STUDENT TEACHERS' UNDERSTANDING OF RATIONAL NUMBERS

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Previous research has shown that secondary school students' understanding of fractions is dominated by the part-whole concept to the detriment of their understanding of a fraction as a number in its own right. The present paper reports on an investigation into the understanding of intending primary teachers in this area. Four representatives of a cohort of sixty students on a PGCE course specialising in the lower primary age range were asked detailed questions probing their knowledge of fractions. The conclusion was that the part-whole concept dominates. All of the students were familiar with the numerical concept from their work on the PGCE course, but they reverted to the more familiar part-whole ideas in attempting to solve problems.

INTRODUCTION

Student teachers studying for the primary age phase have the difficult task of gaining sufficient mastery in a range of academic subjects to be able to teach each subject effectively to the full ability range of children, including the most able. This study focuses on one aspect of mathematics, rational numbers, in which children's understanding has been found to be limited, (Hart et al, 1981, Kerslake, 1986). The purpose of the study was to investigate whether student teachers demonstrated the same conceptual limitations as primary and secondary age children. The students in the study were graduates training to teach in the lower-primary age range. The particular focus was the concept of a fraction as a number in its own right, as opposed to a description of part of some well defined whole, for example a pizza or a rectangle.

RATIONAL NUMBER CONSTRUCTS

"Rational number thinking is not a natural thought process" (Pitkethly and Hunting, 1996, p5), and rational number understanding is different from whole number understanding. There have been many attempts to identify the sub-constructs whose integration is required for effective knowledge of rational number. The following four sub-constructs are attributed to Kieren (1976): ratio number; quotient; measure; and operator. Behr, Harel, Post and Lesh (1992,1993) and Kieren (1980) also consider a 'part-whole' sub-construct, which Kieren (1993) subsumes into his measure and quotient constructs. Carpenter, Fennema and Romberg (1993) regard these 'personalities' of rational number as distinct, although Pitkethly and Hunting (1996) note that mature understanding of rational numbers involves both differentiation and integration of the separate sub-constructs. Behr et al (1992) consider that the five sub-constructs named have 'stood the test of time'. The

different interpretations put upon each, however, and the ability to subsume the whole-part construct within others, suggest that the exact distinctions may be a matter of interpretation.

The National Numeracy Strategy Framework for Teaching Mathematics (DfEE 1999) provides non-statutory detail to the statutory mathematics curriculum in England. The Key Stage 1 (5 years –7 years) curriculum refers only to halves and quarters, with a strong emphasis on the fraction as an operator, finding halves and quarters of shapes and sets of objects. Reference to equivalence is in terms of two halves making one whole, either a measures or a part-whole construct within the above analyses. Mental mathematics stresses the halving and doubling of numbers, but this is not explicitly linked with fractions. At Key Stage 2 (7 years – 11 years), the emphasis on the operator continues, but there are significant references to numerical constructs such as ordering fractions on a number line and comparing fractions to decimals. Kerslake (1986) has noted that placing numbers on a fraction line may invoke a measures or a part-whole construct rather than a numerical one. The equivalence between fractions and decimals does allow a focus on the quotient construct, although the many references to division do not, since they emphasise the fraction as an operator on a set or a shape rather than as a number resulting from division. There are references to solving problems involving ratio at the upper end of the primary age range. Such problems may provide a context for children to develop Kieren's (1993, p54) ratio number construct, which he describes thus:

> As ratios, rational numbers are also intensive in character; they are a relational property of a quantity and a unit….

The noteworthy term here is "relational property". This definition, emphasising the relationship between quantity and a 'unit', is about proportion as much as ratio. The curriculum focus on equivalent fractions, if taught in terms of the relationship between quantities, would seem to have potential to develop the ratio number construct.

STUDENT TEACHERS' SUBJECT KNOWLEDGE

Prestage and Perks (2000) draw the distinction between "learner knowledge" and "teacher knowledge". The former is what is required to pass examinations, while the latter overlays this with professional knowledge, practical wisdom and reflection so that teacher knowledge is informed by experience, and in turn influences pedagogy. It is evident, firstly that teacher knowledge can only be acquired with practice, and secondly that it relies upon learner knowledge as one of its foundations. Notwithstanding the complexity of teacher knowledge, Rowland, Martyn, Barber and Heal (1999) report on a study indicating that there is a positive association between mathematics subject knowledge and competence in teaching number among primary PGCE student teachers.

The present study investigates one aspect of the learner knowledge of student teachers in the area of rational numbers, namely whether they have a well developed

concept of a fraction as a number in its own right. Four students from a cohort of sixty training to teach in Key Stage 1, were interviewed in depth about their understanding of fractions and the techniques they used to solve problems. These 'clinical interviews' were recorded on videotape and the responses were used to gain insight into the subjects' thinking. The results reported are from a sub-set of the questions and tasks used.

RESULTS

Students were asked:

- how they would explain fractions to somebody who did not know what they were;

- what came into their head when they heard the words "a quarter";

- to place the numbers three-fifths and one and one-fifth on a number line which ran from zero to five with the points zero to four already marked.

