

# CYPRIOI CHILDREN'S INTERPRETATION ON A PIAGETIAN TASK ON VOLUME

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*The present study investigates the ideas (~lCypriot Primary School children, (~lage ten to twelve years, on Volume and Capacity. The research is still on going but up to now findings suggest that children's answers are similar to those described by Piaget in his work on Volume. Here we present the answers (~l eight pupils who were involved in the study. Only three (~lthe tasks used are described. In these particular tasks some (if the children's answers were very similar to those used by Piaget to demonstrate the absence (~l conservation (~l 'volume occupied'.*

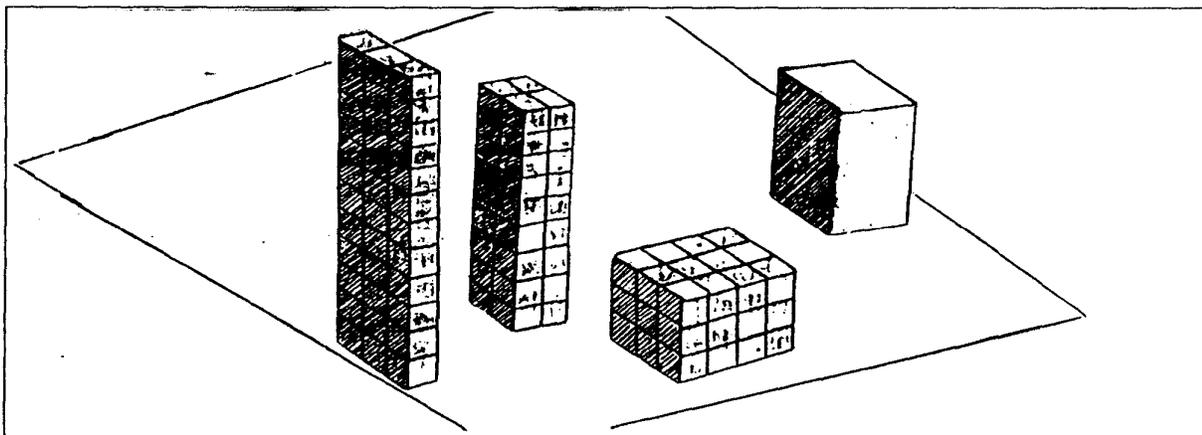
## A. Definitions and brief description of Piaget's work on volume:

Conservation is the recognition that an amount remains invariant under a transformation.

The concept of conservation or invariance of volume is developed according to Piaget later than the conservation of number, quantity and weight. Two aspects of volume were distinguished: *interior volume* and *occupied volume*. Interior volume is that defined by the boundary surface of a block and occupied volume is defined in relation to its surrounding in space.

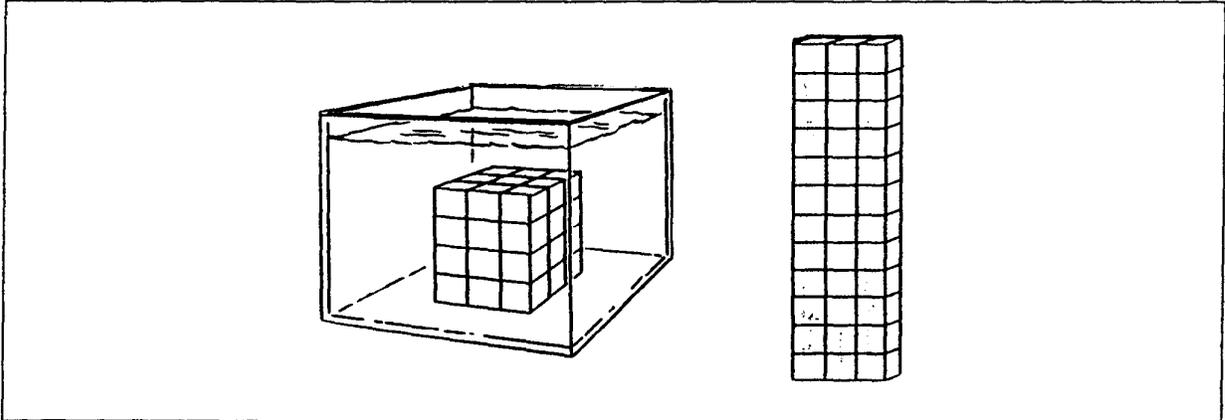
Depending on the two aspects of volume, two kinds of conservation of volume were distinguished. Piaget and his co-workers demonstrated experimentally the difference between conservation of interior and occupied volume referring to some cases of children of six to eight years old. These children were presented with the two tasks concerning volume.

