Clausen-May, T. and Vappula, H. (2005), 'Context in maths test questions – does it make a difference?', in D. Hewitt and A. Noyes (Eds), *Proceedings of the sixth British Congress of Mathematics Education* held at the University of Warwick, pp. 41-48. Available from www.bsrlm.org.uk.

CONTEXT IN MATHS TEST QUESTIONS – DOES IT MAKE A DIFFERENCE?

Tandi Clausen-May and Hanna Vappula

National Foundation for Educational Research

There has been some debate about the extent to which 'stories' and graphical images can help pupils to think their way through numerical problems. The degree to which contexts stimulate useful 'models to think with' may vary considerably. Some contexts seem to encourage pupils to reason more effectively, but others have little impact either on pupils' performance, or on the way in which they tackle the computations required.

MODELS TO THINK WITH

In recent years contextualisation has been given great emphasis in mathematics teaching. According to Sharp and Adams (2002), the U.S. has seen a 'transition away from the behaviourist view' which had focused on determining the best algorithm to use for any particular mathematical operation. Teaching has become more constructivist, and aspects of the 'real world' have been introduced into many mathematics classrooms. Similar changes have taken place in the UK. As Hennessy (1993) explains,

> significant progress has…been made in developing mathematical curricula which build on children's existing knowledge and foster a flexible problem-solving approach (p. 8).

These changes have been made partly in response to claims that school mathematics is disconnected from people's use of mathematics in the real world. Lave (1988) found that adults were highly successful in solving number problems in practical situations but less successful when faced with comparable calculations in a written test. Similarly, it has been suggested that pupils may underachieve because they try to adopt what they perceive to be the acceptable approach to mathematics problems, rather than using their own understanding to devise their own methods of solution (Carraher, Carraher and Schliemann, 1987). In order to bridge this gap between intuitive knowledge and taught algorithms it would seem important to recognise and endorse pupils' own successful methods.

Partly to this end, the National Curriculum and its assessment promote the use of contextualised tasks in mathematics teaching and learning in England. Faced with a contextualised problem rather than an abstract, numerically-presented calculation, pupils may be more likely to think their way through it without resorting to a rote-learnt algorithm. However, such questions do not always have the effects intended. Cooper and Dunne (2000, p. 41) found that some pupils who could carry out uncontextualised calculations correctly were likely to base their responses on their own experience rather than on the data given in a ‘realistic’ problem, and thus lose the mark. Similarly, Boaler (1999, tables 6.2 and 6.3, p. 67) found that between 31 and 42% of Year 9 pupils could solve only an abstract or only a contextualised problem, but not both.

The situated cognition perspective may go some way to explain these differences in pupils’ ability to solve abstract and contextualised problems. Cognition and learning are thought to be dependent on various factors in the learning situation, and people are not always able to transfer skills from one situation to another (Brown, Collins and Duguid, 1989). For example, Hendricks (2001) found that Year 7 pupils who had studied the topic of causality and responded correctly to questions relating to it in one school subject failed to answer similar questions when they were presented during the lessons of another school subject. In relation to mathematics test questions, therefore, it is possible that some pupils struggle to answer abstract mathematics questions even when they are able to solve a contextualised problem, while others might be able to carry out mathematical operations in the abstract but are unable to create the mathematical model needed to solve a contextualised problem.

In some cases, however, the context itself may provide a mental image that could help pupils to solve the problem. For example, in the course of development of a series of age-standardised mental mathematics tests a sample of 1111 rising 13-year-olds were given two questions involving the division of a mixed number by a fraction (Clausen-May, Claydon and Ruddock, 1999). One was contextualised and the other abstract.

Contextualised: How many quarter hours are there in three-and-a-half hours?

Abstract: Divide two and a half by a quarter.

Although they are mathematically similar, the two questions make very different demands. The context used in the first may have offered some pupils a mental ‘model to think with’ of quarters, wholes and halves, while the purely numerical presentation of the second, abstract question was more likely to encourage an algorithmic approach. In the trial 73% of the pupils were able to solve the first question correctly, but only 35% – less than half as many – were able to answer the second (Clausen-May, 2001, p. 29). It seems possible that in this case a mental image was triggered by the context, and this provided effective scaffolding for a number of pupils.

