

TUTORS' REFLECTIONS UPON
THE DIFFICULTIES OF LEARNING AND TEACHING MATHEMATICS
AT UNIVERSITY LEVEL: A REPORT OF WORK-IN-PROGRESS

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ABSTRACT

Following a doctoral study on the learning difficulties of 20 Oxford first-year mathematics undergraduates in their encounter with mathematical abstraction, a study is currently being carried out in which the mathematics tutors who participated in the initial study have been asked to reflect upon samples of its data and findings in semi-structured interviews. The interviews address three areas: clarifications and explanations regarding the pedagogical and psychological language used in the samples, a validation/critique of the interpretations in the thesis regarding the students' learning difficulties and a reflection upon the events in a tutorial from a teaching point of view. The focus of the discussion alternates between a specific (sample-centred) and a general addressing of issues relating to the learning and teaching of undergraduate mathematics. Here short extracts from the interviews are presented and commented upon. Transcription and analysis are now in progress.

The research reported in this paper is a brief follow-up * to the author's doctorate (Nardi 1996) and a pilot to a one-year ESRC-funded project due to begin in October '98 (Jaworski and Nardi 1998). In the following I present the doctoral thesis briefly; then the aims, methodology and data extracts of the current project; finally I link this with the upcoming research.

1. BACKGROUND: THE THESIS

The doctorate (Nardi 1996) set out to explore and elaborate upon the documented view that abstraction is possibly the main inherent epistemological characteristic of mathematics to which its didactical complexity can be attributed. The study originates in the idea that, in order to formulate a didactical discourse on advanced mathematics, microscopic psychological studies are needed which identify the students' needs and then engage in trying to cater for them.

The doctorate was a qualitative study of 20 first-year mathematics undergraduates' learning difficulties in five mathematical topics: Foundational Analysis, Calculus, Linear Algebra, Topology and Group Theory. The students were observed for two terms during the tutorials given to them once a week on a 1: 1 or 2: 1 basis and were interviewed at the end of each term. The minimally participant observation sessions and the semi-structured interviews were audio-recorded. The recorded material was analysed, using a combination of techniques from Data Grounded Theory (Glaser and Strauss 1967) and Discourse Analysis (Dijk 1985) in terms of the students' explicit or implicit articulations of their difficulties. In the analysis, dominant theories in the field of the Psychology of Advanced Mathematical Thinking (an

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account of most of these theories can be found in (Tall 1991) and a theoretical framework that coordinated aspects of the developmental theories of J. Piaget - specifically his concept of Reflective Abstraction (Dubinsky and Lewin 1986) - and the sociocultural theories on learning of L. Vygotsky (1978) were employed.

The aim of the study was to offer a psychological profile of the students' difficulties across the above mathematical topics. The novices' encounter with mathematical abstraction was described as a personal meaning-construction process and as an enculturation process: the new culture is Advanced Mathematics introduced by an expert, the tutor. The novices' concept-image construction was described as a construction of meaningful metaphors and an exploration of the 'raison-d'être' of the new concepts and the new reasoning and was characterised by the tension between the informal/intuitive/verbal and the formal/abstract/symbolic - which was discussed in terms of semantics and reasoning. The novices were in difficulty with the mechanics of formal mathematical reasoning as well as with applying these mechanics in a contextualised manner. This decontextualised behaviour was linked to the fragility of their knowledge with regard to the nature of rigour in formal mathematics. Issues related to teaching were touched upon when it was deemed necessary to the analysis of the learning instances that had been selected.

Some of these issues inform the theoretical basis for the current research. On the basis of the evidence in this study, to integrate teaching styles that the students are familiar with from school into their initial experiences at university level and to present mathematics in a way that is more revealing about the mechanisms that characterise mathematical thinking seem to carry significant didactical potential. From this study several conjectures relating to teaching mathematics at university level emerged and can be seen as indicators of good practice:

A to clarify the rules of the formal game of advanced mathematics, that is to provide clarity with regard to the increased requirements of rigour in the new course that the novices have to confront - for instance with regard to the knowledge that they are allowed to assume;

B to co-ordinate intuitive and formal practices, yield didactical control to the learners and view interaction with the students at a reflective, meta-topical level as a legitimate part of the teaching; C to explore the varying efficiency of types of visualisation and to develop flexible metaphors to support the understanding of abstract concepts; and,

D to support the learners' existential, meta-mathematical enquiries and to launch new concepts as epistemologically useful and necessary apparatus.

In the research, currently being conducted, the tutors are being invited to reflect and comment upon samples of data and analysis relating to the findings of the above. In the following section I describe

briefly this research as a bridge between the strictly psychological concerns of the doctorate and the directly pedagogical concerns of the upcoming ESRC-funded research.

2. THE CURRENT PROJECT

The research described in this section is a short follow-up of the doctorate described above and is currently being conducted. Its primary aims are:

- to provide feedback to the tutors who participated in the doctorate,
- to enrich the findings from the doctorate, by including the participant tutors' point of view,
- to introduce a pedagogical dimension in the psychological discourse developed in the doctorate, and to
- inaugurate the collaboration between mathematicians and mathematics educators involved in the proposed research. **In** this sense the project described in this section is a Pilot to the upcoming research, in the development of discourse and methodology.

