Learning to Write Mathematically

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Recent developments in school mathematics involve school students in more extended writing than was the case in the past, but, although it is recognised that this is an area of difficulty for many students, little attention has been paid to how they might learn to produce more effective texts. Drawing on my research into the writing, reading and assessment of GCSE coursework texts, this paper considers issues related to the development of modes of writing likely to be judged to be appropriate, including the question of the role that may be played by explicit linguistic knowledge.

Traditionally very little writing has taken place in mathematics classrooms. Major surveys of writing across the secondary school curriculum in the seventies and eighties were either unable to identify enough examples of writing in mathematics to analyse separately (Britton et aI., 1975) or, in spite of finding substantial quantities of 'writing', reported that most of it was copied and that none of it was 'extended' (Spencer et aI., 1983). The introduction of GCSE coursework and, more recently, the requirement to assess MAl have involved mathematics students and teachers with extended writing in what is effectively a new genre: the investigation report. While it seems to be generally recognised that many students find it difficult to write effectively in this genre and that they may thus be disadvantaged in the assessment context (Ball & Ball, 1990; MacNamara & Roper, 1992), little attention has been paid to remedying this situation.

While there has been increasing attention to the use of writing as a means of promoting learning in mathematics, particularly in the United States and Australia (e.g. Connolly & Vilardi, 1989), this has not been paralleled by attention to the forms of writing produced by students. Even where curriculum developments have made strong arguments for the inclusion of writing activities, it appears to be assumed that students will develop the necessary skills 'naturally' (e.g. NCTM, 1989). This view of natural development of writing is challenged by Martin et aI. (1987) who argue that writing, unlike speech, is unlikely to develop 'naturally' because of the lack of the possibility of 'immersion' in a written language environment. While curriculum developments involving continuous, long-term written interaction between teacher and students (e.g. Clarke et aI., 1993) have shown some development in students' writing, leading to more positive teacher-researcher evaluations, it appears unlikely that the present UK mathematics curriculum allows enough time and space for such development to take place in the writing of reports of investigative work.

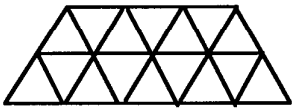
There is currently some debate in the domain of literacy education about whether students should be explicitly taught the characteristics of specific genres of writing (see, for example, Reid (ed.), 1987). It appears to be widely assumed that this is an issue for language teachers, concerned with general language development, rather than for teachers in other curriculum areas, concerned with communication within their subject area. An exception to this is Kress (1990), who points out that subject teachers are unlikely to be aware of the ways in which their judgements are affected by

students' use of particular forms of language. Kress argues that, if teachers were more explicitly aware of the forms that are highly valued within their discipline and of the effects that may be achieved by various linguistic choices and could pass this awareness on to their students, this would not only help students to conform to the conventional expectations of the genre but would also empower them to make informed choices to break the conventions in order to achieve deliberate effects, including to demonstrate 'creativity'.

In studying the discourse of reports of investigative work, I have analysed students' GCSE coursework texts and have interviewed teachers reading and evaluating these. Through this process I have been able to identify a number of features which appear to be significant to the ways in which teachers read coursework texts. The teachers themselves, however, appeared not to have adequate linguistic resources to specify these features or to pass knowledge of their expectations on to their students.

Narrative

Most of the teachers expected to see some personal narrative of the student's actions and thought processes. This is clearly related to their need to "judge what's going on in someone's head" (Charles: 79) in order to undertake the assessment process. Not all types of narrative, however, were acceptable. One student, for example, was heavily criticised:

> *Once Suzanne and I had completed tasks 1 and 2, we set out to discover if there was any connection between the triangular area and the lengths of the sides. First we lettered the sides:*
>
> 
>
> *a = slant line*
> *b = base line*
> *c = top line*
> *d = area*
>
> *In a very short time we had discovered a relationship between the lengths of the sides and the area (triangular). We were able to put this into a formula: -*

The temporal themes *(Once ... , First ... , In a very short time ...*) and the use of the past tense structure this extract as a story of what the author and her collaborators did. This did not, however, satisfy its teacher-readers' expectations. Andy, for example, found the lack of specificity alarming:

> *.. it's this "In a very short time" I must have read that a thousand times and "We suddenly discovered that" and that always rings alarm bells. (Andy: 335-337)*

Similarly, Joan identified the "rather anecdotal description of what they did" as a sign of immaturity. Although Joan's own written advice to her pupils included an admonition to "Write down what you did in the order that it happened" (Joan: 444), this student's attempt to do so was not acceptable.