In the task of building a house of the same room on a new island (fig. 1 ), the child is shown a block which is solid and measures 4 cm in height on a square base of 3x3 cm so that its volume is 36 cm<sup>3</sup>. He/she is told that this is a house built on an island and is asked to build a new house which is to have exactly *as much room* as the old, although, it is being built on another island. The child is shown the other islands which are pieces of card measuring 2x2 cm, 1 x3 cm and 3x4 cm and asked to build the house. In case of failure to understand what the expression *as much room* meant, an explanation was suggested (that every cube is a room, each of the inhabitants had a room of his own in the old house and each wants to have a room of their own in the new house).



**Figure 1.** The task of building a house of the same room on a new island

The second task included immersing equal volumes of different shape into water (fig.2). The children responded correctly that the room remains invariant even if the shape of a block was changed into another shape. But when the question was whether the two constructions take the same amount of room in the water the children's responses depended on the shape of the block.



*Figure 2. The task of immersing equal volumes of different shape in water*

According to the results in these experiments Piaget categorised the understanding of volume in stages. The development of the conception of conservation of volume is completed at around twelve years of age. These stages and the main behaviours observed at each stage can be summarised as follows:

Stage 1: Prior to the fourth year of age the techniques of the above mentioned experiments were impracticable.

Stage 2 (4 1/2 years until 5-7 years): The children show no conservation - if a group of bricks is arranged differently the children think that the 'amount of room' is changed.

Substage 2a: The children think in terms of one dimension only and usually stop building when the house reaches the same height as that of the model, irrespective of the size of the base.

Substage 2b: The children's models begin to vary in height in relation to the size of the base. There is a shift from consideration of one dimension only to relationships between dimensions.

Stage 3 (6 1/2 - 7 1/2 years until 11-12 years):

Substage 3a: At this stage the children cannot dissociate the three notions of height, shape and volume. They work out relationships among the three dimensions (at the beginning they can not handle more than two), but without measuring.

Substage 3b: Children now can measure correctly. They attempt to measure the interior volume either by giving the number of unit cubes on one of the boundary surfaces or the number of unit cubes which they need to surround the model.

Stage 4 (The level of formal operations, 11-12 years): There is conservation of occupied volume. Children can conserve volume relative to the surrounding spatial medium.

## B. The Present Study

The aim of this study is to investigate the ways employed by ten to twelve years old Cypriot children to calculate volume and capacity and subsequently determine their stage of conservation of volume defined in Piagetian terms. Capacity refers to what Piaget terms interior volume.

The sample of the original study consists of 90 children of the 5th and 6th class attending Primary schools in Cyprus. Children were selected from three schools in different areas of the island. Thirty children from each school were first interviewed individually. Then they answered written questions in teams of fifteen. All tasks were presented and answered in Greek.

The tasks used included some numerical questions in the written part. Questions on calculation and comparison of volume of different blocks as well as the capacity of containers were set in both written and interview form. Different expressions like 'number of cubes', 'volume', 'amount of room', 'number of cubes that can fit in', 'volume of air space inside' or 'capacity' were used in defining these questions. The Piagetian tasks on conservation of volume were included as well.

Presently we will be referring to the performance of only eight children of the original sample to three different tasks and we will compare their answers. The first interview question is that of building a house of the same room on a new island (fig. 1) where the old house was a  $3 \times 4 \times 3$  cm construction and the new island a  $2 \times 2$  cm card. The second is the skyscraper question where the child is told that the old house is dismantled and the cubes are put in a pile and he/she is then asked to calculate the height of the tower that can be built using the same cubes on a new island (fig.3). This task was set in written form. The third task is the question of immersing equal volumes of different shape into a bowl of water (fig.2). A  $3 \times 4 \times 5$  cm construction was presented at the beginning of the question. The child was asked to write what will happen if the construction is immersed in the water and give a reason for this answer. He/she was then asked to write what will happen if the cubes from the first construction are used to make a new one, which was shown in a diagram as well, with dimensions  $3 \times 2 \times 10$  cm (the height being 10 cm).

