THE USE OF PRACTICAL AND EXPERIMENTAL TASKS IN GEOMETRY TEACHING: A STUDY OF TEXTBOOKS BY GODFREY AND SIDDONS

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In this paper, I consider the following issues: how were experimental tasks being developed after being introduced in the early 20th Century, and how did people at that time regard the relationship between practical and formal geometry? To address these issues, I focus on the geometry textbooks (1903, 1912) and their revisions by Godfrey and Siddons. As a result of the examination, the following things can be concluded. First, the development of the experimental tasks can be described as a modification of initial introduction of experimental tasks after 1903. Secondly, Godfrey considered that experimental tasks should not be excluded from deductive stages in geometry, because, the 'geometrical eye' should be developed through experimental tasks at any stage of geometry.

INTRODUCTION

In the teaching of geometry, practical and experimental work such as observation, drawing or measurement is very important, because these tasks often make students familiar with geometrical figures as well as leading them to discover various geometrical properties. In fact, in today's geometry curriculum in primary and secondary schools, the early stages of geometry often comprise such experimental tasks before deductive geometry, e.g., geometry in the Japanese 'National Course of Study' (Japan Society of Mathematics Education, 2000). On the other hand, the transition from practical to formal is one of the major concerns in the study of the teaching of geometry. For example, Kuninume stated that students often confuse the validity of experiments and mathematical proofs in secondary schools, when they start to learn deductive geometry (Kunimune, 2000, pp. 48-9). To tackle these issues, it might be important to employ an historical approach, since it often provides us with an opportunity to reflect on today's teaching. In particular, I focus on the early 20th Century in England, when the introduction of experimental tasks was particularly advocated by various individuals and organisations: examples include in Perry in 1901, Godfrey and Siddons *et al* in 1902, and the Mathematical Association in 1902). In this paper, the questions I shall address are a) How were the experimental tasks developed after being introduced?, and b) How did people at that time regard the relationship between practical and formal geometry?

THEORETICAL FRAMEWORK

I achieve the issues in this paper by analysing experimental tasks in textbooks used in the early 20th Century. As a theoretical framework, I employ Schubring's scheme (1987), which suggested that the historical study in mathematics education examine a given textbook author and his collected works, i.e., textbooks, because school textbooks are often revised and therefore examining their revisions would give us various information about changing trends and ideas in teaching (Schubring, 1987, p. 41). The subjects are the geometry textbooks by Godfrey and Siddons, the prominent textbook authors and major players in the reform in the early 20th Century in England. They published their first geometry textbook *Elementary Geometry* in 1903, and then published *A Shorter Geometry* in 1912. Even though their textbooks were considered as important texts in the history of mathematics education in England (Howson, 1982), the detailed examinations have yet to be undertaken (Fujita, 2001).

The procedure for the analysis of their textbooks is as follows: First, I briefly describe experimental tasks in *Elementary Geometry* and *A Shorter Geometry*. I then focus on the revisions between these textbooks with regard to the experimental tasks. Thirdly, I consider why they revised *A Shorter Geometry* in such ways, and thereby I attempt to understand the development of experimental tasks after their introduction in the early 20th Century. Finally, I discuss what Godfrey and Siddons considered the relationship between practical and deductive geometry.

EXPERIMENTAL TASKS IN THE GEOMETRY TEXTBOOKS BY GODFREY AND SIDDONS

In *Elementary Geometry* (1903) by Godfrey and Siddons, which was the textbook written as a result of the Perry movement in the 1900s and one of the most popular and important texts at that time (Howson, 1982, Quadling, 1996), plenty of experimental tasks can be seen. For example, through the exercise below, students would learn how to measure angles (Godfrey and Siddons, 1903, p. 12):

> **Ex.37.** Measure the angles of your set square (i) directly, (ii) by making a copy on paper and measuring the copy.

