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Focus groups to ascertain the presence of formative feedback in CAA

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First year mathematics undergraduates were asked about their experiences of using computer-aided assessment (CAA) in their mathematics modules. It forms a small component of their summative scores in some modules. The aims of these focus groups were to establish how students use CAA systems and how they respond to its feedback. This paper discusses why these should be of interest, how students responded, and the implications on future work.

Keywords: activity theory, computer-aided assessment, formative feedback, undergraduate mathematics.

Introduction

The aim of this PhD project is to examine the effectiveness of the feedback offered by computer-aided assessment (CAA). Many CAA systems feature practice tests to attempt formative feedback, which in literature is described as effective, as we discuss later. First we discuss what formative feedback is and why it is considered effective.

In defining formative feedback, we observe that student response is a necessary component: how students act and react during practice tests is intrinsic to our study. We use activity theory to inform and direct our study of students' activities when undertaking practice tests and we discuss the merits of doing so in this paper.

Formative Feedback

Defining formative feedback is no trivial task: indeed it is one that is often challenged – particularly when we seek a distinction between formative assessment and formative feedback. Taras (2002, 505) believes that for feedback to be effective there are three conditions: "(1) a [community] knowledge of standards, (2) the necessity to compare these standards to one's own work, and (3) taking action to close the gap".

When discussing formative feedback, Taras (506) insists there must be evidence to demonstrate improvement, indicating formative feedback satisfies the third condition of effectiveness. Wiliam and Black (1996, 543) further insist that formative assessment requires a comparison between an accepted standard and the student's work, as well as there being evidence an improvement has been made: satisfying Taras' first and second conditions. Indeed Sadler (1989), citing Ramaprasad (1983), repeats the condition that feedback needs to be acted upon in order for it to qualify as formative. However, while Ramaprasad writes of 'feedback', Sadler discusses 'assessment'.

It would appear that formative assessment and formative feedback are used almost interchangeably. We settle on the phrase 'formative feedback' and, following definitions hitherto, we define formative feedback to satisfy three conditions:

1. A judgement is made against a given standard.
2. Advice is offered to bridge the gap between the judgement and the given standard.

3. Evidence is available to demonstrate that an improvement has made with respect to this gap.

Having established what formative feedback is, it must be noted that it is not only useful but its impact is profound. Concluding from the Beaton et al. (1996) analysis of the TIMSS (Trends in International Mathematics and Science Study) 1995 study, Black and Wiliam (1998) suggest that nationwide best-effect formative feedback would propel England from mid-table of those countries measured for mathematical achievement to the top five of the forty-one studied.

Due to the definition of formative feedback, it follows that the process involves bilateral input: educators offer advice; and students provide evidence that they have improved. If successful, students reach the expected standard and educators no longer need to provide advice at the culmination of this process. Yorke (2003, 496-7) argues that this is a path towards independence: students ought to be able to complete the assessment without the support they were offered previously.

Additionally, Hattie and Timperley (2007, 87-90) argue that feedback has a role in teaching self-regulation. Part of this process involves establishing sources to obtain feedback. They suggest that too many students neglect this responsibility and “view feedback as the responsibility of someone else” (Hattie and Timperley, 101).

Having established what formative feedback is, its power, and how it encourages independence and self-regulation in students, our attention turns to computer-aided assessment (CAA).

Computer-Aided Assessment

One difficulty with assessing mathematical knowledge is determining the best way to communicate mathematics between the student and computer. Mathematics-based CAA systems use computer algebra systems to compare a student’s response to stored solutions (Sangwin 2007; Beevers et al. 2008) alongside multiple-choice questioning, numerical input and selection-based responses (Greenhow and Gill 2004; Pidcock, Palipana, and Green 2004).

Each has its own qualities, but one of the more common traits is of interest to us: the implementation of formative feedback. Many of them feature a practice system, whereby a student may make unlimited attempts at practice tests, improving on highlighted weaknesses, before attempting a summative test (Pidcock, Palipana, and Green 2004). It has also been used to allow lecturers to tailor their early lectures in light of students’ initial performances (Greenhow 2000).

It is through practice tests that students obtain feedback. Sangwin (2007) argues that providing immediate feedback gives students the motivation to attempt the assessment again. It is clear that the intention is formative but, as we observed in the works of Ramaprasad (1983), Sadler (1989), Wiliam and Black (1998), Taras (2002) and Yorke (2003) in the preceding section, without a demonstrable reduction of the knowledge gap it is not sufficient for this feedback to be deemed ‘formative’.

Though CAA systems might record practice test attempts, we do not believe these records reflect students’ current level of performance: for example, a student might omit questions he previously answered correctly when attempting subsequent tests. Therefore it is not possible to judge whether students’ scores are improving through analysis of system data. That is, without introducing a pre-test to the existing tests, we are unable to determine whether CAA is formative without discussing the issue with the students themselves.

Methodology

Choosing methods that yield a representation of the majority of students undertaking CAA in various institutions and subjects across different systems was not possible at this early stage of the project. Methods, such as questionnaires and large-scale individual interviews, require questioning techniques that examine the central issues in detail, whilst also following a structure that facilitates comparisons and generalisations. It follows that determining the central issues must be a priority.

Focus groups appeared to be the best way to invoke discussion of the key ideas: they permit an open-endedness of questioning and responding, and the opportunity to probe further as required. On this basis, focus groups were chosen to initiate the investigation with the intention of using questionnaires at a later time to test the generalisability of focus group conclusions.

Before the Easter vacation in 2011, students attending a lecture of a first-year calculus module with CAA tests were invited to volunteer. They each had similar exposure to the Questionmark Perception CAA system over the first semester and the start of the second semester. Seventeen students of 237 students registered for the course filled in a response slip to indicate times when they were free to attend. From these, we invited twelve students that chose the most popular time slots. They formed two groups (seven in the first, five in the second). On the days of the interviews six of the first group attended; only two attended the second group. Convenience payments were not offered on either occasion.

Both focus groups were asked questions from a prepared list with the current question displayed on a laptop computer. In attendance, beyond the participants, were an interviewer (the first author) and a moderator (a research student) that made notes of the discussion. Two audio recordings were made in each focus group and were transcribed by the interviewer. Each focus group lasted forty-five minutes.

The transcriptions were coded according to the research questions. Essentially, it was the purpose of this coding to help us answer the research questions using quotations from the students, accompanied by commentary and interpretation.

For the sake of brevity and consistency, we discuss only the findings that relate to the following research questions that address formative feedback and self-regulation.

1. Can the feedback offered by CAA be considered “formative”?
    - To what extent does CAA allow students to ascertain their learning gaps?
    - Is there evidence to suggest that students reduce their knowledge deficits by means of this assessment?
2. What reactions and behaviours do students exhibit when they receive feedback through CAA?

Analysis

With respect to these research questions, we examine the feedback that CAA offers and compare this to our definition of formative feedback. Naturally our focus turns to student activity and we discuss how our findings relate to activity theory.

CAA as formative feedback

Some students see CAA as the opportunity to identify learning gaps. As Hattie and Timperley (2007) propose, it is only through assessments that some students identify problems: one student said, “sometimes you think you understand something and then assessments bring up stuff that maybe you're struggling with”. Others find CAA helpful to consolidate what has been explained in lectures: “it's drawn my attention to bits I probably missed in the lectures and… it's causing me to learn it again”.

The participants disagreed whether the feedback given by the CAA systems they used are adequate for them to improve. On the one hand, the feedback is sufficient for them to complete the summative assessment, “if there is a question I can't do I generally have a look at the worked example... and then just go back through until I can get 100% on the practice test”. However, there are several complaints with this approach.

First, the prescriptive nature of some feedback removes the challenge from the assessment: “The information in the practice tests is possibly in such a way that anybody without having attended any of the lectures... could probably answer and do fairly well in the actual test”.

Secondly, the quality and detail of the feedback is inconsistent: “sometimes you can click... and it will say slightly more than ‘this answer is wrong’. Occasionally it will say 'consider this'... or 'this is the kind of concept involved'... but there's equally a chance you get there and there's just a big cross”.

