YOUNG CHILDREN'S REPRESENTATIONS OF NEGATIVE NUMBERS Rute Borba & Terezinha Nunes

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Previous studies (Nunes, 1993) have observed that uninstructed children and adults have a basic understanding of negative numbers but have difficulties with their representation. This study aims to further investigate the conditions under which meanings for negative numbers are understood and represented by uninstructed children. Sixty 7 and 8 year-olds played one of two games and represented the numbers involved in either an oral mode or by means of manipulative material. The children were able to solve more items when requested to do so orally than when the materials were used. The understanding of measures was greater than that of relations. We provide further evidence of young children's understanding of negative numbers and an effect of number meaning on performance. Attention must be drawn to the different performances when explicit or implicit forms of representation are used and when distinct meanings for negative numbers are involved.
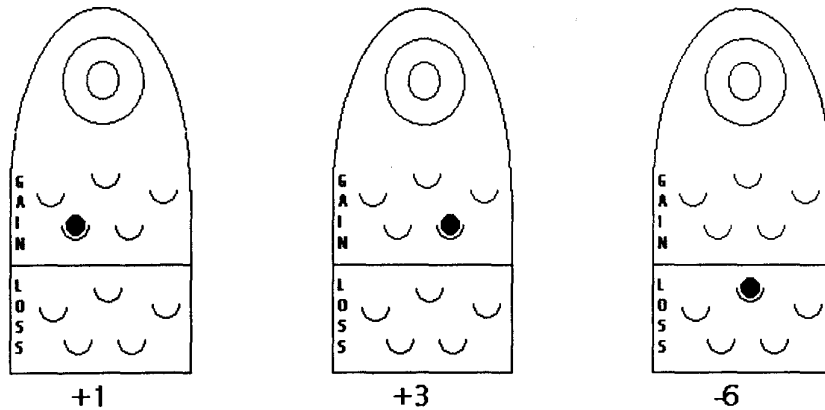
Understanding negative numbers

The aims of this study are to investigate which meanings for negative numbers are more easily understood, and how children's own representations for negative numbers affect their problem solving performances.

Nunes (1993), exploring the role of representations in the understanding of negative numbers, interviewed adults and children before, immediately after, and one year after instruction. The adults and children were randomly assigned either to a group that solved the problems in an oral mode or to one that had to give their answers in a written mode. Understanding of negative numbers was observed even before instruction.' The groups that solved the problems orally performed significantly better than those that used written representations. After an intervention which made clear that the minus sign had distinct meanings negatively marked magnitudes" operation of subtraction and inversion - the performances in the oral and the written mode were not significantly different. Nunes (1993) drew the conclusion that the written representation was the cause of most misunderstandings.
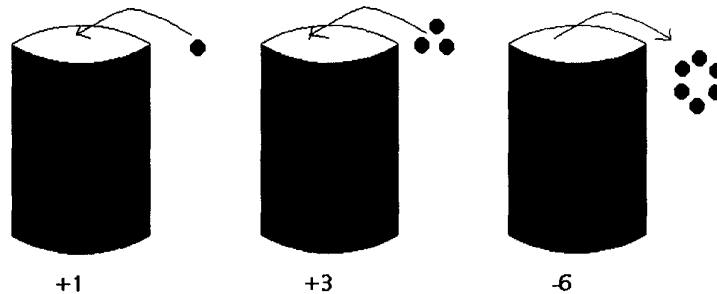
Meanings for negative numbers and the role of representations

The present study aims to extend previous findings. Other forms of representation are involved. The children either solved the problems with no use of external representations (denominated the *oral groups)* or they were designated to groups that had the choice of material they preferred to use (the *material groups).* For this choice the experimenter presented to the children marbles, sticks, sheets of paper, pencil, coloured pens, coloured cards and rulers. Another difference between this study and the previous ones was that the problems involved two meanings for negative numbers: *negatively marked measures* and *negative relations.* In the

context of the *Pinball Game* the children were asked to work out the final score (a measure) when three balls had been shot. In the example below the children were told that the first time the ball was shot there was a gain of one point, followed by a gain of three points and, finally, a loss of six points. They were asked if the final score, after these three transformations, consisted of a gain, a loss or neither and were asked what would be the final score. This situation is represented formally as $(+1) + (+3) + (-6) = (-2)$ but in the interviews the children represented the result, either orally or in some other form of explicit representation, as *a loss of two points.*