This result, which emerged from the trialling associated with one test in a series, led us to look for similar pairs of mathematically equivalent questions with different formats that could be trialled in other test development projects. The data that are available are, in a sense, a by-product of the test development process, and as such they are clearly limited. However, while it is often frustrating to be unable to follow up issues that arise from the research, it may none the less contribute some worthwhile results.

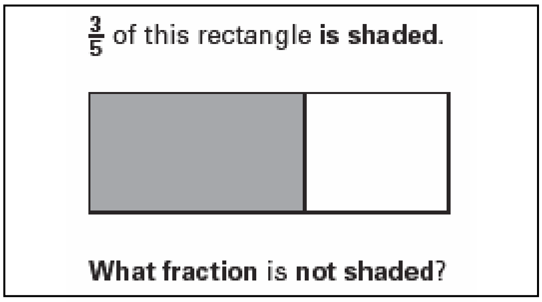
DIAGRAMS AND STORIES

In the course of development of another series of age-standardised tests, two sets of four questions were trialled (Clausen-May, Vappula and Ruddock, 2005). In each case the four questions were in two matched pairs, and about half of the pupils in each year group in the total sample answered each pair of questions. The two sets assessed:

- *Subtraction of a fraction from a whole.*
  Four questions were trialled with a total sample of 2717 pupils in Year 4 to Year 9, aged from 8 to 14 years. Two of the questions had a diagram printed on the page while the other two were presented in a numerical format.
- *Subtraction of a four digit number from another four digit number.*
  Four questions were trialled with a total sample of 930 pupils in Year 8 and Year 9, aged from 12 to 14 years. Two of the questions had 'stories' and pictures printed on the page while the other two were in abstract format.

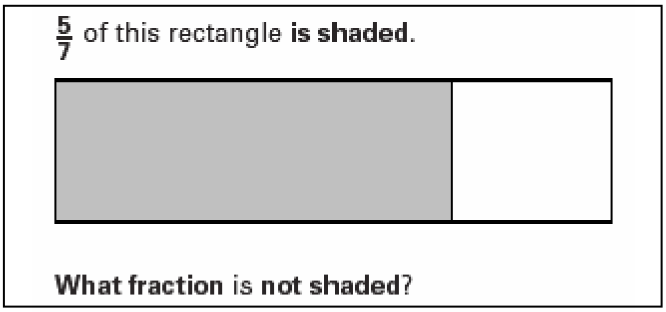
These sets of questions enabled us to compare pupils' performance on matched pairs of questions, with different formats and different degrees of contextualisation. The first set of questions are given in Figure 1.

Questions for Sample A



$1 - \frac{5}{7} =$ ____________

Questions for Sample B



$1 - \frac{3}{5} =$ ________

Figure 1: subtracting a fraction from 1

As the results in Table 1 show, for each year group in both samples the version of the question with the graphic was easier than the version without. Figure 2 indicates the proportion of pupils who answered the question in only one format, who answered it in both formats, and who did not answer it at all. Overall, 40% of the pupils answered both questions correctly, while another 30% answered neither. 23% were able to answer the contextualised question but not the abstract, while only 7% were the other way round.

The rectangular diagrams appear to have changed the nature of the question. They offered a visual model on which pupils could, perhaps, base their reasoning. This may have worked in the same way as the image of quarters, halves and wholes triggered by the mental test question described above. Thus the results from the first set of four questions do support the argument that a ‘model to think with’ may help at least some pupils to solve some sorts of arithmetical problem.

		YR4	YR5	YR6	YR7	YR8	YR9
	Number of pupils	≥222	≥234	≥205	≥218	≥239	≥219
	Question						
Booklet A	3/5 rectangle	22%	51%	68%	78%	87%	93%
	1 – 5/7	8%*	24%	36%*	36%	55%	68%
Booklet B	5/7 rectangle	14%*	38%*	58%	68%	84%	86%
	1 – 3/5	12%	32%	51%	55%	72%	74%

**In order to monitor the possible effects of question order, a star has been inserted whenever the second question in a pair showed an omission rate 5 to 13% higher than that of the first.*

Table 1: The percentage of pupils in each year group who got each fraction question right.