An important aspect of formative feedback is improving students' knowledge. When asked, students were quite scathing in their interpretation of their gains in knowledge and understanding: “with computer assessments... generally if I don't understand something going into it I don't really understand it when I come out”.

Another respondent said that the feedback in practice tests allowed students to copy the given solution and make appropriate numerical adjustments to mechanically retrieve answers for similar questions. He suggested that feedback should give “suggestions on how to solve it so people will still have to use a bit of brain”. A fellow participant lamented being able to “write down the answer without thinking”, explaining, “you're not actually learning any good skills”

One student suggested that there is not enough feedback in CAA for it to be deemed “formative”. He believed that CAA was useful to revisit key points at the end of a topic but that it did not guide his learning. This student also complained about the lack of feedback from the summative test: “it would be nice to have feedback on the questions – the ones that you get wrong where you've gone wrong rather than just a tick or a cross”. Others echo this sentiment.

As Hattie and Timperley (2007) suggest, students place the responsibility of finding learning gaps on their assessors. One might pose the question: do students stop trying to improve if they are no longer receiving feedback from their assessors? If that were the case it might suggest that students believe there is no need for independence once the summative test is completed. However, it might also appear that this student has not acquired the independence to obtain feedback independently or to self-regulate learning.

Despite their apparent dissatisfaction with the feedback they receive, they use it nonetheless. They believe the feedback from the practice tests is detailed enough to enable them to answer the summative test questions. Indeed, some students will not attempt the summative test until they achieve perfect scores in the practice tests. To that end, students described CAA as formative: “a computer test is a formative assessment. Whilst it doesn't always sort of guide learning it can flag up useful things which you'll pick up on in terms of preparing for your [exam]”; “straight away that made me think that computer-based assessment is formative”.

Conflict and its manifestation in activity

Participants used phrases like ‘simple’, ‘not complicated’, ‘easy’ and ‘lazy’ when discussing CAA. They like CAA, but admit this is out of laziness: “Again, the lazy part of me – because it's easy marks – would [choose] computer assessment [over traditional assessment]”; “If you want to make your life easy, then everyone will go for computer assessment. If you want to learn something and gain something out of the subject, then traditional assessment is better”.

This is an indication of the conflict that lies within students. Students feel the need to acquire good marks in order to progress and ultimately gain a good job after university; however, these students have chosen a challenging subject that rewards exploration and perseverance. It seems students find the two incompatible: a student noted “I would like to be comfortable with the material and then as a result get good marks”, but appeared not to be convinced that this is a viable path. However, they appreciate the need for mathematical understanding: “You want to be able to get to the next year of the course so you want the marks to be able to do that… but in later years you’ll fall down if you don’t know what you’re talking about”.

These students remain unconvinced that CAA is making them more adept at mathematics: “If you want to learn something and gain something out of the subject, then traditional assessment is better”; though they concede that CAA tests more of the module content than traditional forms of assessments: “You keep focussing on little bits, it might be a better way of learning”.

It may seem a trivial point that students have a confused view of what CAA does for them and what their overall objectives for the course are. However, it is this confusion that we must understand in order to interpret how students are adapting to CAA. We turn to activity theory to structure these confusions and to inform our study.

Kaptelinin and Nardi (2006, 199) explain the associations between motives, needs, the students as subjects, and their activity. An intrinsic point they make is that subjects and their motives are not directly observable and these can be discovered through analysing their activity.

What is also clear is that students interpret their needs differently and their motives are influenced by this interpretation. We might, therefore, expect students to behave differently – perhaps uniquely – when presented with a CAA task to complete. However, these students appeared to act in similar ways. The participants all said that they practised until they achieved perfect scores, with variations such as note taking and using lecture material. We might conclude that their motives are similar.

Conclusions

Students believe that CAA offers formative feedback. It identifies areas of weakness and tests more of the module content. They do not believe that it is the best method of testing mathematical ability, but they believe it narrows learning gaps.

They feel that CAA is easy and describe their preference for CAA over traditional assessment as a ‘lazy’ choice. The feedback that CAA offers is close to the feedback they want and expect; but concede that the feedback is prescriptive and removes some of the challenge that mathematics ought to present. Students feel that CAA is effective for improving their procedural adeptness, but there are better methods for assessing their conceptual understanding.

We might conclude that students like CAA because it is compatible with their motives, but it is to this point that we must turn our attention. There is a need to study how the activity system interacts with individuals' subjectivities; and activity theory suggests that to do so requires that we observe students' activity when undertaking CAA tests. As such, activity theory directs us towards observations in our future work to understand students' conflicts, motives and independence.

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Where has all the beauty gone?

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Bertrand Russell famously talked of mathematics as possessing an "austere beauty". It would seem though that the capacity to appreciate the aesthetic aspects of our field is not necessarily the preserve of the mathematical elite. Indeed, a number of educators believe that such considerations have, in conjunction with various cognitive factors, the potential to play a significant role with respect to the student learning of mathematics in the classroom. We consider here the notion of the mathematical aesthetic within this context, drawing on the work of a number of key thinkers in this area. Our preliminary explorations focus on a number of lesson observations, and the intention at this stage is merely to ascertain whether or not aesthetic considerations are playing any part in students' mathematical development in the classroom. We provide a brief discussion of our findings thus far, highlighting potential issues and dichotomies that would appear to arise as a consequence of the current climate of test-score-driven schooling.

Introduction

Aesthetics is generally considered to be a sub-discipline of axiology, which is itself a branch of philosophy concerned with the nature of values and value judgments. More specifically, aesthetics is associated with the nature of beauty in all its many forms. Those who study it are interested in both the creation and the appreciation of objects of beauty. Aesthetics often has connotations of physical beauty (and hence of pleasurable visual sensations), but this is an unnecessarily restricted viewpoint. Indeed, Yuri Borev, a former Professor of Aesthetics at the University of Moscow, gives the following very broad definition of his field of study:

> Any human activity has, besides a purely utilitarian purpose, the grains of what makes it universally important for mankind. It is these grains which lend human activity its aesthetic flavour. (Borev 1985)

However, our primary purpose here is to consider the situation with regard to the mathematical aesthetic in our school classrooms. The above is therefore possibly a little too all-encompassing for our present needs. The somewhat narrower perspective we adopt in this paper in keeping with the following:

> A student's aesthetic capacity is not simply equivalent to her ability to identify formal qualities such as economy, unexpectedness or inevitability in mathematical entities. Rather, her aesthetic capacity relates to her sensibility in combining information and imagination when making purposeful decisions regarding meaning and pleasure. (Sinclair 2004)

It ought to be mentioned at this point that the title of the present paper is intended simply to stimulate thought and discussion; we are certainly not claiming here that the consideration of the mathematical aesthetic has ever played a prominent role in the way that mathematics is taught and learnt in the classroom. In fact, no assumptions whatsoever are being made in this regard, and, as a consequence, we do not attempt to make any comparisons with the past; all our observations concern the current state of play. An alternative title might have been: Is there evidence in our classrooms for the presence of, or appreciation for, the various aesthetic aspects of mathematics?

In this paper then we make an initial foray into the notion of the mathematical aesthetic within the context of the modern mathematics classroom, bearing in mind all the baggage that accompanies this milieu in terms of the different constraints and pressures that pupils and teachers continually have to work under. We ask a number of questions in this regard, and, in particular, consider the potentially inhibiting influence that current test-score-driven schooling may have on the development of both teachers' and students' aesthetic sensibilities.

The mathematical aesthetic

Let us now consider in a little more detail what is actually meant by the mathematical aesthetic. It is highly likely that any reader of this paper will have experienced some aspect of the beauty that is inherent in mathematics. Besides the obvious visual appeal of depictions of mathematical objects such as the Mandelbrot set, there are several ways in which our aesthetic sensibilities may be touched by mathematics. For example, every one of us will have encountered *beauty in mathematical method*. This is able to manifest itself through the appreciation of an elegant proof of a theorem such as the irrationality of $\sqrt{2}$ or the infinitude of the primes, both of which are extremely succinct and based on very simple notions. Notice how often the word "elegant" appears both in discussions between mathematicians and in mathematical writing.