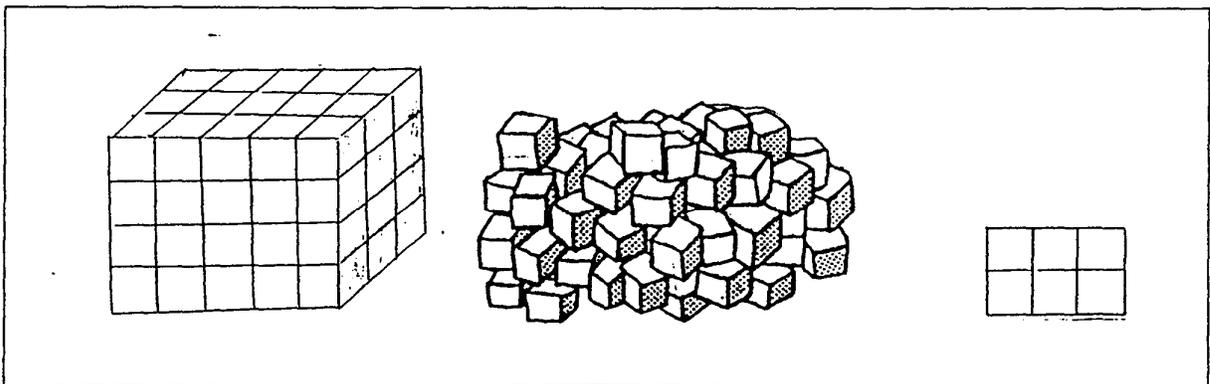


Figure 3. The skyscraper question

C. Children's responses

This section presents the answers of the children to the tasks described above. These are summarised in table 1.

TABLE 1  
SUMMARY OF THE CHILDREN'S ANSWERS

	task1	task2	task3
Simos 10 years 3 months	Multiplies number of cubes on a layer by height.	Multiplied lengths of three dimensions.	Level of water will rise equally. Houses take equal room inside and outside the water.
Stylianios 10 years 8 months	Thinks of how the columns can be rearranged on the new island.	Adds cubes in layers.	Level of water will rise the same. Houses take equal room inside and outside the water.
Thalia 10 years 6 months	Adds number of cubes in columns	Adds number of squares on top, front and two sides. (59)	Level of water will rise the same. Taller construction takes more room in the water.
Christos 11 years	Adds cubes on front, sides, back and middle avoiding double counting.	Adds layers. (60)	Level of water will rise the equally. Taller construction takes more room in the water.
Louiza 9 years 11 months	Adds area of top, sides, front and back faces. (54)	Finds area of surface. (94)	The two houses take the same room outside but not inside the water.
Konstantinos 10 years 6 months	Adds area of sides around the house. (42)	Adds cubes shown in the diagram. (Avoids double counting.) (36)	The two houses take the same room outside but not inside the water.
Elena 10 years 2 months	Calculates total area of surface. (66)	Calculates total area of surface. (94)	Level of water will rise more. The taller house takes more room in the water.
Maria 9 years 10 months	Adds number of cubes on first column of front layer with number of cubes of first row of top layer. $6+6+6=18$	Same way as with task 1. $6+6+6+6+6=30$	Level of water will rise more. The taller house takes more room in the water.

Here is the answer of Simos to the interview task:

Sim.: We will have to make it higher because the island is smaller. So the new house has to be thinner but taller in order to have the same room inside it.

I.: Very good. Can you tell me how tall it will be?

Sim.: The old house has 36 cubes. So the new house will be 9 cubes high because 4 times 9 is 36.

I.: Good. Can you tell me how you have calculated the number of cubes that make the old house?

Sim.: It has 12 cubes on the top. There are 3 pieces of 12. So the cubes are 12 times 3, 36.

I.: Well done.