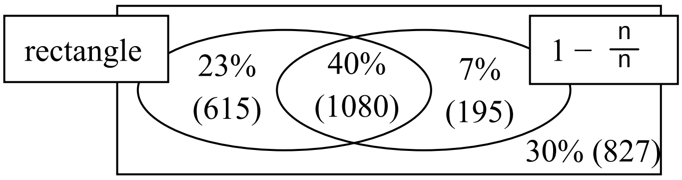
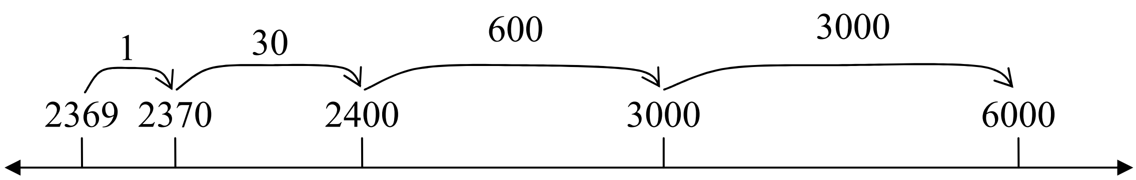


Figure 2: The percentage and number of Year 4 to Year 9 pupils who answered the contextualised and abstract fraction questions.

The results from the second set of four questions, however, were much less clear. In the 1995 TIMSS study, rising 14-year-old pupils in England were found to perform badly in comparison to pupils from other countries on a multiple choice question involving a vertically presented subtraction (Whitburn, 1999, p163):

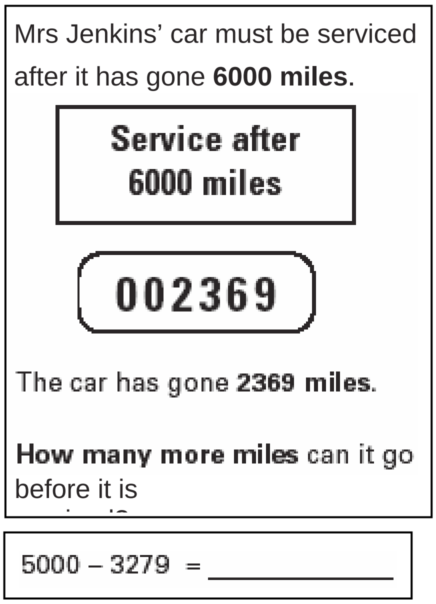
6000
–2369
_____Options: 4369; 3742; 3631; 3531

In England, only 59% of 13-year-olds and 65% of 14-year-olds were able to select the correct option, as compared to an international mean of 86% for each group. It seemed possible that this performance might be improved if a suitable context, which might encourage pupils to use an appropriate ‘model to think with’, were offered. In particular, a number line could provide a visual structure to guide pupils through the steps involved in subtracting 2369 from 6000.



Two contextualised questions were trialled. Here again, each one was paired with the uncontextualised computation found in the other, as shown in Figure 3.