We may also gain pleasure from the very act of participating in mathematical activities, particularly those of an exploratory nature. In engaging with mathematics this way and adopting a 'hands-on' approach we become intimately acquainted with its beautiful structures and are able gradually to unlock its secrets. This might be termed *beauty via mathematical experience*. In addition, there are many results in mathematics that could be deemed to possess a certain aesthetic quality. One of the most oft-quoted examples in this regard is the equation $e^{i\pi}+1=0$ relating five numbers that play a central role in the field of mathematical endeavour. The Prime Number Theorem, a result giving a degree of order to the apparently-erratic distribution of the prime numbers, is another instance of this. Indeed, we may say that such examples provide us with the opportunity to experience *beauty in mathematical results*. It is of course possible also to enter into any number of debates about the nature of mathematics. Is it, for example, the case that mathematics is invented or simply discovered? Considerations of this type might be referred to as *beauty through philosophical aspects of mathematics*.

An elitist concept?

Of course, some of the examples given above might not necessarily be suitable for the mathematics classroom. At this point then we may ask ourselves a rather pertinent question: Is the mathematical aesthetic an elitist concept; something that only a select few have the capacity to appreciate? Some professional mathematicians would no doubt answer in the affirmative. In order seemingly to reinforce this point of view, we provide the following relatively well-known quotes from Henri Poincare, Godfrey

Hardy and Bertrand Russell, respectively, each of whom was a leading of mathematician of their time:

> The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful. (Huntley 1970)

> The mathematician's patterns, like the painter's or poet's, must be beautiful. The ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test: There is no permanent place in the world for ugly mathematics. (Hardy 1999)

> Mathematics, rightly viewed, possesses not only truth, but supreme beauty. (Russell 1988)

There are, however, a number of distinguished mathematics educators who do not share the belief that only expert mathematicians are truly able to experience the mathematical aesthetic. This more inclusive viewpoint is encapsulated in the following quote from Mary Beth Ruskai:

> We cannot hope that many children will learn mathematics unless we find a way to share our enjoyment and show them its beauty as well as its utility. (Ruskai 1995)

Natalie Sinclair is possibly the most prolific contemporary author on matters associated with aesthetics in mathematics education. In Sinclair (2004) she makes a strong case for the vital role that aesthetic processes play in the development of mathematical knowledge and in the course of mathematical enquiry (in addition to any accompanying cognitive processes that may be taking place). This is irrespective of whether the learning is taking place in a classroom, a lecture theatre or in the rarefied confines of a research mathematician's office.

On the basis of the theoretical work carried out in Dewey (1934), Sinclair identifies three fundamental roles that are played by the aesthetic:

- motivational;
- evaluative;
- generative.

Let us consider each of these in turn. It is important first to emphasise the fact that the motivational aspect of the aesthetic is intrinsic rather extrinsic. That is to say, the reward for pursuing some mathematical activity is the inherent pleasure and personal satisfaction one derives from it rather than any type of material gain one might receive. This role may be considered at least partly responsible for situations in which a person is attracted to a certain mathematical problem or area. The evaluative role is concerned with making judgements about the beauty of the mathematics one is engaged in. For example: Is a particular proof more elegant than another? Is one result deeper than another? Which of the possible lines of enquiry looks the most promising? The answers to these questions will frequently be influenced by aesthetic responses. Finally, the generative role may in some sense be linked with intuitive modes of thought. The aesthetic would appear in this case to operate at a subconscious level, providing tacit guidance for the mathematician. It might be partly responsible for leading to new ideas that would not necessarily have arisen easily from deductive reasoning alone, thereby facilitating generative learning. For more in-depth discussions concerning generative learning see Wittrock (1974a and 1974b).

From the above it would appear that the mathematical aesthetic has the potential to play a significant role in mathematical development at many levels, and that it is not therefore the preserve of the mathematical elite. This might in turn imply that there are tangible benefits to be gained through planning for the presence of aesthetic elements to lessons. Indeed, a number of educators concur with this sentiment. For further research and commentary regarding the aesthetic in the mathematics classroom, see Betts (2005), Gadanidis and Hoogland (2003), Mack (2006); Papert (1978). Furthermore, Sinclair (2009) argues that aesthetic awareness ought to be both a connective and a liberating force in mathematics education.

Aesthetics in the mathematics classroom

By way of an initial exploration in this area, a number of lesson observations took place (each with a different teacher). At this early stage we simply report on what might be seen as ‘the current state of play’. We note that in most cases the observed lessons contained several good features, and some would in many respects have been regarded as good to outstanding. It was the case, however, that there was not one single explicit reference to the beauty inherent in the mathematical topics being taught, and in many of the activities provided there seemed to be virtually no scope to facilitate anything even implicitly in this regard.

Instead, the dominant discourse, in terms of motivation at least, concerned forthcoming examinations, despite the fact that in some cases these were many months away. Phrases such as “The examiner will be looking for …” or “We are doing this because you will get asked about it in your GCSE” were frequently to be heard. In fact, we recorded 28 comments of this nature (remembering, from above, that there were none at all associated with aesthetic considerations). This would appear to indicate that the current climate of test-score-driven schooling is very much driving the way that the curriculum is being delivered. It is a question of deciding whether or not this is an entirely good thing.

In addition to the fact that there were no real opportunities for students to experience or explore mathematics in a way that would allow their aesthetic sensibilities to develop and even flourish, there appeared to be a lack of vocabulary associated with the aesthetic. Although plenty of good learning certainly did take place in many of the lessons that were observed, the atmosphere and learning environment often seemed somewhat flat in the sense that there was a complete absence of any “Aha!” moments amongst the students.

Given that aesthetic considerations do indeed have a part to play in the teaching and learning mathematics at school, then some might be a little concerned that, from our limited observations at least, test-score-driven schooling is dominating classroom discourse to such an extent that motives for teaching and learning are purely utilitarian as opposed to aesthetic. There are thus some stark questions to be asked in this regard: Is there the will amongst stakeholders to remedy this situation, or have we now gone beyond the point of no return? Is it now the case that only the mathematical elite will have the opportunity to appreciate the aesthetic aspects of mathematics in the classroom? Are we, in schools, simply disregarding aesthetic values in mathematics? If so, is this process self perpetuating? Will this affect the mathematical creativity and engagement of future generations to the extent that most students become ‘mathematical parrots’, capable only of regurgitating the work of others as opposed to being creative themselves?

Concluding remarks

In this paper we are merely highlighting some findings and expressing possible concerns over what might be perceived as just one of many potential issues in contemporary mathematics education. At this preliminary stage we are certainly not claiming to offer any solutions, although several suggestions will be mooted here; there is indeed plenty of scope for future research in this area. As mathematicians and teachers, we occupy a privileged position in the sense that each of us is likely to be endowed with a heightened awareness of just what it means for a piece of mathematics to possess ‘beauty’. This raises yet another question: Are students benefitting from this in their learning? Surely, in our roles as teachers and educators, it is our duty to pass on the sheer delight our subject gives us at so many levels.

In order to nurture students’ aesthetic sensibilities with respect to mathematics, the learning environment must be conducive both to facilitating the requisite intellectual involvement and to encouraging the independent and creative exploration of mathematics. It does seem that the current climate of test-score-driven schooling is in fact inhibiting teachers’ natural tendencies toward the aesthetic, thereby making the establishment of such learning environments very rare events indeed. Since this phenomenon would appear to be here to stay, some thought may be needed as to how to counteract some of its potentially negative effects.

First, this issue might be made explicit on PGCE programmes. Some educators believe that teachers should plan for aesthetic activity to take place in any mathematics lesson. There is of course the potential here for a dichotomy between the educators’ ideals and student teachers’ realities, and some open discussion in this regard would be helpful in preparing trainees for the dilemmas and different pressures they will encounter when attempting to cater for aesthetic dimensions in their lessons. Indeed, specific guidance may be needed in this area.