To calculate the number of cubes of the house he multiplied the number of cubes of the top layer by the height of the construction. He used the same procedure for the calculation of the height of the new house to be built. He also had no problem with the pictured model of the skyscraper question. In the question of immersing equal volumes in water he answered that the level of water will rise equally as both constructions take the same room in the water.

Stylianios is another child who answered correctly to all three tasks. He decided about the height of the new house without calculating the number of cubes that make it up. He thought of how the columns of the old house can be rearranged to fit on the new island. To give the correct answer in the 'skyscraper' question he first calculated the number of cubes that made the construction by adding the cubes on each layer. He then multiplied 6 by 10 to find that the new house will be 10 cubes high. In the question involving equal volumes immersed in water he answered exactly as Simos.

Thalia realised that the new house must have an equal number of cubes with the old one only after the explanation. She then counted the cubes of the old house in columns 3 by 3. To count the number of cubes used to build the new house she added them in layers. Thalia had difficulty with the pictured construction in the 'skyscraper' question. She concluded that the number of cubes that make the 5x3x4 cm construction is 59, adding the number of cubes shown on some of the surrounding surfaces ( $20 + 12 + 12 + 15 = 59$ ). In the third task she answered that the water in the bowl would rise more if the new construction was immersed as it was higher and was taking more room in the water.

Christos also had a difficulty in seeing how a house of the same room can be built on a smaller island. Here is his answer before the explanation of the interviewer.

C.: I think that they can not make it.

I.: Why do you think so?

C.: Because here they can have more cubes. (showing the old house)

I.: And why is that?

C.: Because the old island is bigger therefore it can take more cubes.

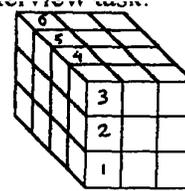
It was only after the explanation that he realised that he could make the new house taller and keep the number of cubes the same. To find the total number of cubes he added the number of cubes on all the surrounding faces and then added the number of the remaining cubes in the middle. To decide about the height of the new house he divided the total number of cubes (60) with the number of squares which were shown on the new base(6).

Louiza and Konstantinos added the area of some of the surrounding surfaces instead of the number of cubes that make up the house in the interview task. In the skyscraper question, Louiza found the total area of surface of the construction involved while Konstantinos made an effort to count cubes rather than squares but he managed to count only the cubes shown in the diagram. In

the task of immersing equal volumes in water both children thought that the two houses take equal amount of room outside but not inside the water.

The remaining two children, Elena and Maria had difficulty in all three tasks. They used the same procedures in the calculation of the total number of cubes for the real and pictured models. Here is how Maria calculates the total number of cubes in the intervention.

M.: There are 6 here (showing the first column of the front face and the first row of the top). And 6 here and here (showing the second and third, in the same way). So 18 cubes in total.



To answer the skyscraper question she dealt with the pictured construction involved exactly as with the real model of the previous question. Maria also thought that the two houses of the task of immersing equal volumes in water took the same amount of room outside the water. But she thought that when the taller one was immersed in the water it would take more room and the level of water would rise higher.

This is exactly what Elena thought for the third task, another child who calculated the area of surface of the constructions involved in the first two tasks.

#### D. Conclusions

The answers of Simos and Stylianos were those described in the Stage of formal operations. Their answers to the three tasks suggest that they conserve both interior and occupied volume. They had no difficulty with pictured models. They used addition in layers as well as multiplication of the lengths of the three dimensions to calculate the number of cubes that make up a given construction.

The answers of the remaining children are similar to those Piaget demonstrated to show the difference between conserving both interior volume and volume occupied and conserving interior volume only. Some of these children calculated the number of cubes of the real model correctly and had difficulty with the pictured models. Others calculated the area of surface or the area of some of the surrounding surfaces both in the pictured and the real model. But in the task of immersing equal volumes in the water they all had difficulties in understanding that both constructions take the same amount of room in the water. These answers are similar to those described by Piaget as indicative of children who have not yet reached the stage of formal operations and who conserve interior but not occupied volume.

The tasks on volume and capacity in the textbook used in the fifth and sixth grade of the Cypriot Primary Schools, include calculation of volume of different blocks and the capacity of different containers. There is a concentration on the formula of finding volume, while not a lot is included about the relations between the lengths of the sides and how volume remains invariant if the lengths of the sides are transformed.