Questions for Sample A



Questions for Sample B

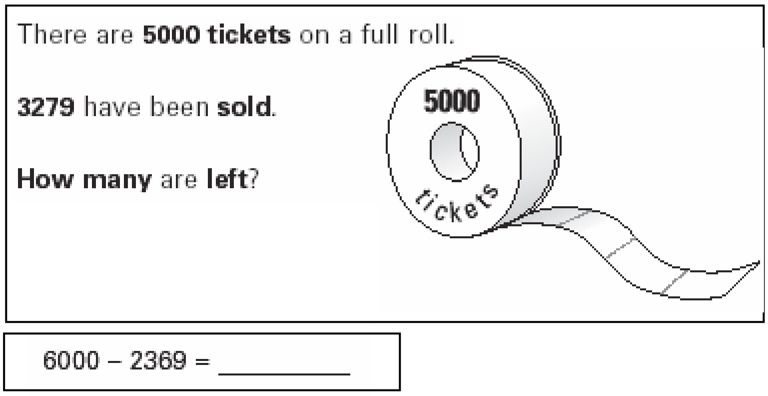


Figure 3: Subtracting whole numbers

Each of these contextualised questions would allow the image of a number line to be used. In *Service*, for example, pupils might visualise the total distance of 6000 miles as a long, straight road, with 2369 already travelled. In *Tickets* the roll of tickets could be unrolled to create a continuous strip, like a number line, 5000 units long, with 3279 units cut off. However, the results indicated that this attempt to encourage the use of a 'model to think with' was unsuccessful.

The questions are open response, rather than multiple-choice. This would be likely to lower facilities in comparison to the TIMSS questions since it would be expected that, if pupils guessed the answer to the multiple choice question, about a quarter of them would get it right. This being the case, the results on the four questions were broadly comparable to the international means in the TIMSS study (see Table 2).

	Booklet A	YR8	YR9
	Number of pupils	252	220
Question	Service	64%	78%
	6000 − 2369	68%	77%

	Booklet B	YR8	YR9
	Number of pupils	239	219
Question	Tickets	62%	77%
	5000 − 3279	61%	76%

Table 2: The percentage of pupils in each year group who got each four digit subtraction question right.

What was not apparent, however, was any difference in the facilities for the questions set in and out of context. Roughly the same number of pupils could do either.

Furthermore, most of the pupils who could do a question in one format could also do one in the other. The number of pupils who got each of the four questions correct is shown in the Venn diagram in Figure 4.

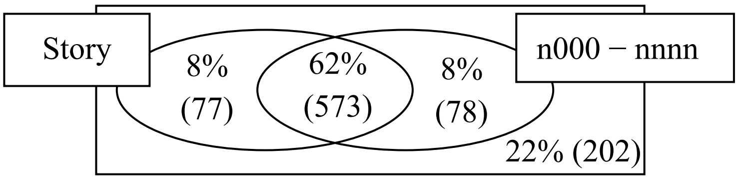


Figure 4: The percentage and number of Year 8 and Year 9 pupils who answered the contextualised and abstract whole number questions.

It is clear that the presence or absence of a 'story' in these questions made very little difference to pupils' performance. Overall, 62% of the pupils answered both questions correctly, while another 22% answered neither. Only 8% were able to answer the contextualised question but not the abstract, while another 8% were the other way round. Thus the great majority of pupils either answered both the contextualised and the abstract question in whichever test they took, or they answered neither.

Furthermore the contextualisation of the questions, or the lack of it, had little effect on the methods that pupils used to answer them. Data were collected on the number of pupils who used formal written algorithms, more informal written or drawn methods, and mental strategies for each question in the two sets. Very few pupils showed any working at all for the subtraction of a fraction from a whole, but Table 3 shows the results for the whole number subtraction questions, in and out of context.

	YR8 (12-13 yrs)		YR9 (13-14 yrs)	
	In context	Abstract	In context	Abstract
Formal written algorithms	58% (286)	60% (296)	64% (280)	65% (287)
Informal strategies	29% (142)	26% (126)	28% (123)	26% (116)
Mental strategies	8% (41)	10% (50)	5% (20)	4% (19)
Omitted	4% (22)	4% (19)	4% (16)	4% (17)

Table 3: The number of pupils attempting the whole number subtraction questions by formal, informal or mental strategies; or omitting the question.

It was slightly more common for pupils to attempt the uncontextualised questions using a formal written algorithm, and the contextualised questions using an informal written method or by drawing a sketch. Overall, however, putting these subtraction questions into context had little impact either on the pupils' performance, or on the way that they approached the problems.

SO, WHAT DIFFERENCE DOES IT MAKE?

