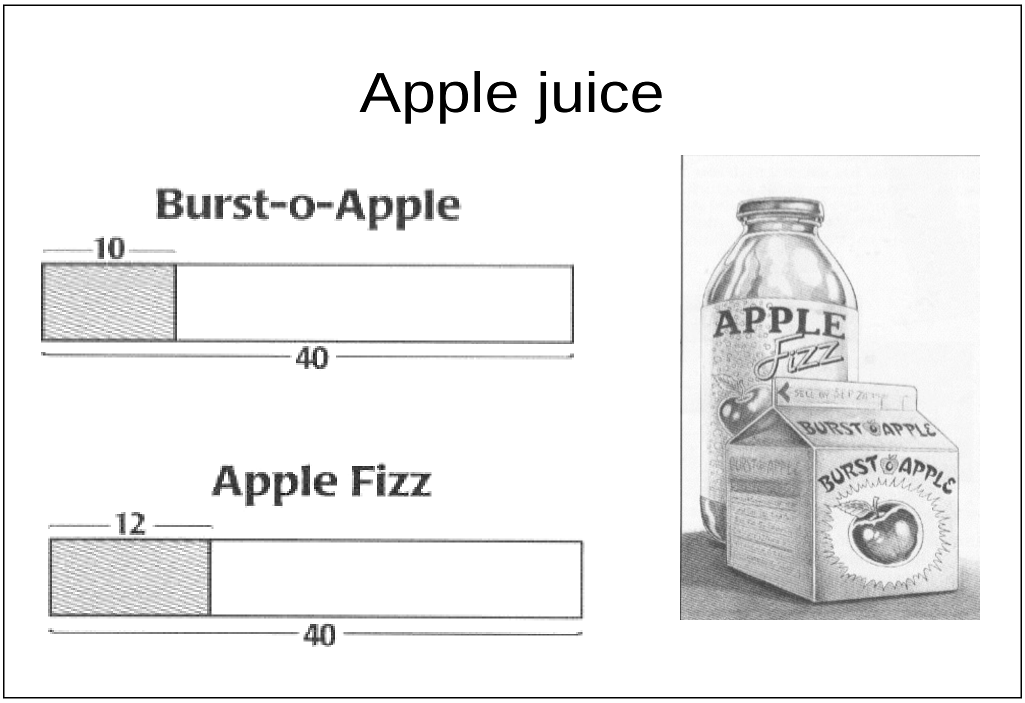


Figure 2

It is through this progressive formalisation of models (Figure 3), from picture of a context to more abstract mathematical diagrams, that pupils make progress towards the formal notions of equivalence and the use of common denominators. But this happens in a way which allows pupils to stay close to the ‘reality’ of the situation, and to return to more informal, primitive strategies as the need arises.