So what kind of narrative did the teachers expect? An examination of the examples they approved and disapproved and the suggestions that they provided of forms they would prefer to see suggests that mental processes such as noticing or predicting were valued while discovering or finding were not. In addition, most of the examples that were condemned by the teachers were simple, single

clause statements of single actions or strings of such statements connected only by 'and'. Many of them also merely declared the discovery or the existence of a result. The examples that were recommended tended to be more complex, suggesting causal relationships between the actions. Unfortunately, such examples did not appear in the texts considered; this analysis is based on teachers' reports of their preferences rather than on observation of their reading practices.

The advice offered to students to write down what they have done does not provide knowledge of the forms of narrative that are likely to be valued. An analysis of the full set of five coursework texts produced over a period of two years by a single student revealed his language to be remarkably consistent, including repeated use of the expression "I have found a formula". without any further elaboration of the student's mental processes. This suggests that the student was not aware that this phrase and a general lack of a narrative of mental processes were specifically condemned by teacher-readers or, if he was aware of this, that he did not have adequate resources to construct a more acceptable text. Any feedback he may have received about his writing had not effectively addressed this aspect.

Explanation

Explanation (in the sense of an argument providing some justification for results achieved) is highly valued in mathematics generally and in the school curriculum. This valuing is associated with a belief (expressed in curriculum documents and assessment schemes, e.g. ULEAC, 1993) that it is 'difficult' and hence that many if not most students are unlikely to be capable of producing it. Although the tasks set had asked students to explain their results, there were few examples of explanation in the student texts and even these appeared to give rise to some difficulty for teachers in reading and assessing them. It appeared that the form of a passage of explanation was more important to the teachers' evaluations of its worth than the validity or completeness of its argument. Thus, although Steven's explanation of why a pile of rods would fall over

> *The reason that the pile topples could be because the weight over the starting pile becomes too much and gets pulled down by gravity. . .*

was not seen as adequate, it was nevertheless taken as a sign of understanding and high 'ability'.

> *He's obviously quite a good pupil in terms of his thought processes about it, but could have explained more.* *(Grant: 148-150)*

The valuing of form over content was even stated explicitly by one teacher, perhaps in recognition of the 'difficulty' of producing a valid argument:

> *1 think any argument would be good so long as they got a good explanation and a good argument as to why it might be the case. I'm not too worried about it necessarily being correct but I would be more interested in them whether they could actually argue the point for themselves. So I would be looking more in the assessment on the strength of the argument than actually what they're trying to put across.*
>
> *(Charles:* 29-34)

One feature of Steven's text which may have influenced the teachers judgements (and which was not present in other, less highly valued, passages of argument in the student texts) is the explicit signalling of its status through the use of the terms *the reason* and *because.* Whereas some other passages of explanation were dismissed as "intuitive" or identified as narrative rather than explanation, Steven's text is unambiguously about reasoning and causality.

Use of a 'mathematical register'

On a number of occasions, teachers appeared to find features of students' texts incongruous, commenting explicitly on the style or merely marking their discomfort with laughter or other paralinguistic means. As the following example illustrates, many of the features marked in these ways were in opposition to conventional characteristics of academic mathematics writing such as abstractness and conciseness. Dan compared two extracts:

No.2:

> *If you add the top length and the bottom length, then multiply by the slant length, you get the number of unit triangles.*
>
> *For example:*
>
> $3 + 5 = 8$ *and* $2 + 4 = 6$
> $8 \times 2 = \underline{16}$ $6 \times 2 = \underline{12}$
>
> *This, therefore is the formula:*
>
> *(TOP LENGTH + BOTTOM LENGTH) × SLANT LENGTH = No. OF TRIANGLES*

and No.3:

> *If you add together both the top length and the bottom length and times it by the slant length, you will end up with the number of unit triangles in that trapezium.*
>
> *You can write this as S(T + B)*

The comparable parts of the two texts, as Dan remarked, say almost the same thing in slightly different words.

> *Number* 2 *gives me the impression they obviously know what they're talking about whereas this one, although it says almost exactly the same thing in different words, er, it doesn't give me the same impression.* *(Dan:* 361-364, *No.2 & No.3)*

Obvious differences include the fact that No.2 had included two examples and had used verbal variable names, while No.3 had used algebraic symbols for her formula. Dan had commented on these differences earlier, claiming that they did not greatly affect his assessment of the students (see Morgan (1994a)). His "impression" appeared to be based rather on the verbal descriptions of the procedure which, as he said, appear very similar in form. There are, however, a number of aspects of the two texts which may have affected Dan's reading:

- The use by No.3 of *times* rather than *multiply* is less formally 'mathematical' and may be read as a remnant of the early years of mathematics schooling and hence a sign of immaturity.