Also, exercises below would lead students to discover various geometrical truths which are proved deductively in the later stage (Godfrey and Siddons, 1903, p. 28):

> **Ex. 123.** Cut out a paper triangle; mark its angles; tear off the corners and fit them together with their vertices at one point, as in fig. 50. What relation between the angles of a triangle is suggested by this experiment?

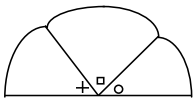

Fig. 50

Also, we can recognise the exercises which students apply the theorems practically. For example, the following exercise would be undertaken after they learn that 'there is one circle, and one only, which passes through three given points not in a straight line'.

> **Ex. 1162.** (Inch paper.) Draw a circle to pass through the points (0, 3), (2, 0), (-1, 0), and measure its radius. Does this circle pass through (i) (0, -3), (ii) (1, 3), (iii) (0, -2/3)

In summary, the roles of these tasks can be summarised as follows: a) to make students familiar with geometrical instruments and figures, b) to lead students to discover geometrical facts, and c) to apply the theorems to practical problems. Also, Godfrey and Siddons include exercises to justify geometrical facts by the experimental tasks, and Godfrey discussed its importance from a general educational point of view (Godfrey and Siddons, 1931, Fujita, 2001, p. 17). In 1909, in accordance with the recommendations of the *Circular 711 on the Teaching of Geometry and Graphic Algebra in Secondary Schools* by the Board of Education, they published their second geometry textbook *A Shorter Geometry* in 1912. Like *Elementary Geometry*, experimental tasks appeared in all parts of this textbook. When we refer to the roles of experimental tasks, it can be said that they were almost the same as those in *Elementary Geometry*. However, it should be noted that some revisions of the experimental tasks can be recognised on closer examination. In the next section, I describe these revisions in *A Shorter Geometry*.

REVISIONS AND NEW PROBLEMS IN THE TEACHING OF GEOMETRY

When we compare *A Shorter Geometry* (1912) to *Elementary Geometry* (1903), three main revisions can be recognised with regard to the experimental tasks. First, as Godfrey and Siddons stated in the preface in *A Shorter Geometry* (Godfrey and Siddons, 1912, p. vii), the number of experimental exercises, particularly drawing and measurement with instruments, were reduced. Secondly, in front of some exercises, the symbol '¶' was added in *A Shorter Geometry*. According to Godfrey and Siddons, the '¶' exercises were intended for discussion in the classroom (Godfrey and Siddons, 1912, p. vii). Finally, drawing figures freehand was recommended in some exercises in *A Shorter Geometry*. The important question to be considered, now, is why Godfrey and Siddons revised in such ways in *A Shorter Geometry*.

From the preface in *A Shorter Geometry*, it is clear that these revisions are related to a new problems in the teaching of geometry, i.e. Godfrey and

Siddons considered that the time required was emerging as a problem in the teaching of geometry as a result of the introduction of experimental tasks‘(Godfrey and Siddons, 1912, p. vii). As to the introduction of the discussion, if students would carry out experiments without knowing what they were aiming at, it would be time consuming. They stated that class work would improve this situation because it would make students aware of what these experiments aim at (Godfrey and Siddons, 1912, p. vii). In fact, Godfrey had already been aware of the class-teaching in 1910. He pointed out that learning mathematics individually had been too much emphasised in English classrooms and it would be a good idea to introduce the class-teaching methods in mathematics (Godfrey and Siddons, 1931, pp. 28-30). He found that the class-teaching was effective for the teaching of mathematics through his teaching experiences in secondary schools. It is, of course, open to question whether the class-room teaching or students' discussions would be effective in mathematics lessons, and this issue is beyond of this paper's contents.

