PATTERN V STRUCTURE

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Abstract

This paper discusses the nature of the mathematical activity which may be expected to accompany the developing understanding of pattern - in particular an understanding that admits of abstraction of structure and thus classification, in this case, of frieze patterns. Some activities involving the use of COOri Geometre to explore pattern are described. These activities and a review of some of the pattern literature are used as a basis for discussion of the process of objectification which is held to characterise understanding in this context. The pedagogical implications of acknowledgment of a process/object distinction are illustrated through consideration of some work done by undergraduate students.

I want to situate this discussion in the context of work I've been asking some students to do at South Bank University. These ate Primary Education students who have chosen to do mathematics as their specialist subject. The students have generally very little mathematics beyond O-level when starting the course. The work I'm going to describe was done as part of a third-year unit on Geometry. The unit, as inherited, was entirely focused on transformation geometry; it had been conceived in terms of some specially written software which was not actually available to me and appeared to have little relation to any context in which the study of transformations might be well motivated. Starting from scratch, as I had to, I saw an opportunity to bring in ideas of pattern and tiling which I had been playing about with on Cabri Geometre for some time. In particular, I welcomed the chance to work with the students using Cabri.

I have now taught the unit twice, to successive cohorts. I changed the approach quite a lot the second time through in the light of experience with the first cohort. I want to talk partly about these different approaches and why I thought changes were needed.

As part of the unit I wanted to work with students towards achieving a real understanding of pattern. By real here, I mean an understanding sufficient to be convinced, say, of the classification of frieze patterns.

Both approaches used the idea that Cabri could aid the apprehension of the generality of pattern its structure; that which makes a pattern a pattern. Both, at some stage, invited students to explore such generality by trying, to use the Cabri language of Healey et al (1994), to 'mess up' patterns which were - eventually and, perhaps regrettably - provided ready made. Both approaches also provide a particular focus for thinking about the inter-relationship of geometry and algebra and I shall return to this later. Figure I gives some idea of what I mean here by messing up a frieze pattern. This particular pattern is formed by repeated half-turns of the base unit **abc** and its images about centres **01** and **02** (Figure II.)

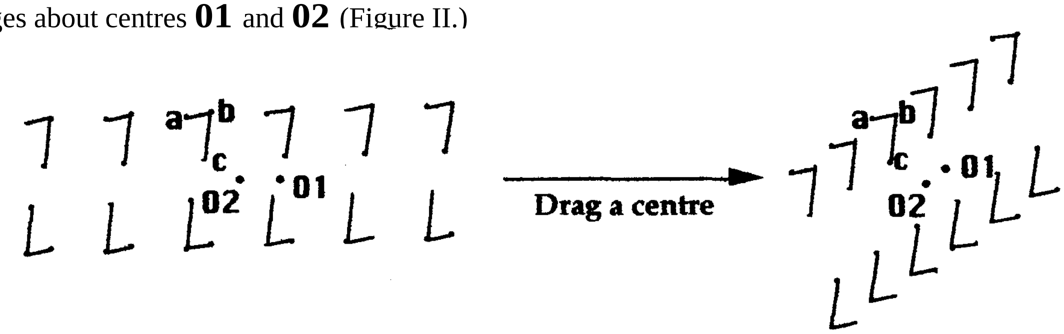

Figure I

I say that I regretted providing ready made patterns because I had hoped that students would be able to play naturally with the idea of defining for themselves a limited set of transformations and applying them ·again and again'; they might then be in a position to note the stable properties of their resulting patterns. Those people who are familiar with other packages (Sketchpad, Geometry Inventor) may wonder why I didn't use them and thus their built-in transformations. There were two reasons: firstly I had no usable copies of these other packages; secondly, I believed that there was much to be gained from learners defining for themselves ·transformations as macros'. This act of definition seems to me to be potentially a powerful act of rendering transformations concrete and tangible. Love talks nicely about this idea of turning processes into objects in a geometrical context (1995).