The interviews were conducted individually, but in presenting results I will group the responses.

The part-whole idea was dominant in explaining fractions.

Int: How would you explain to somebody who didn't know what a fraction is?

Karen: It is a number: it can be represented in many different ways, and it's usually thought of as being part of a whole.

Sinead: As part of something.

Mary: It's part of a whole. Something broken into pieces.

Kathy: Part of something whole.

At this point Kathy made a gesture with her hands, indicating a large circle.

It is fair to note that the phrasing of the question validated a simplistic description of fractions. Karen's opening sentence shows an awareness that fractions are numbers, although she immediately then focuses on the non-numerical, part-whole aspect. All of the students had undertaken a course on fractions, including how to teach fractions in primary school, where the numerical aspect of fractions had been emphasised. There were instances where they acknowledged a mismatch between what they now know as a result of teaching on their PGCE course, and their prior understanding, which still dominates their thinking.

Karen: I know we've done the sessions on fractions, and I can remember what was said, but when you actually sit down and do it I revert to the old method.

I had used the word "number" to describe fractions on a worksheet and asked students whether that had surprised them.

Karen: No, but that may be because of the course, because before I started the course I did think of a fraction as being part of something else, and it wasn't really a number by itself.

Sinead: ...just because I've heard you in class talking about them as numbers.

Mary: It's interesting what you said in one of our lectures about thinking of them not as part of a whole but as numbers in themselves, and that's certainly not what we've been taught, any of us, in the generation that we are. We don't see them as numbers

So at an intellectual level the students know that one aspect of fractions is as numbers in their own right, not as operators on an object or a set, but their initial images of fractions continue to be as part of a whole.

In response to the question about their image of a quarter, two of the students responded with a picture of the numeral and the other two with familiar part-whole images in the context of circles.

Int: What comes into your head when I say this word – "a quarter?"

Karen: A quarter of a pizza or a cake.

Mary: A quarter of a cake.

Kathy and Sinead initially said "One over four." Kathy later amended this to "half of a half", and then explained that she was seeing it as a line, dividing the line into two and then into four.

All of the students were presented with a number line five units long, with the points from zero to three marked and labelled. There were no other marks on the line. In a replication of Kerslake's (1986, p32) work with secondary students, the students were asked to mark the number four, and then the number three-fifths. All of the students marked the number four correctly, Mary also marked the number three-fifths. Sinead initially marked the number three, but then spotted her own error.

Sinead: Three-fifths is....it has nothing to do with that three has it? Three-fifths of what?

Both Kathy and Karen marked the three-fifths at the same place as the three, that is, three-fifths of the way along the number line. Karen was later able to describe her error,

Karen: That's three-fifths of the whole line, which isn't what you asked me to do.

These responses suggest that the number line is being seen as a whole that is then divided into parts, and confirm Kerslake's (ibid) doubts that simply placing fractions in order upon a line is evidence of numerical thinking. Had the line been between zero and one the correct result would have been obtained through part-whole methods.

DISCUSSION

>From the interviews it seems clear that student teachers have a strong visual, part-whole image of fractions, and do not naturally regard them as numbers. Furthermore, all of the students recalled having been recently taught about the numerical aspect of fractions and felt that they had understood. Two questions arise. Why have the students developed such limited rational number understanding, and how will it affect their teaching?

It is possible to form hypotheses about both questions. In informal discussions all of the students had talked about their memories of fractions at school. They had only vague memories of primary school, mostly involving cutting and colouring. All described fraction work at secondary school as being about adding, subtracting, multiplying and dividing fractions. It is possible that a primary school curriculum focused on the visual interpretation of fractions, and taught by teachers whose own concepts are overwhelmingly part-whole, leads into a secondary curriculum taught by mathematics specialists whose concepts of fractions are largely numerical. Between the two systems is a discontinuity, and the switch to numbers is not made explicit to the students. If this hypothesis is correct then two actions are indicated; a more integrated approach to fractions in primary school, with some reference to their existence as numbers; and a smoother transition between primary and secondary curricula, with greater acknowledgement of the level of students' current understanding. The National Numeracy Strategy attempts to address both of these issues, but, as has already been noted, even those aspects of it which appear to present numerical ideas, for example comparing numbers on a line, can be interpreted as part-whole constructs.

The second question concerns the ability to teach effectively when one's own constructs are limited. As a result of their learning within the PGCE course all of the students were aware of the numerical aspect of fractions, and the importance of teaching this; their responses make this explicit. They have, however, had little or no experience of thinking of fractions as numbers, and in problem situations revert to their familiar part-whole images. It seems likely that in planning work for children they will be able to reflect upon the constructs that they want the children to develop, including a numerical construct, and prepare appropriate experiences. Within the classroom, however, when responses have to be made with no time for reflection, it is likely that they will fall back exclusively upon their favoured images, for example comparing two-thirds and three-quarters with pictures but without noting their positions on a number line, or translating them into decimal notation by dividing. The challenge for teacher educators is to find ways to help their students to integrate appropriate numerical constructs into their existing rational number schemas so that their teaching is initially founded upon secure learner knowledge with the potential to develop into profound teacher knowledge.

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