Pupils generally found it easier to subtract a fraction from a whole when a rectangle with some shading was printed on the page. However, the same effect was not apparent when a picture and a story required the pupil to subtract a four digit number from another four digit number. The results from the two sets of questions also differ on how common it was for pupils to be able to solve only the contextualised or only the abstract versions. In the fraction questions the diagram was clearly helpful, even for those pupils who could not solve the abstract version. In the story problems, on the other hand, a small number of pupils seemed to find the story helpful, but an equally small number did not. Pupils' working out did not reveal any clear differences in how they approached the contextualised and abstract versions of the whole number subtractions, although there could be a tendency for pupils to choose an informal strategy when solving contextualised problems.

With the two whole number subtractions discussed here, there was no evidence that pupils had difficulty transferring their knowledge from the abstract to the contextualised and visa versa. The fact that only 16% could answer either only the contextualised or only the abstract question seems to contradict Boaler's (1997) and Cooper et. al's (2000) findings. Since Boaler's and Cooper's samples were drawn a few years earlier than the current sample, these results could indicate a general improvement in pupils' ability to apply their skills in these types of mathematical question. However, in the fraction questions a proportion comparable to that in Boaler's study answered only one type of question.

A number of factors could help to explain why the two sets of four questions discussed here gave such different results. First, it could be that the level of scaffolding provided in the contextualisation is significant. In the fraction questions the relevant image was printed on the page, whereas in the whole number subtractions the pupil had either to draw the image or to visualise it mentally – and this, perhaps, they did not do. Second, it could be that calculations which involve fractions make qualitatively different cognitive demands to those dealing with whole numbers alone. It is interesting to note that the mental mathematics question where a context seemed to make a positive impact on pupils' success (Clausen-May, 2001, p. 29) also involved fractions. Third, it might be worth considering pupils' familiarity with the question presentations. Lowrie and Kay (2001) have shown that pupils tend to use visual methods to solve novel problems, while they are more likely to use non-visual strategies if the question looks familiar. Word problems are perhaps more familiar to pupils than diagrammatic presentations of fractions, so it could be that they triggered well-practised strategies in pupils' problem-solving rather than engaging them in more intuitive processes.

REFERENCES

Boaler, J. (1997) Experiencing School Mathematics: Teaching styles, sex and setting. Buckingham: Open University Press.

Brown, J., Collins A. and Duguid, P. (1989) 'Situated cognition and the culture of learning', *Educational Researcher*, **18**(1), 32 – 42.

Carraher, T., Carraher, D. & Schliemann, A. (1987) 'Written and oral mathematics', *Journal for Research in Mathematics Education*, **18**(2), 83 – 97.

Clausen-May, T., Claydon, H. & Ruddock, G. (1999) Mental Mathematics 6-14. Windsor: NFER-Nelson.

Clausen-May, T. (2001) An Approach to Test Development. Slough: National Foundation for Educational Research.

Clausen-May, T., Vappula, H. & Ruddock, G. (2005) Progress in Maths 4-14. London: NFER-Nelson.

Cooper, B. & Dunne, M. (2000) Assessing Children's Mathematical Knowledge: Social class, sex and problem-solving. Buckingham: Open University Press.

Hendricks, C. (2001) 'Teaching causal reasoning through cognitive apprenticeship: what are results from situated learning?', *The Journal of Educational Research*, **94**(5), 302 – 311.

Hennessy, S. (1993) 'Situated cognition and cognitive apprenticeship: implications for classroom learning', *Studies in Science Education*, **22**, 1 – 41.

Jaworski, B. & Phillips, D. (Eds) (1999) Comparing Standards Internationally: research and practice in mathematics and beyond. Oxford: Symposium Books.

Lave, J. (1988) *Cognition in Practice: Mind, mathematics and culture in everyday life.* Cambridge: University Press.

Lowrie, T. & Kay, R. (2001) 'Relationship between visual and nonvisual solution methods and difficulty in elementary mathematics', *Journal of Educational Research*, **94**(4), 248 – 255.

Sharp, J. & Adams, B. (2002) 'Children's construction of knowledge for fraction division after solving realistic problems', *Journal of Educational Research*, **95**(6), 333 – 347.

Whitburn, J. (1999) 'Why can't the English learn to subtract?', In B. Jaworski & D. Phillips (Eds) Comparing Standards Internationally. Symposium Books 61 – 78.