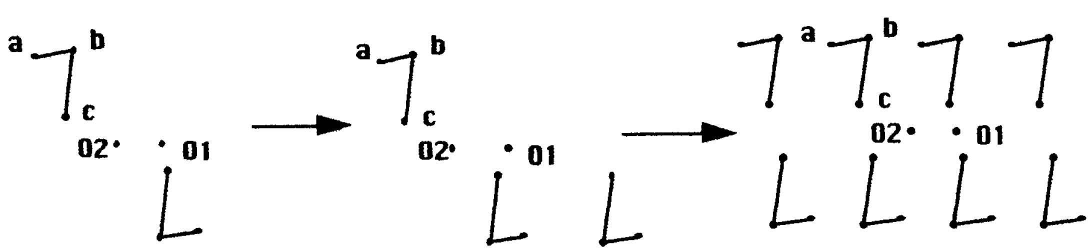

Figure II

I will shortly turn to a description of the two approaches I mentioned earlier. But I want first to set these in the context of some of what might be called mainstream pattern literature - that is writing that sets out to explain the mathematics of pattern. I think that there are some stark and interesting contrasts to be found. We might place beautiful and weighty mathematical treatments (Grunbaum and Shephard 1986) at one end of a spectrum and, at the other, small articles with small scale and specific teaching objectives (McLeay and Abas 1991a, 1991b, 1992). Such a spectrum of writing is to be expected, and I might seem to be making a very obvious comment. There is, however, some point in drawing attention to what I think is a significant difference in emphasis in the way in which patterns are conceived, or at least written and learned about. If you read almost any authoritative text on pattern you find very quickly that the units of the pattern, which appeared, quite naturally, initially to be the objects of study, recede and give way to those less tangible objects of study which I've tried to capture through the macros - namely the symmetries of the pattern. Mathematical understanding demands this shift of emphasis, of focus. You need to be able to shift focus from the tangible unit of pattern, to the intangible generating symmetries. Grunbaum and Shephard (1986) assume that their readers are familiar with this shift of focus; McLeay and Abas (op cit) appear to make no such assumption, nor do they direct or guide the reader to objectify the underlying process. Their focus is identification of the motif, the unit tile (a composite made up of sufficient application of the generating symmetries to the basic motiO which when tiled in one direction produces the pattern. They provide algorithms for generating patterns which work, but following an algorithm is not understanding.

It is true that students can use this idea of repetition of a motif, for example with a computer package like Mosaic, to stamp and study patterns. I think however, and this is based on my own experience using Cabri (and those of my students), that 'stamping' activities can obscure the fact that we do better - in terms of mathematical understanding - to view the pattern, the repetition, as repeated application of generating symmetries. Perhaps we should say that the motif focus supports an inductive account of pattern (a train spotters' account.) If you want to be able to deduce any sort of classification, then you must shift your focus to the process of generation.

The sort of process/object shift I have been describing here is related to process/object, procedural/structural distinctions made elsewhere in the algebra literature (Sfard and Linchevski, 1994, Gray and Tall, 1994.) In this particular context, however, the pedagogical implications of such distinctions may be clearer. Problems of understanding pattern, it seems to me, are possibly related to expected readership and context of explanations. Grunbaum and Shephard must assume in their audience the facility to make this shift of focus - it is a given for this kind of text; similarly, in Washburn and Crowe's lovely book (1988), it is actually sufficient for many readers not to make the step to objectification; decision charts enable patterns to be deciphered algorithmically, and proofs of a combinatoric nature - again made possible only once processes have been made objects - are provided in appendices. Problems arise in reading articles such as those of McLeay and Abas where the intention is to foster mathematical understanding, but there is no recognition of any process/object distinction. We read these texts with the same feeling of incomprehension (at least I do) as I suspect is felt by many of our students when 'reading' algebraic texts without similar facilitation of a shift of attention. Where is the mathematics in such texts? Maybe it's in the learning of the international pattern notation?

Let me briefly describe the work I've done with the students and some of their responses.