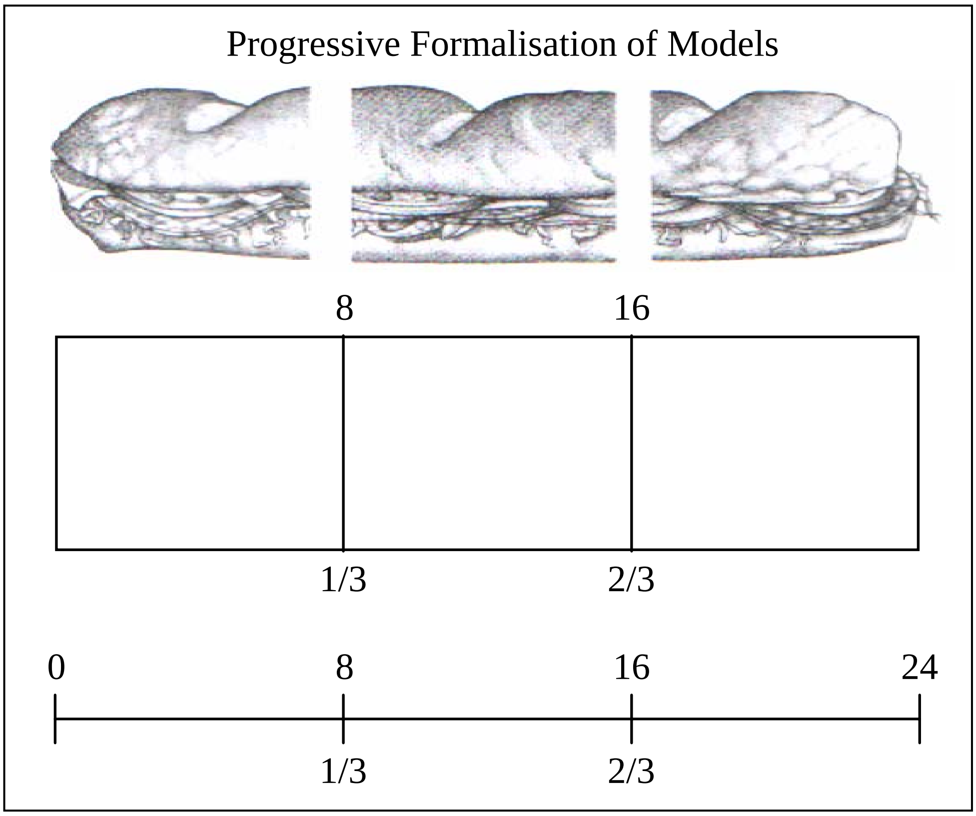


Figure 3

We shall now look at some examples of this, and in particular evidence that the lower and middle achieving pupils in the project and control groups use significantly different strategies when attempting problem solving questions. We will concentrate on three particular questions. The questions and exemplars of pupil work are shown in Appendix 1.

Onion Soup Recipe Question

for 8 people

- 8 onions
- 2 pints of water
- 3 chicken stock cubes
- 2 desert spoons of butter
- $^{1}/_{2}$ pint of cream
- 1 teaspoon of parsley

If you were cooking for 6 people, explain how you would work out:

a) How much water you need

b) How much parsley you need

c) How much cream you need

Part c is the most difficult and when utilised as part of the Chelsea Diagnostic Mathematics Tests (Hart (1985)) it was found that about a quarter of the pupils got this correct and a similar fraction obtained $^{1}/_{3}$ as the answer, perhaps believing that the only fraction between $^{1}/_{4}$ and $^{1}/_{2}$ is $^{1}/_{3}$.

In our test the results are summarised in Table 1.

Recipe (part c)	L.A.		M+	
	P	C	P	C
Correct	0%	0%	26%	13%
Recognising Structure or answer 3/8	16%	0%	44%	26%
No working	48%	56%	28%	34%

Table 1

From the table it is clear that although the lowest attaining pupils found this difficult, 16% of the project pupils provided evidence of understanding. Examples of lower attaining project pupils' responses are shown in Figures 4 and 5. Figure 6 is the most sophisticated response from a lower attaining control pupil.