Second, are the curriculum and the accompanying resources (textbooks, for example) also conspiring against the aesthetic? To deflect the focus from this constant test-score-driven approach to teaching, it would seem important for teachers to select tasks that give students the opportunity to have aesthetics experiences. There are in fact a number of examples of good practice in this area. In Griffiths (2010) is an activity that was designed to allow students to experience the mathematical aesthetic in several different ways, and at several different levels. The starting point is something as simple as the logo of the Fibonacci Association. In Unal (2008 and 2009) may be found examples relating diagrams to sums of infinite series in surprising and beautiful ways. An example of a beautifully simple visual model describing the way the universe is expanding in given in Griffiths (2009). In Lesmoir-Gordon (2010) can be seen some wonderful exposition and accompanying graphics associated with fractals. For further specific examples aimed at younger students, see Gadanidis and Hoogland (2003) and Sinclair (2002 and 2006).

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The use of conversation analysis in identifying creative approaches to mathematical problem solving

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Techniques of conversation analysis have been used in an effort to better understand the thought processes of adults engaged in a range of mathematical tasks. Participants were asked to provide a commentary during problem solving, in a non-judgmental environment with minimum intervention from the researcher.

Interesting outcomes from the work are: an inability to link arithmetic and algebra in problem solving, a lack of specialised mathematical vocabulary, misuse of standard algorithms which have been learned in a superficial manner without full understanding, and a preference for justification by concrete example rather than through abstract reasoning. Distinct differences in approach to problem solving are observed between participants with different preferred learning styles.

Keywords: Conversation analysis, adult numeracy, preferred learning styles.

Introduction

This investigation came about through a desire to better understand the problems faced by adults in improving their numeracy.

A first step when new students join a numeracy class is often to undertake an assessment of their mathematical skills through a written test. However, assessment of written answers may give limited insight into the thought processes of students. Where solutions are incorrect, this might variously be a result of: misinterpretation of the problem, lack of knowledge and understanding of solution methods, or inaccuracy in applying formulae and algorithms.

In this small project, six adults living in a town in North Wales and ranging in age from 20 to 60 were selected as a convenience sample. None was in employment requiring specialised use of mathematics or numeracy beyond Adult Numeracy level 2, and none had undertaken any formal study of mathematics or numeracy since leaving school. Occupations included: office worker, computer technician and school teacher, and one participant had been unemployed for approximately a year. Although a limited sample, the group appeared to be fairly typical of the adult population as a whole.

Techniques of conversation analysis were used in an effort to better understand the thought processes of the participants (Ginsburg 1981). Subjects were asked to provide a commentary whilst engaged in a range of mathematical tasks, in a non-judgemental environment with minimum intervention from the researcher.

Investigating the arithmetic-algebra connection

An initial objective was to investigate the extent to which participants were able to make connections between arithmetic and algebra by substituting arithmetical values where appropriate to clarify algebraic expressions, and by formulating simple algebraic expressions to help in the solution of arithmetic problems (Lee and Wheeler 1989).

Task 1

Researcher: Would you look at this expression and say whether it is definitely true, definitely not true, or possibly true:

$$\frac{2x + 1}{2x + 1 + 7} = \frac{1}{8}$$

Participant: I would say it is not true... no, it is true, but it would have to be nought. That is my quick answer.

Four of the subjects spotted that the first expression was true for x = 0, but all then assumed that the value of x had to be zero so the expression was always true.

One participant commented: 'If x could have any value, then there are millions of answers and there is no way of checking if it's true. '

Task 2

Researcher: Could you do the same with this expression... say whether it is definitely true, definitely not true, or possibly true:

$$\frac{1}{6x} - \frac{1}{3x} = \frac{1}{3x}$$

Participant: I would say that would be definitely true because the sum of those two would be the other one.

Four participants made the error:

$$^1/_6 - {}^1/_3 = {}^1/_3$$

No-one attempted to substitute numerical values to test the equality, relying instead on first impressions of the pattern of the equation.

Task 3

Researcher: Now try this question. Add and subtract numbers from 10, and see if the final total would always be the same for different starting values.
Participant: I would have to try this out with a range of numbers. If you use 7 you get 17... Then take away 7 is 3... So it comes to 20.
Let me go for 5. That is 15... and 5... and it's 20 again...
Well, yes, I think so. It would be the same.

The general approach was to try out several examples. The number of values chosen varied between two and four. Only one participant produced an algebraic expression:

(10+ N) + (10-N) = 20 as a proof of the assertion.

The outcome of tasks 1-3 suggested that the group of adults saw little connection between algebra and arithmetic, treating these as two entirely separate compartments of mathematics which could not be used together in any meaningful way for problem solving.

Using real objects

It has been suggested by Nunes, Light and Mason (1993) that the direct use of concrete objects in solving numeracy problems is less intellectually demanding than the use of mathematical methods, so will generally be preferred. To test this hypothesis, several tasks were devised using everyday physical objects. An example is given below:

Task 4

Researcher: I am going to show you this plastic lid and this tin. Could you say how you would work out whether the area of the plastic lid is bigger or smaller than the area of the paper label - without removing the label?

Participant: If I was to start with that line on the label at the edge of this sheet of plastic, and roll it very carefully like that.See where it goes...
I can see that it comes over the edge.
Whether it's got a smaller area –
It's got a smaller width, hasn't it?
So, as an estimate, although it goes longer here, its actually shorter there, so it's similar I would guess. About an inch wider here, and it seems about an inch shorter.

Researcher: If you wanted to be a bit more precise, how could you make some measurements?

Participant: I would roll the tin, starting with this edge on the label here, and see where it stops. If I can use a ruler... Then compare the two.

Researcher: Could you just imagine the label being unwrapped, so the circumference of the tin is the length of that rectangle.

Participant: Well, yes. So that would become a rectangle, and you could find the area of the rectangle.

Researcher: Would any of these formulae be true?

area of label = height of tin × circumference

area = π × radius^2 × height

area = π × height × diameter

Participant: Area of the label.... Height of the tin times circumference...
Yes, that's true, isn't it.
Area is pi r squared h… No, it's 2 pi r… I'm not sure.

When undertaking tasks involving real objects, the participants almost always attempted to solve the problem by physical measurements alone, without recourse to mathematical reasoning. When mathematical methods were suggested, there was an evident lack of recall of geometrical and algebraic techniques.

Investigating approaches to problem solving

It was felt that different individuals might have different approaches to problem solving, and some insight might be gained from assessing their preferred learning style. A variety of taxonomies of learning style have been proposed, but that of Roger Felder (1993) was chosen. Participants were evaluated using a questionnaire instrument to determine their positioning in respect to four dichotomies:

- How does the subject prefer to process information:
  actively—through engagement in physical activity or discussion, or
  reflectively—through introspection?
- How does the subject progress toward understanding:
  sequentially—in a logical progression of small incremental steps, or
  globally—in large jumps, holistically?
- What type of information does the subject preferentially perceive:
  sensory— sights, sounds, physical sensations, or
  intuitive— memories, ideas, insights?
- How is sensory information most effectively perceived by the subject:
  visually—pictures, diagrams, graphs, demonstrations, or
  verbally—sounds, written and spoken words and formulas?

Testing revealed a range of personal profiles for the six participants, indicated on a scale from 1 (low preference) to 11 (high preference) within each dichotomy:

Subject	Sensing	Intuitive	Visual	Verbal	Sequential	Global	Active	Reflective
1	2	9	9	2	3	8	2	9
2	4	7	6	5	7	4	7	4
3	9	2	11	0	5	6	6	5
4	11	0	8	3	8	3	6	5
5	10	1	2	9	6	5	2	9
6	2	9	4	7	6	7	5	6

Moderate preference   Strong preference

Participants were then asked to undertake a series of tasks, and an attempt was made to relate their approach to problem solving to the assessed learning style preferences. Examples are presented below:

Task 5

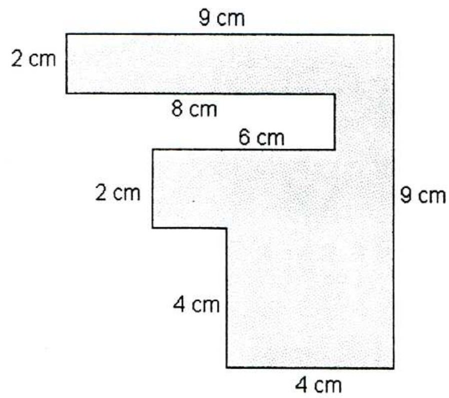
Researcher: Could you read this problem and have a go at solving it:

Participant: The teachers at Cwm Coed School decide to organise a fund to pay for gifts to teachers who leave the school.
They decide to pay £5 a year into this fund. When a teacher leaves, he or she is given £30 plus £3 for every year the teacher has been at the school. After how many years at the school would then amount paid in by the teacher equal the amount of the gift received on leaving?
Well, if they stayed for six years they would get £30 plus ... they would also get £3 for every year they had been teaching. Ur... for six years that would be £48.
(pause … 6 seconds)
The difference between 3 and 5 is £2. Uhm...
In 15 years they would get £75 and pay £75. I see it, yes! So it's 15 years.