In the context of the *Marbles in the Tin Game* the children were told that marbles had been put in or taken out of a tin that already contained some marbles. In the following example they were told that one marble had been put in, then three marbles had been put in, and finally, six marbles had been taken out. The child would have to judge if there were more, less, or the same amount of marbles in the tin and how many more or how many less there were after the three transformations. This judgement involved the understanding of positive relations *(more than)* and negative relations *(less than).*
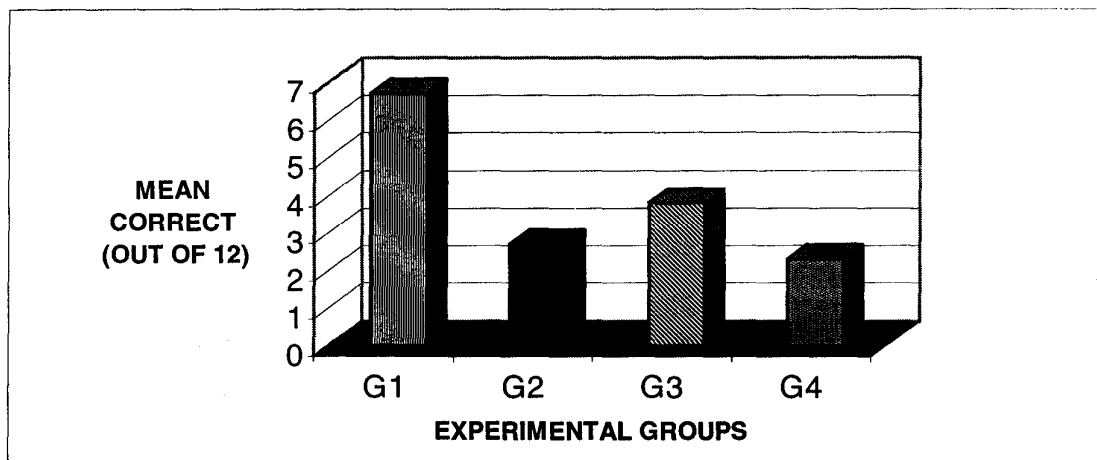
Children in year 3 of a North London mainstream school were interviewed. The 60 children were randomly assigned to one of the four experimental groups, accordingly to the form of representation requested (oral or use of material) and to the kind of negative number involved (measures or relations).

	Oral	Use of material
Measures	G1 (n=15)	G2 (n=15)
Relations	G3 (n=15)	G4 (n=15)

The task-oriented interview consisted of six direct problems and six inverse problems. The direct problems involved three transformations that resulted either in a measure or in a relation, depending on the experimental condition. In the inverse problems the first transformation was unknown.
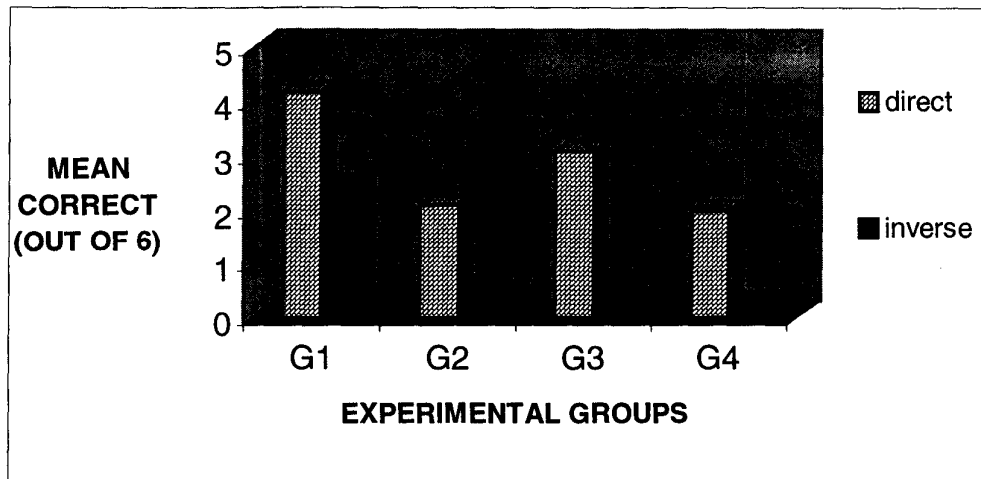
Meanings and representations more easily understood

The experimental groups that solved the problems orally (Gland G3) performed better than those that used external representations (G2 and G4), such as the use of manipulative material or in a written mode.

G1 -   ORAL REPRESENTATION – RESULT: MEASURE
G2 -   USE OF MATERIAL – RESULT: MEASURE
G3 -   ORAL REPRESENTATION – RESULT: RELATION
G4-   USE OF MATERIAL – RESULT: RELATION

For all the experimental groups, solving direct problems was easier than solving inverse problems.

One of the main characteristics of the solutions presented by the children in the oral groups was the flexible way in which they dealt with the numbers presented. The following transcription describes how an 8-year-old child (D.), that solved correctly 9 of the 12 problems, dealt with the gains and the losses separately.