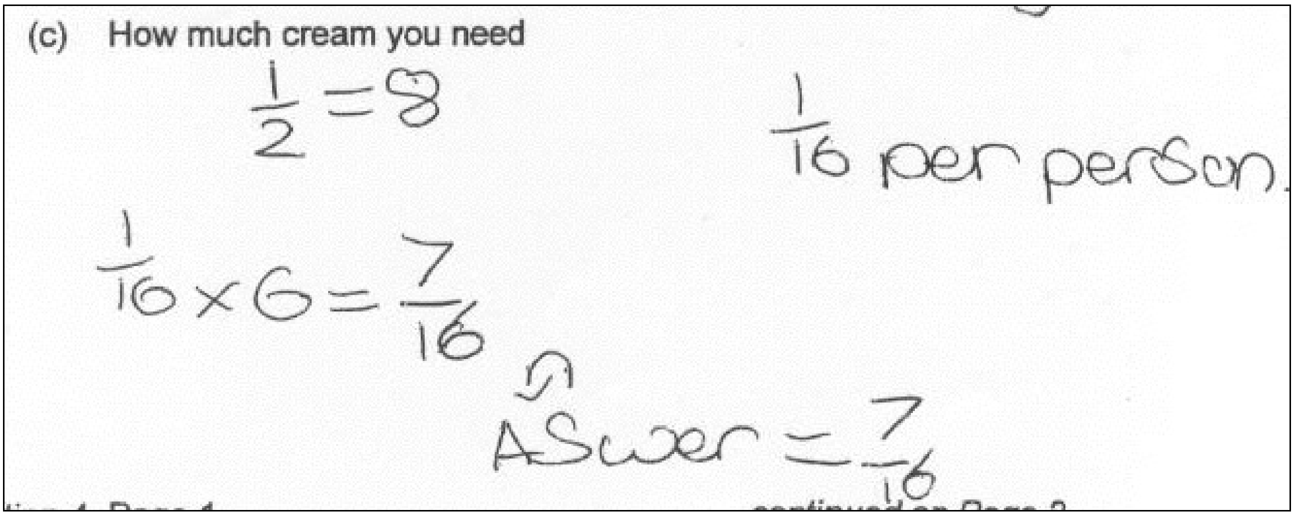


Figure 4

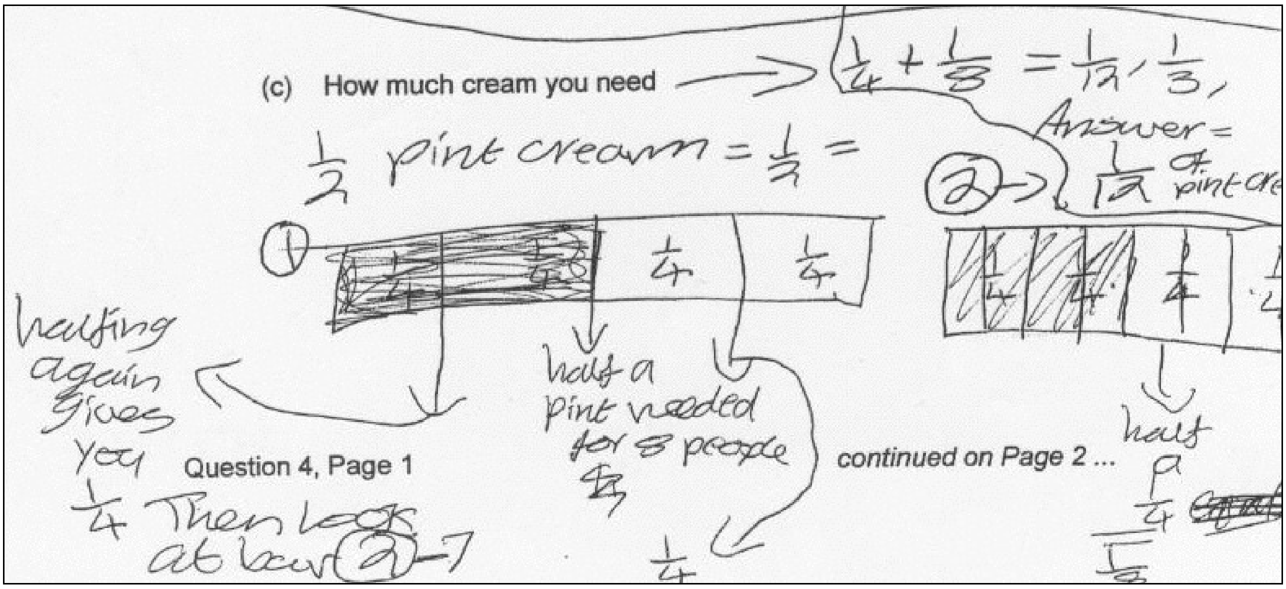


Figure 5

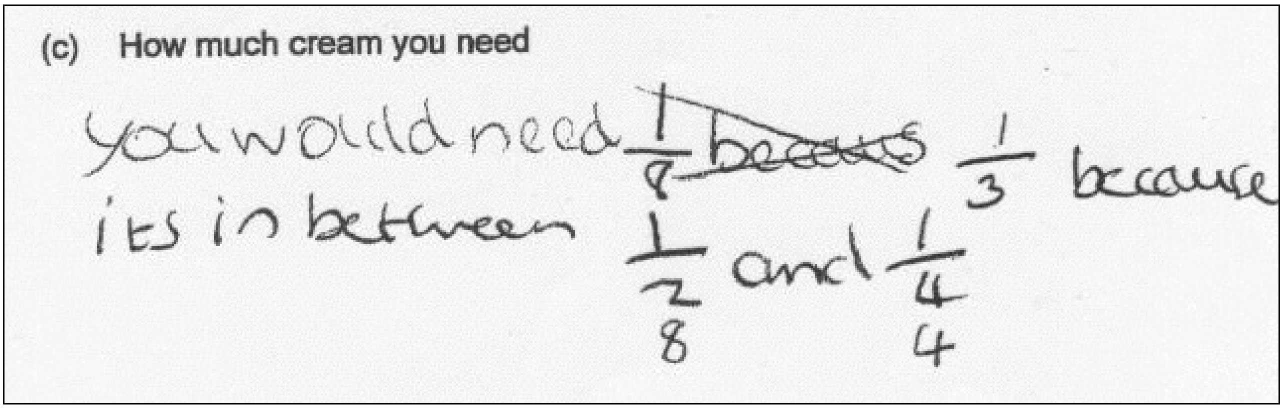


Figure 6

Trapezium Question (Fig 7)

This question was thought to be particularly useful as neither project nor control pupils are not taught to find the area of a trapezium

Trapezium	L.A.		M+	
	P	C	P	C
Correct	15%	0	18%	16%
Sensible drawing	35%	8%	36%	18%
% correct/drawing	76%	0%	45%	72%
Numerical only (all incorrect)	50%	70%	46%	60%

Table 2

The results summarised in Table 2 suggest that not only do the drawings support correct solutions but also provide evidence that the control pupils are more inclined to work at a purely numerical level and this working is not helpful in arriving at an appropriate solution. Figures 7, 8 and 9 show low attaining project pupils' solutions and Figure 10 shows a typical numerical solution of a control pupil.