EXPERIMENTAL TASK AND 'GEOMETRICAL EYE'

Godfrey and Siddons considered that the wasting of time was one of new problems in the new methods of teaching, and that is why they revised *A Shorter Geometry* in such ways. It should be, however, noted that experimental tasks were not excluded completely in theoretical stages in *A Shorter Geometry*. When we referred to comments by Godfrey and Siddons, it can be recognised that they were dissatisfied with other teachers' way of using their textbook *Elementary Geometry*, i.e. teachers strictly distinguished practical and deductive geometry. For example, Siddons wrote that 'Godfrey and I were very dissatisfied with the way in which some teachers were using the book [*Elementary Geometry*] – working though all the Practical part first before staring through all the Theoretical' (Siddons, 1952, p. 9).

Now, the important question is why they considered that disconnection between experiment and deductive geometry would be inappropriate in the teaching of geometry. Godfrey considered that mathematics would not be undertaken by only logic (Godfrey, 1910, p. 197). He wrote that another important 'power' would be necessary for solving mathematical problems, i.e. 'Geometrical power', which was 'the power we exercise when we solve a rider' (Godfrey, 1910, p. 197). To develop this 'geometrical power', it would be essential to train students' 'geometrical eye', which was 'the power of seeing geometrical properties detach themselves from a figure' (Godfrey, 1910, p. 197). When we reflect on solving geometrical problems, Godfrey's view is quite right. Let us consider an example 'if A, B are the mid-points of the equal sides XY, XZ of an isosceles triangle, prove that AZ=BY' (Godfrey and Siddons, 1903, p. 94). When we consider this problem, we would not be able to prove this statement unless we can 'see' that, for example, triangle AYZ and triangle BZY are likely to be congruent first of all. Godfrey stated that this kind of

'power' would be essential to solve geometrical problems, and it was experimental tasks that would make possible to train 'geometrical eye' at any stages in geometry:

> There must be a good foundation of practical work, and recourse to practical and experimental illustration wherever this can be introduced naturally into the later theoretical course. Only in this way can the average boy develop what I will call the "geometrical eye". (Godfrey, 1910, p. 197)

Thus, for example, before the theorem 'A straight line, drawn from the centre of a circle to bisect a chord which is not a diameter, is at right angles to the chord', the exercises, which would make students aware of the symmetry of the circle as well as leading them to discover the theorem, are required. Also, the exercises would help students to see the congruency of the triangles, which would be necessary for the proof of this theorem. Now, we can see experimental tasks as developing the 'geometrical eye' in the teaching of geometry, and it can be said that Godfrey considered that the practical and deductive geometry should be combined in the latter in the teaching of geometry.

CONCLUSION

In this paper, I have examined the experimental tasks by looking revisions in the textbooks by Godfrey and Siddons in the early 20th Century. In particular, I have explored the development of these tasks, and the relationship of deductive geometry. After being introduced, the experimental tasks were being settled in proper places in the teaching of geometry in secondary schools. However, through their teaching experiences, Godfrey and Siddons noticed that the number of experimental exercises would be too many and that students carried on these tasks aimlessly, and the time for deductive geometry, still a major emphasis in geometry, was sacrificed. Thus, they reduced the number of such tasks and introduced the freehand drawing of figures, and also suggested discussions in the classroom. During 1903-12, the development of the experimental tasks can be described as a modification of the initial introduction of experimental tasks, accompanied by the development of new ideas of teaching methods. In particular, it is worth considering that discussions about their experiments would be effective in classrooms. In facts, a study found that such discussions, e.g. what would advantages and disadvantages of experimental verifications be?, often motivated to appreciate verifying geometrical facts deductively (Kunimune, 1987, pp. 154-5). It can be said that the teaching by Godfrey and Siddons was very progressing in the early 20th Century. I also examined that what Godfrey and Siddons considered the relationship between practical and deductive geometry. It can be seen that these two types of geometry were by no means separated in the teaching of geometry. As we have seen, Godfrey considered that 'geometrical eye' would be necessary for solving geometrical problems, and that it should be trained by experimental tasks at all stages of geometry. Although Godfrey's view should be verified in

empirical studies, it can be said that this point of view gives an insight as to the relationship between practical and deductive geometry.

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