- No. 3's procedure is more 'wordy' using *add together* rather than simply *add,* and *times it by* rather than *multiply by.* The number of unit triangles is also qualified as being *in that trapezium.* These additional words include reference to the concrete lengths, numbers or shapes. The procedure may thus be read as being at a lower level of abstraction.
- The use of *you will end up with* rather than *you get,* by using the future tense, also suggests a more concrete procedure, located in time.
- The presentation of the final fOreJ.ula by *You can write this as ...* presents the symbolic formula merely as an alternative to the verbal procedure. NO.2's announcement *This therefore is the formula,* on the other hand, displays the formula as a product in its own right which follows logically from the procedure rather than merely being equivalent to it. This may be read as an indication that No.2 has a better understanding of the importance of the relational formula in mathematics, even though she has not used algebraic symbols to express it. The contrasting modality of these two statements also suggests that the two students differ in their levels of confidence.

Any of these features might have contributed to an impression that No.3 is less competent mathematically. While it is not possible to say precisely which aspects of the writing contributed to Dan's assessment, there is clearly a mismatch between the student's texts and Dan's expectations which appeared to be affecting his evaluation of the whole of the students' performance.

This analysis of possible sources for Dan's different evaluations of the two extracts points to the subtle nature of the relationship between the linguistic form of the text produced by the student and the teacher's evaluation of her general intellectual 'ability'. As Dan himself was unable to identify the features of the text which gave rise to his impressions it seems unlikely that he would be able to provide advice to a student on how to produce an acceptable text. Moreover, his identification of the style of writing with 'intellectual level' and with 'knowing what they're talking about' suggests a view of language as the transparent representation of thought. It is a logical consequence of such a view that 'improvements' to the text can only follow (and will necessarily follow) developments in the students' thinking about the mathematics. A teacher holding this view of language is unlikely to consider it necessary or useful to address the form of writing with his students.

Teaching students to write?

The only language available to mathematics teachers to describe the language of coursework texts appears to be a simple list of features such as tables, diagrams (Morgan, 1994b) or descriptions of "what you have done". This is not adequate to describe the characteristics of a text that will be highly valued in the assessment process and is thus unlikely by itself to help students to produce such highly valued texts. Moreover, one of the effects of using such a list to guide the writing of investigation reports is likely to be to reinforce the hold of the stereotypical 'investigation', stifling any possibilities of creativity, as Dixon (1987) argues in his attack on the idea of 'teaching genre'. On the other hand, as Kress (1990) argues, knowledge about the different effects that various

linguistic choices can achieve could provide students with the power to manipulate their own use of language to produce such effects deliberately. Students need to be aware of the ways in which their texts will be read and assessed. Such awareness does not necessarily arise simply from being informed of the criteria. By focusing on the different effects that may be achieved through different linguistic choices, the significant features of student texts identified here should provide a basis for explicit discussion with teachers and students of the ways in which language and other forms of communication may be used most effectively to produce reports of investigative work that is likely to be judged to display highly valued mathematical processes.

References

Ball,B. & Ball,D., 1990, 'How do you cheat at coursework?', *Mathematics Teaching* 133: 9-12 Britton,J., Burgess,T., Martin,N., McLeod,A., Rosen,H., 1975, *The Development of Writing Abilities* (11-18), School Council Research Studies, London: Macmillan Educational

Clarke,D.J., Waywood,A. & Stephens,M., 1993, 'Probing the Structure of Mathematical Writing', *Educational Studies in Mathematics* 25(3): 235-250

Connolly,P. & Vilardi,T. (eds.), 1989, *Writing to Learn Mathematics and Science,* New York: Teachers College Press

Dixon,J., 1987, 'The question of genres' in Reid (1987)

Kress,G., 1990, 'Two kinds of power: Gunther Kress on genre', *The English Magazine* 24: 4-7

MacNamara,A. & Roper,T., 1992, 'Attainment Target 1 - Is all the evidence there?', *Mathematics Teaching 140: 26-27*

Martin,J.R., Christie,F. & Rothery,J., 1987, 'Social processes in education: A reply to Sawyer and Watson (and others)' in Reid (1987)

Morgan,c., 1994a, 'Teachers Assessing Investigations: The Role of Algebra', *Proceedings of the 18th Conference of the International Group for the Psychology of Mathematics Education,* Lisbon: 295-302

Morgan,c., 1994b, 'What is the role of diagrams in comunication of mathematical activity', *Proceedings of British Society for Research into Learning Mathematics,* Institute of Education, London

NCTM, 1989, *Curriculum and Evaluation Standards for School Mathematics,* Reston, V A: National Council of Teachers of Mathematics

Reid,l. (ed.), 1987, *The Place of Genre in Learning: Current Debates,* Centre for Studies in Literary Education, Geelong: Deakin University

Spencer,E., Lancaster,J., Roy,J., Benvie,J., McFadyen,l, 1983, *Written Work in Scottish Secondary Schools: A Descriptive Study,* The Scottish Council for Research in Education

ULEAC, 1993, *Coursework Assessment* 1993: *Mathematics A,* London: University of London Examinations and Assessment Council