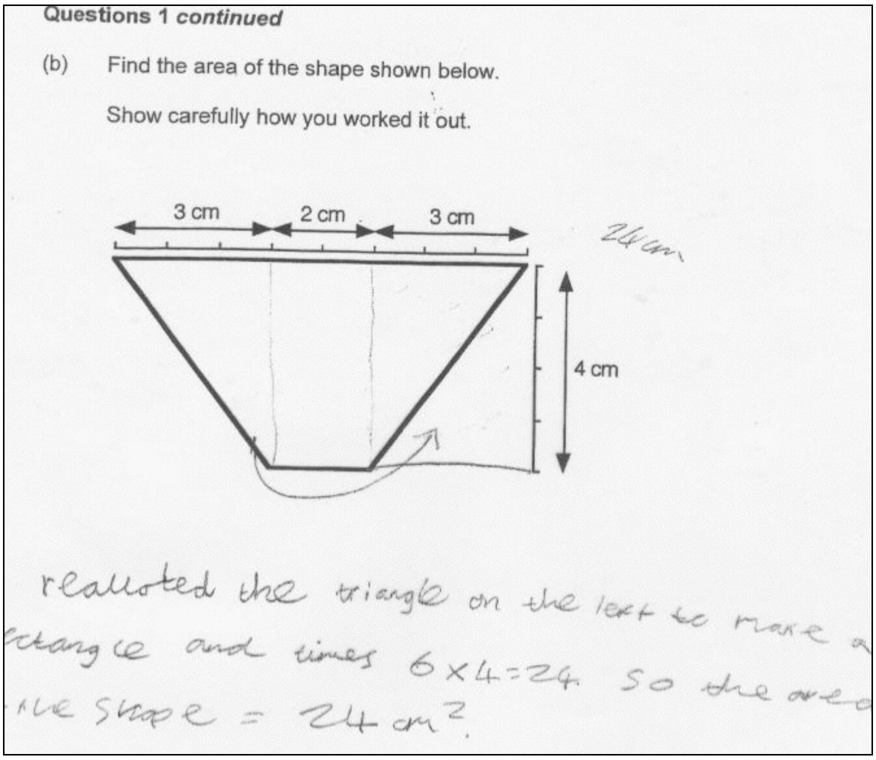


Figure 7

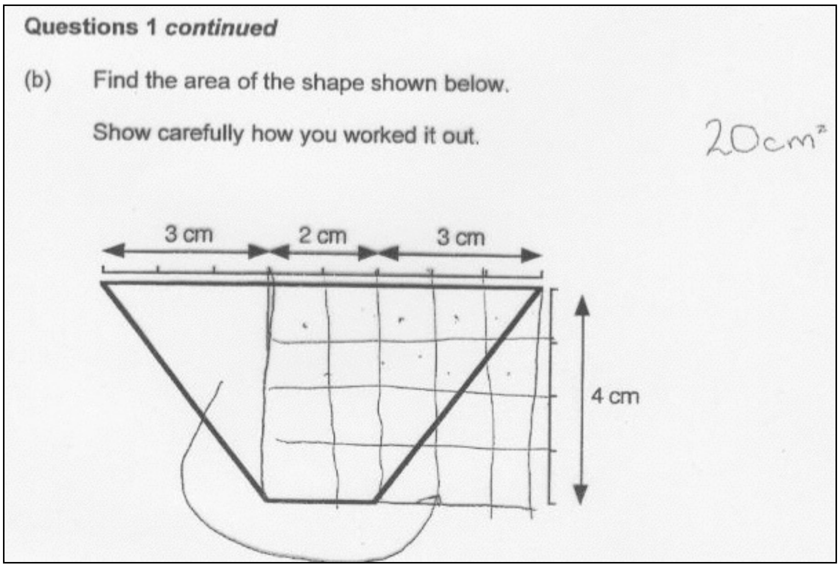
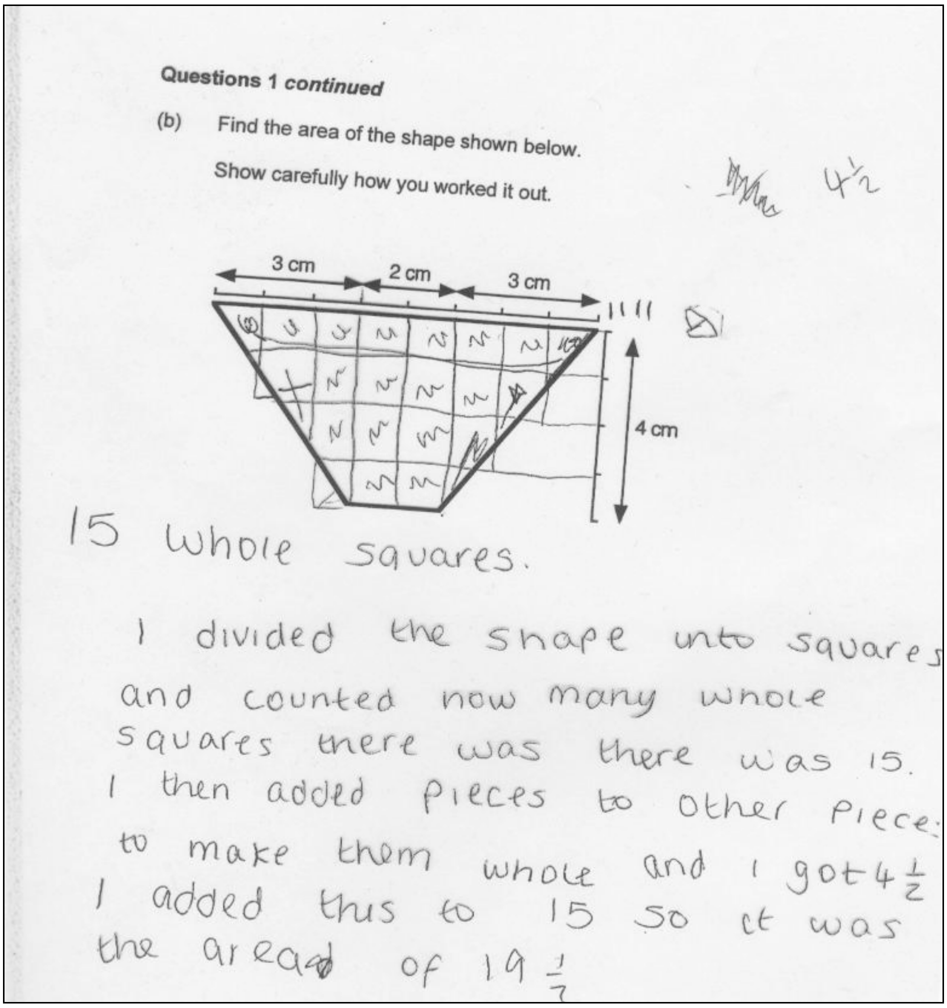


Figure 8



Questions 1 ***continued***

(b) Find the area of the shape shown below.