The participant (subject 6) worked entirely ***verbally*** throughout, writing nothing on paper. There was no recourse to any algebraic technique. The final solution was reached in a moment of ***intuition***. This fitted remarkably with their preferred learning style as strongly intuitive and moderately verbal.

Task 6

Researcher: This is a shape, and I would like you to work out the distance round the edge of it.



Participant: The distance round the edge of it...
(begins counting the distances by mental arithmetic, then stops)
We haven't got how long that is...
We could take that from that... no, that wouldn't work...
It’s not as straight forward as I first thought. Oh dear...
(pause … 11 seconds)
Researcher: Can we do anything with those two measurements?

Participant: Oh, yes. So that's one... That's six....It will work this time..
So that's three... and that's one as well...
(using calculator)……. 48
Sorry, that took a long time!

The participant's difficulty in seeing the solution to this apparently simple geometrical problem may lie in very strong preference for ***sensing*** and ***sequential*** learning styles (subject 4), as opposed to the ***intuitive*** and ***global*** approaches which might have been more successful.

Conclusions

This study is ongoing, but has begun to provide interesting insight into adult numeracy. The conversation extracts above are part of a larger body of data which allows some general conclusions to be drawn:

Within a few years of leaving school, and with no further formal study of mathematics or numeracy, adults lose much of their familiarity with algebra and geometry. They are hampered by a lack of specialised mathematical vocabulary when exploring the solution of problems, and frequently misuse standard algorithms which have been learned in a superficial manner without full understanding.

A particular difficulty arises from an inability to link arithmetic and algebra in meaningful ways during problem solving. There is a strong preference for justification by concrete example and direct measurement, rather than through abstract reasoning. This is particularly evident when problems are presented which involve physical objects.

Distinct differences in approach to problem solving are observed between participants with different preferred learning styles. Individuals appear to develop their own unique mathematical coping strategies which may diverge widely from the standard methods taught in schools. This seems an interesting area for further investigation.

It became apparent during the research that there is a tendency for the teacher to intervene too quickly when a response is not forthcoming, allowing insufficient time for reflective learners to think through the problem and develop their own, perhaps unique, solutions.

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Supporting students in their transition to university mathematics

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Our *Transmaths* projects aimed to understand how different practices in mathematics during the transition to higher education impact on students' dispositions and identity and influence their future success in mathematically demanding subjects. In this paper, we discuss three examples of university transition support mechanisms and how these seem to be helping students, in particular those who are considered mathematically weak, to make a successful transition into university. We discuss implications for pedagogy, curriculum and institutions.

Keywords: transition, undergraduate mathematics, boundary crossing, resilience, learning to learn

Introduction

Our *Transmaths*[1] projects take a socio-cultural theoretical perspective to try to understand how different educational practices in mathematics at College and in transition to University impact on students' dispositions and identity, hence influencing their choices and future success in subjects that demand high levels of mathematics.

In this paper we draw particularly on three of the projects' papers (Williams et al. under review, Hernandez-Martinez and Williams accepted, Farnsworth and Williams under review) to elicit a discussion on different university transition support mechanisms and how these seem to help different students to achieve a successful transition. The papers take different theoretical concepts, such as 'brokering', 'third spaces', 'resilience' or 'learning to learn', to explain how learners interact in different ways with the socio-cultural contexts in which they participate and in particular during transitional moments which pose challenges and obstacles to students, especially in relation to mathematics.

In the following, we briefly describe these papers and discuss the implications that our conclusions might have for pedagogy, curriculum and institutions.

Practices that support the transition to University

Boundary crossers, brokers and third spaces

The Williams et al. (under review) paper approaches the subject of first year university mathematics provision from the perspective of different 'boundary crossers', those who experience moving between two different activity systems like school and university. Some of these can also become 'brokers' (in the sense of Wenger 1998) or facilitate the creation of 'third spaces' where elements of both systems meet and new meanings are created (in the sense of Gutierrez, Baquedano-Lopez and Tejeda 1999). The paper describes the cases of James, an engineering student, Joanne, who teaches mathematics at school and university, and Lilian, a tutor responsible for the mathematics support centre at her university.

James dropped out of an 'elite' university engineering course because mathematics was "too hard". He switched to a less 'prestigious' engineering course in his local post-92 university, where he found himself doing very well: the mathematics was "a lot simpler" and his relative maturity, he said, "favoured" him. At the end of his first year he got 'first class' grades. It seems that several elements contributed to James' successful transition from one university to the other: although mathematics remained something of a problem in a few courses, his perception in general was that it was "a lot easier, more understandable" and "more enjoyable" because mathematics was taught more slowly, in smaller classes and, in general, the course contained more practical work and projects than that in the 'elite' university, something that is closer to how James sees himself professionally, "a more practical engineer". However, it seems that on reflection James regrets the loss in status and the exchange value associated with a qualification in a less prestigious university, and also that he now considers mathematics important: it is difficult, but this makes it valuable.

Joanne teaches mathematics part-time at school (Advanced level Calculus) and university (first year mathematics for engineering), but remarkably the topics are very similar. The university has employed her to 'teach' students that are mathematically weak, mainly those coming with a vocational qualification. Our observations and interviews with her led us to identify several differences in her practice at school versus university: (a) the pace of the work at university is much faster but given that her class is small (around 20), students feel that they are getting a quality, one-to-one time with the lecturer; (b) the expectation that students at university should be more independent in their learning, which was shared both by Joanne and her students, but also the realisation that this independence is harder to obtain in mathematics than in other subjects and that the gap in the mathematics to be learnt was just too big; (c) Joanne's use of formative assessment, by being aware of individual needs and constantly reinforcing students' understanding; and (d) the constraints at school about performance in lessons and exams and how these pressures were totally different at university. We see Joanne's work as one of brokering by introducing elements of school teaching (perceived as good quality teaching by her university students) into the university system.

Lilian works at the same university as Joanne in the mathematics support centre. Her work there involves not just teaching mathematics to anyone that needs help but also helping them "learning how to learn". She is also proactive in dealing with more than just a "sticking plaster job", but talking to lecturers and making them aware if a group of students are having problems in understanding certain topics, and giving lecturers some feedback on their teaching. Crucially, the institutional status of the support centre provides Lilian with the authority to 'broker' between students and staff, making the centre a 'third space', where developmental work takes place.

From the experiences of these three 'boundary crossers' we conclude that many students appreciate extra help in transition, which includes amongst other things smaller, interactive classes, a slower pace when focussed on critical difficulties, a more expert teacher who knows how to identify students' problems and "take them on from there", but perhaps more important, institutional spaces where brokering work is made possible, and that have the potential to generate a cultural change and professional development, and not just a quick fix.

Building resilience

The Hernandez-Martinez and Williams (accepted) paper focuses on the concept of resilience and how some students that are statistically considered 'at risk' because of their cultural and socio-economic background are able to build resilience and persist to achieve a successful transition.

In this paper, we define resilience as a dynamic process of interaction between sociocultural contexts and the agency of developing individuals. Taking Bourdieu's notion of social and cultural capital as representing the capacity to exercise agency in a field, we add a note on reflexivity: that students can develop capital through reflection, particularly on 'critical moments'. This capital can allow for agency in new fields (for example, during transition), and the possibility to negotiate successfully their habituses with the conditions of the new field.