First cohort approach

The first time around, my approach to this aspect of the unit was to go straight for the 'transformation as macro' route for encouraging the shift of focus I wanted. After students had had some time to get used to Cabri (including the use of macros) I asked them to play with figures in which linear transformations had been essentially embedded. They were invited to explore these transformations and, if possible, to reconstruct them as macros. My hope had been that they would then be able, as I have suggested above, to generate patterns through focusing on repeated application of a set of macros. However, fewer than half the students managed this kind of objectification and thus were not able (at least in this way) to gain the kind of insightful appreciation of the generation of pattern and its algebraic representation as an infinite symmetry group of some kind.

Evidence from the students' assignments and the tape of a group interview held at the end of the course suggests that computer experience was insufficient by itself to make the pattern making process tangible enough; there was certainly insufficient focus on repetition of actions.

Second Cohort Approach

My comments about the work of the first cohort actually constitute something of a confession; I had so obviously failed to recognise the importance of practical pattern making activity. The second time around I first asked students to construct and analyse patterns using mats, devising trails, etc. At this time I made no mention of pattern generators at all, and, not surprisingly, none of this kind of language appeared in students' writing. The first attempt to make transformations tangible was through a symmetry card activity, similar to those sometimes used in texts looking at first steps towards group theory. Figure III shows a cut-down version of this activity.

Here is a set of isometries. Cut them out to get separate cards.

- Pick one card. Use it repeatedly on the basic 'hook' shape. What do you notice?

(Note: Each time you use a card, you MUST be sure to use the same isometry. For example, if you use a **rotation**, you must use the **same centre** and **angle of rotation**.)

- Choose two cards (you can do this in lots of ways!)
- Three cards, four cards.....

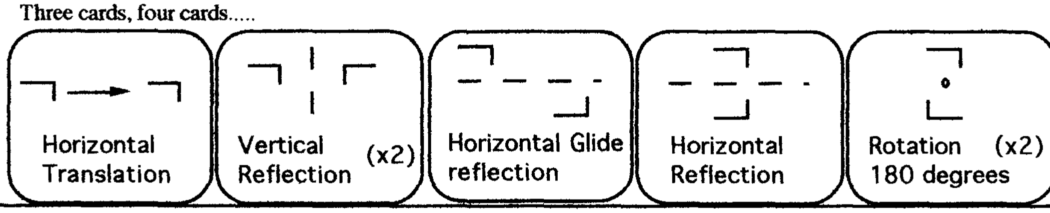

Figure III

I wanted to make the idea of selecting and using a finite set of generators the start point for the activity, though I didn't expect or plan that students might construct ideas of groups quite at this point. In fact, whilst the idea of finitely generated infinite patterns was generally well understood through the activity, students did not, at the same time take the step of recognising sameness of pattern that I wanted to encourage. Perhaps the students recognition of pattern was, in Orton's terms (1993) pictorial rather than propositional.

There was also the problem of adhering consistently to rules which had to be 'held in the mind' whilst the pattern was being produced. It is easy to be tempted to take short cuts and fail to apply these properly as the example below shows (using tracing paper is difficult and became tedious)

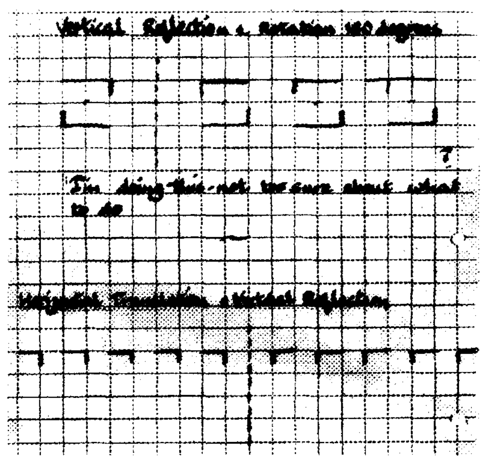

Figure IV

The next step was to ask students to use Cabri to explore pre-constructed frieze patterns (as in Figure I). This time I witnessed the sort of discussion of pattern that I had hoped for.