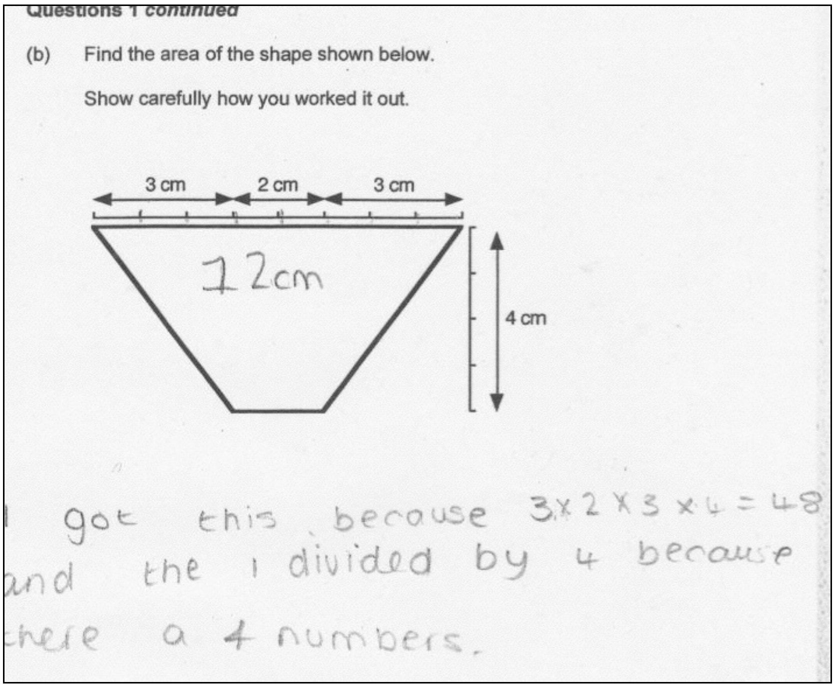
Show carefully how you worked it out.

4½

15 whole squares.

I divided the shape into squares and counted how many whole squares there was there was 15. I then added pieces to other pieces to make them whole and I got 4½ I added this to 15 so it was the area of 19½

Figure 9



Questions 1 ***continued***

(b) Find the area of the shape shown below.

Show carefully how you worked it out.

I got this because 3x2x3x4 = 48 and the I divided by 4 because there a 4 numbers.

Figure 10

Tape Question (Fig 11)

This question requires pupils to demonstrate some understanding of the relationship between $^{1}/_{3}$ and the whole when set in context. Table 3 summarises the results.

Tape	L.A.		M+	
	P	C	P	C
Correct	37%	6%	64%	36%
Drawing	55%	19%	46%	30%
% correct/drawing	55%	33%	59%	39%

Table 3

Again we see that not only are project pupils more likely to draw but the drawings seem to provide some strategic support. Figures 11 and 12 show low attaining project pupils' attempts and Figure 13 shows a typical numerical solution.