We illustrate this concept with the cases of two students in transition: Jenni and John, who have acquired some capital during their schooling which became valuable during their transition. Both of them had negative experiences of mathematics at school, Jenni being in a disruptive class and John being in a "shit school" with no provision for further mathematics. Jenni experienced a 'critical moment' when she reflected on her situation and decided that she had "had enough now", blocking out her disruptive classmates and becoming a more independent learner, changing her 'hate' for mathematics into 'love' for the subject. This reflective development of such educational capital provided her with the necessary agency during transition to make her habitus resonate with the new field and take full advantage of what the new institution had to offer to students that have a more mature and independent approach to learning. In the case of John, his experience of having to undertake distance learning through the Further Mathematics Network provided him with the necessary capital (through a process of inner reflection) to persist during transition at moments where "nothing makes sense", especially in the case of mathematical proofs. His more mature approach to learning (as opposed to his peers that still expect to be 'spoon-fed') ensured that his habitus aligned with a new field that values such capital. In both cases, we emphasise the importance of different sources of capital, in Jenni's case a supportive and encouraging family and in John's case a special teacher who advised him and helped him see what it means to become a 'good' mathematician at university.

Therefore, we claim that resilient students are those who actively engage with a reflective process (which can be a critical moment) in which individuals become consciously aware of their need to break with what is taken-for-granted and therefore are able to develop certain social, cultural (and specifically educational) capital that they can bring to bear in a new field, giving them a certain agency to negotiate the transition successfully. Despite the poverty and other factors that put these students 'at risk' statistically, they show how significant social capital from their family, school or peer group can make the difference in their conscious acquisition of this educational capital.

We conclude that processes that encourage reflexivity in students should be incorporated in school pedagogical practices. This requires spaces to discuss, argue, question, think and connect mathematical ideas, but also spaces where learners can relate appropriately with a peer group, teachers, family and community, which are the sources of valuable forms of capital.

Learning for understanding and self-regulation

The Farnsworth and Williams (under review) paper approaches a first year university Medicine case study, where the main feature is a 'problem-based learning' (PBL) approach. This study provides insights into how students change habits, perceptions and beliefs about learning as they transition into their university studies.

By the nature of their degree, students in this case study reported themselves to be highly motivated to complete the course and to have an imagined future in medicine firmly in mind. The PBL approach to teaching/learning meant that some of these students found the 'transition gap' greater than other types of students who experienced a more 'traditional' teaching approach. In brief, PBL means that students are not directed to particular texts and that they are not directed by a tutor but only mentored and steered if they are going off track. Discussion with peers, more experienced students and tutors, are essential to learning through this approach, and some students realise that talking "about something from the top of your head (…) pushes you to learn it". Students experience a change in the way they see learning and really appreciate that "all that matters is whether I understand it or not and I can explain it to my colleagues". Mathematics learning, in particular, becomes for some an independent, self-directing task as one student expresses:

> Maths, for example, I found I learned a lot better by going on the Internet and looking up things like long division. Somehow, if you do it yourself you actually read it and you actually assimilate the knowledge.

The fact that PBL appeals to students' identification as future doctors, and how this influenced their learning, is clearly expressed by a student:

> [We were] trying to be almost, like, mini doctors when they're looking at the case the first time, because I think that's PBL, but try and diagnose something when you first see it and then linking them together as opposed to going home and say, "Oh, I don't [know] something and someone will pick it up".

The analysis identifies a learning system that is structured around the PBL curriculum and found that, for some students, the different aspects of the system worked together to support their transition. From this case study we conclude that a learning system that aligns students' goals of becoming professional doctors (engagement with future identities) with the norms and rules of the community of practice is better suited to encourage the acquisition of 'learning to learn' skills and a 'self-directed' approach to learning (Gallagher 1997, Rawson 2000). This in turn promotes shifts in students' dispositions and relationships towards knowledge, supporting the transition to higher education where an emphasis is placed on understanding and applying knowledge. We propose that students would benefit from: a) a curriculum that structures the learning system around a common goal, and b) explicit representation of the ways the different aspects of the learning system work together and complement each other to help them reach their goals.

Discussion

There is a concern in Higher Education about high rates of failure in first year courses that are highly mathematically demanding. Almost all universities in the UK have implemented certain mechanisms to alleviate the transition from school to university. However, some of these mechanisms seem not to be effective, or have little effect for those students who are considered mathematically weak but that nevertheless have the potential to become good professionals.

Here we have presented three examples of such mechanisms of support: (1) brokering and 'third spaces' of more school-like 'teaching' practices, such as small(er) classes or mathematics support centres, where effective, more student-centred learning may take place, whether these are institutionalised or not; (2) processes where reflexive work takes place, allowing students to consciously build capital that can give them agency in negotiating challenges such as later during transition; and (3) learning systems that support learning for understanding, 'learning to learn' and engagement with students' imagined future identities in a coherent way.

We believe that these mechanisms of support have important messages to contribute to the discussion of how best to support students in their transition to mathematics at university.

In the case of our 'boundary crossers', the key message we want to put across is that 'third spaces' should serve not as a 'sticking plaster' solution but as a mechanism of cultural change. Such spaces should become a source for professional development and for change based on research and practical experience within the institutional community. For example, Joanne's brokering work is evidently helping students to cope with the multiple changes that occur at once during transition, but because her status within the institution (as not a full-time member of the lecturing staff) does not allow her to influence the practice of the community, the success of her work is limited. In contrast, Lilian's brokering work reaches the community further because of the status that the support centre has within the institution. She is able to provide feedback to lecturers, influencing in this way the practices of some of them.

In the case of our resilient students, an important message is that the 'risk factors', which make these students 'vulnerable', can become central to their development of important educational capital. What we are suggesting then is that learning should incorporate conscious reflective work, and that this work can be best achieved by activities that are challenging, by discussion of different and perhaps opposing ideas, and by teaching content that is authentic and useful. We wonder if this could be possible in a system that prioritises 'exam results' and 'league tables', and if one day this might change to allow the majority of our students to be(come) resilient?

In our medicine students' case, our key message is that pedagogies should be able to 'speak' to students rather than alienate them. Here the notion of identity is vital: 'real' doctors solve problems by discussing with colleagues, by independently researching solutions, by striving to conceptually understand. PBL tries to replicate this and engage students into the community of practice. Students then feel that what they are learning is useful, that they are becoming 'mini-doctors', and that they are being enculturated into the practices of the career that they have imagined themselves doing. We should ask then if this could be applied to other subject areas where future imagined careers are not as clear as those of our medicine students, and where pedagogies might not be able to 'speak' as directly to students' identities? We then must ask ourselves, how much of what we expect students to do needs to be made explicit, and where we can structure the curriculum in ways that encourage students to learn through discovery and self-directed learning (which is actually directed towards a particular goal or imagined future and not a solitary process of the 'self')?

Our current 'knowledge transfer' project is attempting to synthesise our work to impact on policy and practice by creating 'tools' (e.g. policy briefings, new projects, think pieces, etc.) that can inform, persuade, influence and help our project partners and others in implementing changes. The project has been designed around the goal of sensitively transforming substantial research findings, such as the ones presented in this paper, so that they are best positioned to make a difference in mathematics education.

Acknowledgements

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End notes

[1] For more information about our projects go to www.transmaths.org

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The development of Taiwanese students' understanding of fractions: A problem-based learning approach

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Problem-based learning (PBL) was first implemented in medical education at the McMaster University in Canada in the late 1960s. Now, we are seeing an explosion in the use of PBL in its various adaptations across many levels and subject areas. This paper outlines some preliminary findings from a one-year PBL teaching intervention on students' understanding of fractions in a Taiwanese fifth grade mathematics classroom. The purpose of the study is twofold. Firstly, it seeks to investigate the process of implementing PBL in the context of a Taiwanese elementary school. In doing so, it aims to help others to gain some usable insight by showing them this intervention as it really was. Secondly, it aims to understand what impacts a series of PBL intervention has on the students' understanding of fractions and to add to the knowledge base on the teaching and learning of fractions.

Keywords: problem-based learning (PBL); understanding of fractions; teaching and learning mathematics in Taiwan.