(The examiner (Ex) describes a game in which there was a loss of six points, then a gain of two points and finally a loss of three points. A direct problem: (-6) + (+2) + (-3»

D: Can I do it the other way?

Ex: What's the other way? ...

(Child doesn't answer this question).

D: You lost 7. 'Cause 6 and 3 is nine and then I took away those 2.

Initially the child did not answer what he meant by 'the other way' but, as he tells how he solved the problem, it is clear that the child did not deal with the numbers in the order presented. He dealt with the losses separately (6+3=9) and then subtracted the gain to conclude that at the end there was a loss of 7 points.

Another child (J.) from an *oral group,* that solved correctly 11 of the 12 items, was not only able to deal successfully with the negative measures in direct problems but also in inverse situations.

Ex: I played it once but I'm not going to tell you what I scored. Then I won 4 and I lost 7. I ended up with a loss of 1. What did I score in the beginning? (Describing the inverse problem: (?) + (+4) + (-7) = (-1».

J: You scored 2 points in the beginning.

Ex: How did you get to that?

J: I counted 1,2,3,4. Then I counted backwards to 7 (gets to -3). To get to minus 1 (from **-3)** it's 2.
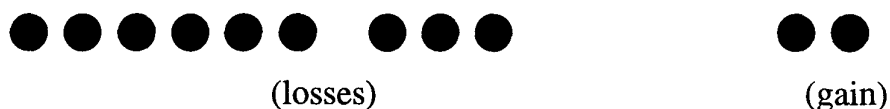
The use of material generally did not aid the children. The procedures below were presented by two children from the second experimental group - that solved the problems with material and that dealt with measures. The problem involved the following transformations: (-6) + (+2) + (-3). Of all 12 problems, the first child (A.) solved correctly 2 problems and the second child (N.) only one.

A. puts 6 sticks on the table (*I I I I I I)* then he puts them back in the pile of sticks. (As if to lose 6 points one must have originally 6). When told that there was a gain of 2 points A. puts 2 sticks on the table *(/1).* The loss of 3 points was represented by putting the 2 sticks back in the pile. The child says that the final score is of "no points".

N. also puts 6 sticks on the table *(111111)* representing the loss of 6 points. Then she puts 2 more sticks on the table *(/11111 II)* representing the gain of 2 points. No differentiation is given between gains and losses, however. When told of the loss of 3 points N. puts 3 sticks back in the pile remaining with 5 sticks *(/1111).* She interprets this as "At the end you stayed with 5 points".

Some of the children were successful in the use of material because they marked differently positive and negative transformations. A child from the fourth experimental group (B.), that is, the group that dealt with relations and used manipulative material, solved correctly 8 out of 10 items. Solving the same problem described above «-6) + (+2) + (-3» he used the sides of the table to mark the transformations.

B. Puts 6 marbles on one side of the table, 2 marbles on the other side and finally 3 more next to the first 6. He interprets his result correctly: "You now have 7 less".

           (losses)   (gain)

Written representations were also successful when the child marked negative numbers in a distinct way. One child put positive and negative signs on the right of the numbers, e.g. 6-, 2+, 3-, representing, respectively, a loss of 6, a gain of 2 and a loss of 3. Another represented these transformations as L6, W2, L3. Still others represented the transformations in a more mathematical formal way. One child (P.), that solved correctly 7 out of 12 items, when answering the same problem proposed above, wrote –6 **+** 2 –3 = 7. Although the answer was not marked negatively he interpreted the result correctly and replied: "At the end you are losing 7".

What can we learn about children's understanding of negative numbers?

The conclusions that can be drawn from this study are:

- Young children may more easily understand negative numbers when they do not have to use external representations.

- The use of manipulative material or written representation is successful when positive and negative values are marked differently.

- Understanding negatively marked measures is easier than understanding negative relations.

The educational implications of this study would be that before being introduced formally and systematically at school to negative numbers, children as young as 7 are able to have some understanding of this complex concept. Their understanding is, however, limited by the form of representation used to solve problems involving negative transformations. Not every meaning for negative numbers is clearly understood and, thus, whilst introducing negative numbers formally the most familiar meanings should initially be used and also it must be made clear the necessity of distinct markings for positive and negative numbers.

Further research is planned to observe how instructional programmes may be designed to aid students in the understanding of negative numbers.

Reference:
Nunes, T. (1993). Learning mathematics: Perspectives from everyday life. In R. Davis & C. Maher (Eds.), *Schools, mathematics, and the world of reality* (pp. 61-78). Needham Heights, MA: Allyn and Bacon.