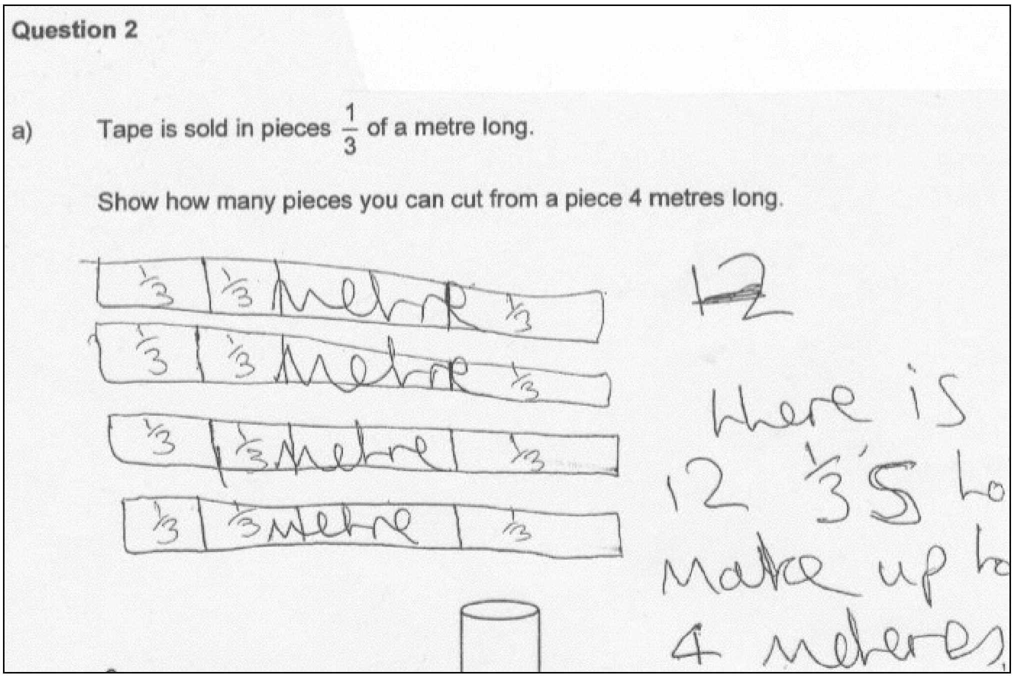


Figure 11

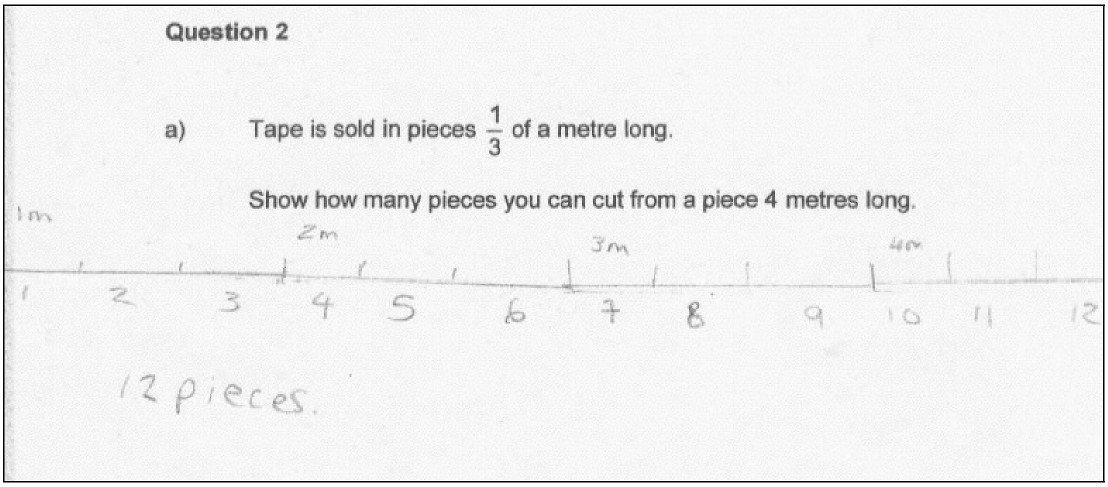


Figure 12

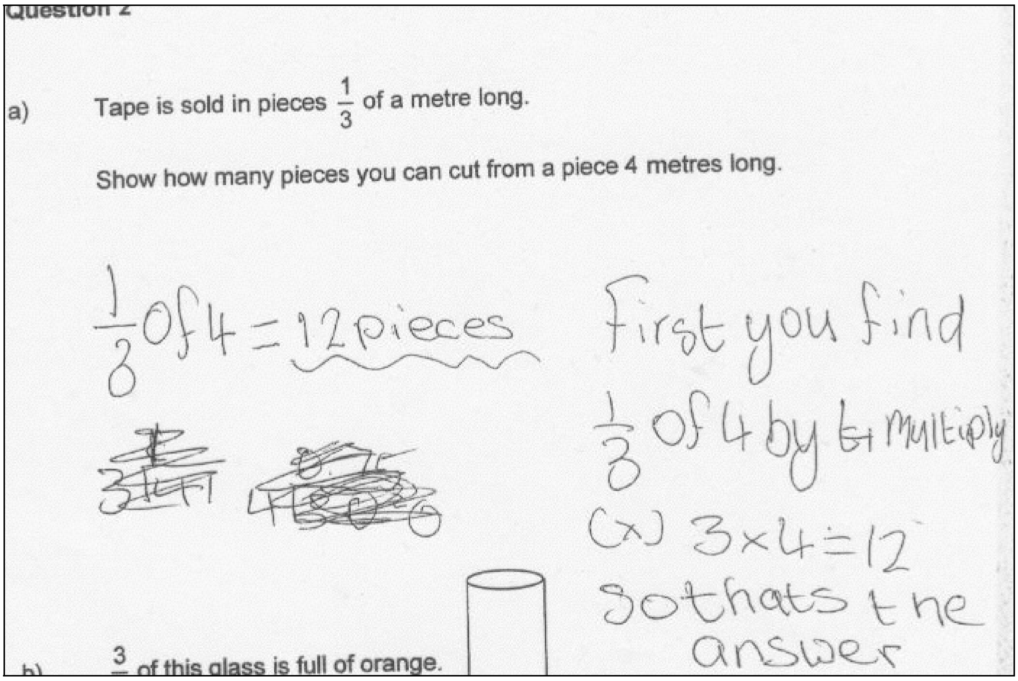


Figure 13

CONCLUSION

Whilst it is rather early in the life of the project to be claiming it to be a success, there is at least tentative evidence to suggest that the approaches to teaching under RME encourages pupils to refine and develop their informal strategies and that these strategies facilitate problem solving in some situations.

APPENDIX: PROJECT DETAILS

RME

RME is an approach to teaching utilised in the Netherlands and developed over a period of thirty years. The approach is significantly different to the approaches used in England in a number of regards:

- Use of realistic situations as a means of developing pupils' mathematics as opposed to using contexts as an introduction to mathematics or as an application of mathematics.
- Less emphasis on algorithms and more on sense making and gradual refinement of informal approaches.
- Emphasis on refining and systematizing understanding.
- Less emphasis on linking single lessons to direct content acquisition and more on gradual development over a longer period of time.
- Greater emphasis on research into learning and teaching and of trialling and refining materials in schools.
- Use of guided reinvention rather than discovery learning or teachers explaining.

Internationally RME is recognised as a very effective approach to teaching content (TIMSS 2003 (IEA 2004)) and mathematical problem solving (PISA 2003 (OECD 2004)).

MiC

MiC is a scheme of work for the middle school age range, based on RME, developed as a result of a collaboration between the Freudenthal Institute and the University of Wisconsin.