For the above purposes, the plan, currently being implemented, is as follows:

- *Re-establish contact* with the tutors who participated in the doctorate and request their collaboration. The request has been met with positive responses.
- *Prepare the feedback material* to be presented to the participants: this consists of samples of the data, transcribed extracts from the tutorials, and the analysis, presented in the doctorate, deliberately chosen so that they trigger off the tutors' reflection upon the students' learning processes as well as their response to the analysis.
- *Introduce the above aims to the participants and conduct semi-structured interviews* in which they are asked to react to a sample of data and analysis with regard to
 - their response to the researcher's interpretation of the students' learning processes in the sample,
 - their own interpretation of the students' learning processes in the sample, and
 - their reflection upon the teaching actions taken in sample.
- *Analyse the interviews*. This aims to be the development of a discourse in which the analysis in the doctorate and the tutors' interpretations, as expressed in the interviews, are juxtaposed. The rationale for this juxtaposition is that it allows the identification of the possible differences of perspectives between mathematicians and mathematics educators. Given that the upcoming research involves the collaboration between mathematics educators and mathematicians who teach at university level, a collaboration which, as explained in the concluding section, will take the form of a clinical

partnership (Wagner 1997), the doctorate, and in particular the current research, set up a forum for this collaboration: they allow establishing and modifying the methodological principles and clarifying the issues to be raised by the upcoming research.

The interviews, which were audio recorded, are now being transcribed. I exemplify the tutors' reactions by citing extracts ** from the interviews and in particular from the parts of the discussion regarding area C of the implications for teaching as discussed in the doctorate and mentioned in Section 1, namely the varying efficiency of types of visualisation and metaphors as supports to the students' understanding of abstract concepts. Preliminary analysis suggests that the tutors' reactions vary from an educated confidence in their long-established practices,

Tutor 1: [...] as a general rule my attitude is that **without pictures we'll never understand anything**. Em, but you can't argue from the pictures. So what I try to teach students is that for expository purposes a picture is worth a thousand words and that the picture should usually be there as a guide to the student to in writing the rigorous proof, writing the formal argument, the mathematical argument. It should also be there as a guide to the reader in reading that mathematical argument but it cannot be there to substitute for the proof. It isn't the proof, it is **the guide to the understanding of the proof**. And that's what... I try to be reasonably consistent about that. **I may get it wrong from time to time** em, but yes I am always trying to get students to draw pictures in order to get an intuitive understanding of what they are doing in order to guide what they are writing but I suppose I am also aware and I mention it to them **sometimes different people think in different ways**. Em, when I am teaching Group Theory for example, I very often draw the sort of picture that you've probably seen me drawing in the past: there's a Venn diagram, a portrait of a group there is the identity element in it, and if we talk about a subgroup, well, here it is, a subset, and everything is neat and orderly and the cosets can be then neatly drawn partitioning the whole group G . Well, it's **the picture that I've got in my mind** and the picture I use for teaching but of course it's **no damn good if you've got a student with a different type of psychology from mine**. One has to be aware of that.

Tutor 2: [...] I try to use it [the metaphor of the plane in Vector Analysis] as a **realisation of a vector space** whereas they are probably **thinking about it as being THE vector space**. Em, ... it's quite difficult because you want them to get some sort of picture of what's going on but not... em, I mean their experience with vector spaces really are with 2 or 3 dimensions, so if you want to go back to their experience then...

** Text in [] is my addition.

EN: So to you talking about the plane or the space is the realisation of your intention to use their previous experiences.

Tutor 2: Yes! That's one of the things I was trying to do, to actually **latch it on to some ideas they had already developed.**

to a certain amount of self re-evaluation.

EN: [the evidence in the thesis] give a quite strong picture of what metaphors do to the learning process for the students. Would you think it is a significant thing to talk about with them?

Tutor 2: Explicitly?

EN: Yes.

Tutor 2: It's an interesting idea! [pause] That ... I guess the question is how I would then do it. But that... because **I do not know what sort of pictures they themselves have - perhaps I should ask them!** And find out what's going on.

EN: ...one of the big things I want to talk about is the use of metaphors in Linear Algebra to talk about abstract concepts like spanning sets and bases. Em, there seems to be a constant misunderstanding for most of the tutorials I observed. That the tutors come up with concrete examples from the line, the plane and the space and the students take this literally. [...] Do you have similar experience with your students?

Tutor 3: I am **not sure I would be aware of it all the time.** This is a problem. This isn't probably what you want to know but I mean to some extent in a tutorial you will tend to focus on examples on the grounds that you feel that in the lectures they got the abstract theory and that's where they will hear it and if you can give them simple examples it will help. But **how to make sure that they don't latch on to that and identify it with the more abstract concept that's harder I think.**

An analytical framework which integrates the learning-oriented perspective of the doctorate with perspectives on the tutors' self-reflective processes is currently under development.

3. THE FUTURE: THE ESRC PROJECT

The broad aim of the upcoming, more action-oriented 1-year research is to explore, in a partnership with university mathematics teachers, current thinking and practices in mathematics teaching at first-year undergraduate level. Participants of the research will be Oxford mathematics tutors. The researchers will observe tutorials in which the participants are involved and then interview the participants. The analysis will produce documentation and grouping of practices, processes and the thinking of participants. Researcher interpretations will be fed back to participants for respondent validation but also, in the spirit of the intended partnership, to initiate discussion on the implications of the research findings for future practices. To exemplify how the tutors have expressed their expectations from the upcoming partnership, I conclude with two remarks made by the tutors in the current interviews:

Tutor 2: So therefore I think **there are other possibilities** [regarding how my students think] **I hadn't had in mind** because on the whole I don't actually know how much tutors think about their role as a tutor and what actually one is doing as opposed to, you know, what end result you would like to get out of it all. A lot of it I suppose is really done by what has worked in the past with our students.

Tutor 3: I think that this sort of thing — certainly for young tutors but really at any age — **to do a little bit of this** [the interview] **and then go through it rather well**, I am not complaining in the delay in this case, **but going through the thing later the same week and seeing where you could have done better, then you have some chance to improve. Because it's got to become a subconscious, automatic thing.** You don't have time in the tutorial to figure out 'well, should I be using some different approach?'. It's got to be almost automatic, your reaction to the student.

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