Helena: (looking at the pattern generated by vertical reflection and glide reflection) 'dragging (the mirror) ml horizontally moves the line and the reflection with it, yet the pattern stays the same ... '

Lorna, whose initial work is shown in figure IV, had been unsure of what was supposed to be happening with the card activity and had been on the point of giving up. However, she revived with the Cabri experience, and was able to see patterns in terms of generators.

Jeanette was one of a number who had provided a complete classification after using the cards

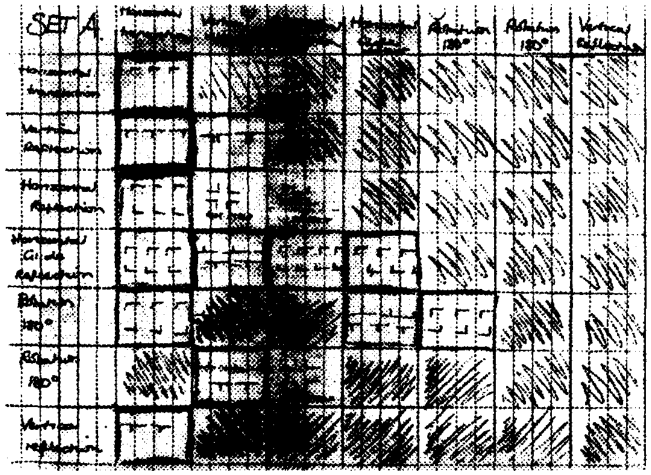

Figure V

Cabri enabled her to take the step to plane patterns 'accidentally' using the idea of pulling centres of rotation 'out of line' .

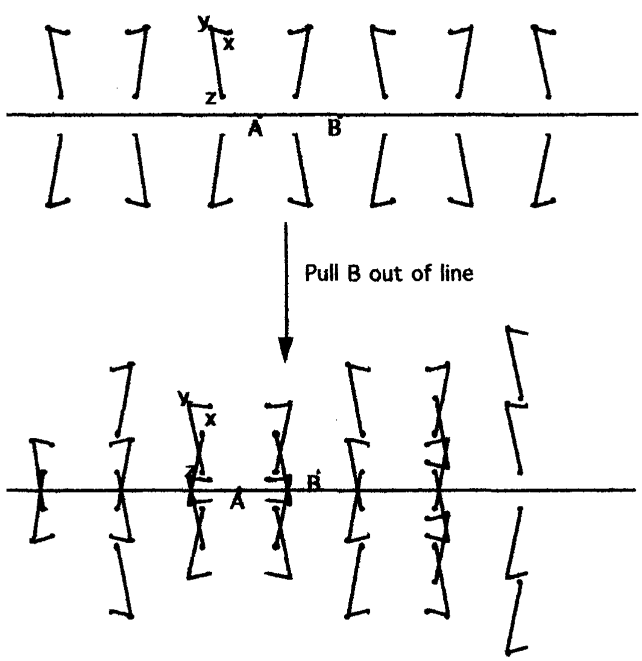

Figure VI

Joanne made the connection between her geometry experience and her previous experience of group theory (in much the same way as I think I did not all that long ago.) She identified a powerful feature of the computer here:

'very importantly, it showed me how some patterns, which I had previously considered to be different, were in fact the same'

Summary

I have tried to point to what I think is an important factor in recognising and discussing pattern namely the development of an ability, in a geometrical context, to shift focus from object to process. I have made connections between this shift and distinctions discussed in the algebra literature. Along the way I have been critical (too harshly, maybe) of some treatments of pattern which have appeared in professional journals. At the same time, through consideration of the work I have done with my own students, I have attempted to indicate how the awareness of process/object shifts in the understanding of pattern does have implications for pedagogy.

I have not attempted, in this short paper, to characterise any of the activities as geometric or algebraic. I think, however, that the development of understanding of pattern requires us to be able to flip from one mode of activity to the other and that this, in turn, is connected with the shift of focus I have described.

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