The Project

The project explores the implementation of MiC in secondary schools with the following timescale:

- 2004-05: six secondary schools, year 7 pupils
- 2005-06: twelve secondary schools, year 7 and 8 pupils
- 2006-07: limited national trial, year 7, 8 and 9 in local schools and in schools clustered around regional universities

Data on Pupils: Data Collection

In the first year the programme involved 6 schools, each school using MiC with two of its Year 7 classes. In all, this involved 315 pupils ranging in ability from Levels 3c to 5a (based on KS2 SATs scores). One aspect of monitoring pupil development involved the identification of 315 'control' pupils working at identical levels to those in the programme. Since in some schools MiC was being used by all pupils at a particular level (for example both top sets in one school), it was impossible to match within each school. Hence, in many cases, a programme pupil in one school would be matched against a non-programme pupil in another. By doing this, we were able to stratify the control group in the same way as the programme group in terms of gender, schools, and KS2 precise levels (4a, 4b, etc).

In order to meet the aims set out in our original proposal, data was collected from programme and control groups in a number of ways:

- End of Year written test
- Assessment of Problem Solving ability
- Work on Proportional Reasoning
- Beliefs about Mathematics.

Details about data collection, sampling and analysis of the results are provided on <http://s13a.math.aca.mmu.ac.uk/DMtC/Updates/ReportAnnual2005.html>.

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