An introduction to problem-based learning (PBL)

PBL has been a popular topic of research and discussion among educators, educational psychologists and researchers. Its importance has been highlighted by many studies and documents. PBL was first implemented in medical education at the McMaster University in Canada in the late 1960s. The model for student-centred, problem-based, small-group learning took shape at McMaster 40 years ago. Now, we are seeing an explosion in the use of PBL in its various adaptations across many levels and subject areas.

PBL stands in the philosophy of social constructivism (Savery and Duffy 1995). That is to say, in a PBL environment students engage in social learning activities that involve hands-on problem-based situations and utilisation of discipline-based cognitive tools, and they work as groups to impose meaning on the knowledge that they construct through the social learning process. Therefore, learning is a process, not a product. The practices compatible with constructivist views through PBL approaches may vary from subject to subject. In general, there are three main characteristics of PBL that contribute to its potential benefits.

The first characteristic of PBL is the problem-driven content structure. The content of a PBL lesson is organised as a problem or a series of problems rather than in a textbook form. These PBL problems are challenging, open-ended and contextualised. The second characteristic of PBL is the inquiry-based and collaborative learning processes in which students work as groups to solve problems and learn from small group collaborative interactions, rather than being taught by the teacher. The third characteristic of PBL is the student-centred situation. In PBL, teachers are not in classrooms to deliver knowledge to students, but are there to facilitate students' learning. Likewise, students are not in classrooms to wait for their teachers to give them instruction, but are there to construct knowledge and establish a new level of knowledge through the process of working with group members.

Students' difficulties with fractions

Davis, Hunting and Pearn (1993, 63) argue that "the teaching and learning of fractions is not only very hard; it is, in the broader scheme of things, a dismal failure." Such a quote is shocking, but, unfortunately, research has agreed that fractions have continued to be a difficult topic of elementary mathematics over the years. The difficulty that students encounter with fractions is largely attributed to the complex relationships between different representations and basic arithmetic operations. Their symbolic form simultaneously represents many concepts (Kieren 1988). For example, 2/3 is part of a whole, a rational, a ratio, a number in its own right (Mack 1990), a location on a number line, a representation of division, an operator and an operand (Dienes 1969); 2/3 of something is different from the number 2/3.

Studies show that many students hold a limited view of fractions and that is one of the causes of their difficulties in this area. It has been long argued that the concepts that children are encouraged to construct in schools lead to a limited view of fractions, since students are, traditionally, taught algorithms about fractions with little attention to any meaningful grounding in this area. Kamii and Dominick (1997, 59) also claimed that "when we try to teach children to make relationships between numbers by teaching them algorithms, we redirect their attention from trying to make sense of numbers to remember procedures." Schoolchildren often have only a brief exposure to many concepts and procedures of fractions. Li (2006) examines Taiwanese students' conceptual and procedural knowledge of fractions at ages 12 and 13 and concludes that there is a perceived inequality in the procedural and conceptual developments of Taiwanese students regarding fractions. Many of them may be able to use algorithms to do a wide range of operations regarding fractions, but they do not necessarily understand the concept that underlies the operations.

The purpose of the study

Before describing the purpose of this study, it is helpful that the reader be familiar with some educational contexts of Taiwan in which the teaching intervention was located. Mathematics teaching in Taiwan is strongly influenced by the constant preparation for examinations. Birenbaum, Tatsuoka and Xin (2005, 175) describe that the examination culture in many Eastern Asian countries reflects "a nationwide obsession with excelling in exams." Such an examination culture may partially reflect the teaching and learning of mathematics in Taiwan. In most mathematics classrooms in Taiwan, students sit individually and rarely interact and work as groups. Lecture-based instruction dominates classroom activities (Lin and Tsao 1999). Further, the majority of Taiwanese teachers also tend to "prefer traditional settings" (Yang, Chang and Hsu 2008, 527).

Earlier, Leu (1996) examined the teaching and learning of fractions in Taiwan and concluded that working with fractions often challenged students and teachers alike. For Taiwanese students, they may have learned a lot about fractions both procedurally and conceptually; however, their ability to use this knowledge may remain under-developed. In view of the potential benefits of PBL to students' development of a higher-order understanding of mathematical ideas, I therefore proposed to undertake a one-year PBL teaching intervention on students' understanding of fractions.

While a great number of studies have offered a wealth of empirical evidence to the potential benefits of PBL, it remains like a "black box" (Hak and Maguire 2000) without sufficient information and details of implementation varied among different programmes. Many challenges and problems may arise to obstruct the development of cooperation and interaction in real-life PBL classroom situations, since cooperation does not necessarily occur when students are put together and asked to work in groups. Recently, a new focus of research attention has been paid to understanding the essential mechanisms underlying the situation being investigated and, therefore, framing research as a case of a more encompassing phenomenon with the aim of providing more educational value to the understanding of the relevance in real-life classroom settings. Therefore, the purpose of the study is twofold. Firstly, it seeks to investigate the process of implementing PBL in the context of Taiwan. In doing so, it aims to help others to gain some usable insight by showing them this intervention as it really was. Secondly, it attempts to understand what impacts a series of PBL intervention has on the students' understanding of fractions and to add to the knowledge base on the teaching and learning of fractions.

Participants

The participants in this study were one fifth grade teacher (Miss Lee) and her thirty-five students (19 boys and 16 girls). Both the teacher and students encountered PBL for the first time. Miss Lee considered her teaching as a primarily lecture-based and textbook-driven style. The results from the student interviews also provided some confirmations of it. They all described that their regular mathematics lessons were that Miss Lee talked and they listened. They hardly asked questions, expressed their thoughts or shared ideas with other students during lessons.

Four teaching units during the intervention

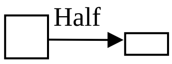
The main structure of this intervention was designed by me, in discussion with Miss Lee, and carried out by her. In the classroom I involved myself in the role of 'observer-as-participant', in which my identity as a researcher was clear to the students and I was there to observe what was going on in the classroom without taking part in the teaching activities.

According to the fraction-related studies and the Taiwanese mathematics curriculum, four teaching units were designed: (1) Basic Constructs of Fractions; (2) Equivalent Fractions; (3) Multiplication of Fractions; (4) Fractions, Decimals and Percentages.

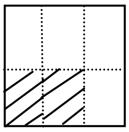
In general, the PBL process in this study followed three steps. I use the following problem, which was drawn from the teaching unit of fraction multiplication, to further illustrate these three steps. The objective of this problem was to help students conceptualise the rule of multiplying fractions (multiplying tops and multiplying bottoms).

> Problem: Jenny's mum made a square-shaped cake for her birthday. At the party, half of the cake was eaten and then the rest put in the fridge. The next day, Jenny's brother ate 2/3 of the remaining part of the cake.
>
> Please first work in a pair to discuss how you would fold the coloured paper provided to represent how much cake Jenny's brother had eaten and then share the pair work in groups.

The first step was to launch a problem. Miss Lee introduced the problem/activity to students. The second step was to solve the problem through group/paired discussion. Students worked as groups/pairs to explore and solve the problems and Miss Lee circulated among groups at all times. The following quotes were drawn from group/paired discussion over the aforementioned problem.

> 'This is easy. Jenny's brother ate $\frac{1}{2}\times\frac{2}{3}=\frac{2}{6}=\frac{1}{3}$, but how do we fold the paper to present it?'
>
> 'First fold the paper in half, like this:  , but what we do next? Erm, a bit challenging.'
>
> 'Miss Lee, are we allowed to use a ruler?'

The third step was to conduct a whole class discussion over the problem being investigated. Students articulated their solutions to the whole class and gave/received feedback to further reason the problem. During the whole class discussion over the problem above, Miss Lee invited students to come forward to show how they folded. Several students raised their hands. Miss Lee picked three students to come forward. Miss Lee used one of their folding approaches, as shown below, to further guide students to discuss why $\frac{1}{2}\times\frac{2}{3}=\frac{1\times2}{2\times3}=\frac{2}{6}$.



Data collection methods

In terms of investigating the implementing process of the intervention, data were collected from school and classroom observations, teacher and student interviews, audio- and video-recordings of all the lessons, field notes, researcher diaries and the post-implementation open-ended student questionnaire. In examining the students' understanding of fractions, I collected data from classroom observations, audio- and video-recordings, field notes, group discussion notes, the post-unit (PU) tests and the post-implementation (PI) test. Group discussion notes were that each group was required to note down about what they had discussed during group discussions. The PU tests comprised a series of questions, each of which contained both open and closed questions that related to the concepts of the teaching unit; they were given to students at the end of each unit. The PI test, incorporating identical questions in the PU tests, was administered to all students three weeks after the intervention had been completed. The comparison between the results from the PU tests and the PI test provide an insight into the retentive effect of the intervention on students' understanding of fraction sense.

Preliminary findings

Some preliminary findings from this one-year PBL intervention are presented here. In terms of the implementing process, there were three main phases: the beginning (Lessons 1 to 5), the middle (Lessons 6 to 12) and the later parts (Lessons 13 to 19) of the intervention. During the beginning part of the intervention, it was evident during classroom observations that after a problem was launched, although some students might remain indifferent or quiet, most students got together for the discussion and looked like they were enjoying the discussion. The classroom atmosphere appeared harmonious and relaxed. Miss Lee also told me that she was satisfied with the involvement of students' participation in the activities during this beginning phase. Nevertheless, many difficulties and challenges were gradually arising and needed to be addressed during the middle part of the intervention; for example, "students talk off topic", "students don't get along", "students' disruptive behaviour" and "teacher being angry".

It was not surprising to know that many things affected the process of the implementation when creating a more learner-driven classroom. However, for Miss Lee who was accustomed to being a traditional teacher, she appeared to be struggling with the aforementioned challenges. She thus tended to use a more custodial pupil ideology and a generally more authoritarian approach to demand immediate compliance from students for maintaining control over the class and running lessons smoothly As Clark (2006) notes, due to the long-held pedagogical beliefs, even the best-intentioned teachers can easily slip into traditional teaching approaches. It was also observed that, during the middle part of the intervention, Miss Lee struggled to balance the teachers' teaching desire and the PBL principles. Often she ran counter to the principles of PBL, of which she seemed hardly aware. For example, a whole class discussion became teacher summary time.

As Reeve et al. (2004) suggest, the possibility of creating learner-driven classrooms is in the hands of the teacher; however, dealing with challenges in a PBL environment is not a single competence which suddenly emerges in the teacher. By the later part of the intervention, it was observed that Miss Lee appeared to gradually gain in classroom experience; suggesting that previous experiences helped her develop some strategies and take more appropriate teacher action. For example, in terms of her classroom management, there was a change from straightforward coercion to a better communication style.

Earlier, when Miss Lee saw dysfunctional group work, she would usually immediately deliver punitive measures to the group (e.g. asking them to stand up until they were told to sit down) as a consequence of their failure. During the later part of the intervention, she would try to explain to students why they were being asked to do the tasks in groups and emphasise the importance of discussion to their learning. The attitude used by Miss Lee to deal with the situation above showed a hint that she was changing from merely 'doing things' to the students towards 'working with' them. Further, instead of acting as only one authoritarian figure in the classroom, she began to share some of her authority with the students to take charge of their group off-task behaviours. The frequency that she gave direct solutions to students' problems also decreased over time. She would redirect the questions addressed to her back to the group. This suggests that she appeared to act as less of a mathematical-content expert in class.

In terms of the impacts of the intervention on the students' understanding of fractions, the results of the students' performances between the PU tests and the PI test show that the students performed better in the PI test than the PU tests. It is important to note here that there could be many factors that would have contributed to the fact that the students generally did better in the PI test than earlier in the PU tests. Nevertheless, as far as the scores were concerned, the increased scores might imply that this PBL intervention may have a positive impact on the students' understanding of fractions. Further results will be offered in the near future. Incidentally, this test result seemed to be a source of encouragement for Miss Lee. She mentioned,

> I am glad to know this result. I didn't expect that they would be able to do better in the PI test. I thought they would forget most of the concepts introduced during

the lessons and perform much lower in the PI. It seems to work, doesn't it? I may involve this kind of teaching more in my teaching, although it is challenging.

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Researching Primary Trainees' Choice of Examples: The Findings

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This paper reports on the findings of a doctoral study exploring primary trainee teachers' choices of mathematical examples and the relationship between this and their mathematical subject knowledge. Through a combination of interview analyses and lesson plans gathered from the final school placement of one cohort of Bachelor of Education trainees, some approaches appear to be commonly held by trainees about the nature and purpose of examples in the planning and teaching process. This paper presents the research design and summarises the outcomes from the data.

Keywords: Primary, Trainees, Examples.

Introduction

When teachers plan to teach mathematics, they draw on many examples to either demonstrate a concept or provide opportunities for learners to practise skills and procedures. The examples used by primary trainee teachers, it is suggested, are often chosen without suitable consideration of learners' strengths, weaknesses or misconceptions. Whilst there has been research on the choice of examples by teachers in secondary mathematics, detailed empirical research of primary mathematics or for trainee teachers is relatively scarce. In this study, two cohorts of final year trainee primary teachers were invited to submit lesson plans for analysis and a sample group was interviewed to try to identify the theoretical frameworks trainees use for planning mathematics and their approaches to choosing examples for learning. The data collected were then analysed using a multiple case study approach against a conceptual framework based on the Knowledge Quartet research of Rowland et al. (2009) and the development of the notion of example spaces by Watson and Mason (2005). The analysis sought to identify commonalities in the way the group of trainees approached planning mathematics and draw insights on their rationales for choosing mathematical examples. Each trainee's planning was scrutinized against the theoretical background in the literature and conclusions were drawn regarding the methods of planning adopted, the examples chosen and the possible links between these actions and the trainees' levels of mathematical subject knowledge.

Literature

Teachers' knowledge for classroom practice was conceptualised in a seminal work by Shulman (1986) in which he sets out categories of teacher knowledge. His work has been used widely in developing approaches to developing and assessing teacher knowledge, particularly in work arising from the implementation of Circulars 10/97 and 10/98 (DfEE 1997, 1998). In these documents, curricular coverage for Initial Teacher Training (ITT) in the United Kingdom was set out. Three of Shulman's seven categories focus directly on what can be called content knowledge, these being subject matter knowledge, pedagogical content knowledge and curricular knowledge.

The role of knowledge in the development of a primary trainee teacher has been examined by mathematics educators and government alike, each arguing for the content of a teacher education curriculum in mathematics. The concern with subject knowledge, particularly for primary trainees, has been growing since Alexander, Rose and Woodhead (1992) felt that strengthening teachers' subject knowledge would contribute to the aim of improving standards in mathematics. A later evaluation of the first year of the National Numeracy Strategy in England (Ofsted 2000) located weaknesses in teachers' mathematical subject knowledge.

This study examined the examples chosen by a number of final year Bachelor of Education (B.Ed) primary trainee teachers. The types of examples were analysed during consideration of the lesson plans, but it was helpful to have a typography and an understanding of the possible types of example that may be encountered. In order to analyse trainee teachers' examples, it was necessary to have a clear categorisation, or framework, against which to relate the trainees' choices. For this purpose, one such categorisation is that presented by Watson and Mason (2005). Whilst they focus on the role of learner-generated examples and the advantages of using such examples to enhance mathematical learning, they set out a number of definitions of examples which help to clarify the differences between types of examples that might be constructed and used by teachers or their pupils.

Using the notion that mathematics is about making general statements regarding the "actions carried out on objects" (Mason 2010), the categorisation of examples which Mason and Watson set out considers that all examples are used to enable a learner to generalise from them. They present the following types:

- Illustrations of concepts and principles
- Placeholders instead of general definitions and theorems
- Questions worked through in textbooks or by teachers (worked examples)
- Questions to be worked on by students (exercises)
- Representatives of classes
- Specific contextual situations

These types can be used in the process of exemplification, that is:

> to describe any situation in which something specific is offered to represent a general class with which the learner is to become familiar – a particular case of a generality. (Watson and Mason 2005, 4)

The data collected was also scrutinized using the framework of the 'Knowledge Quartet' developed by Rowland et al. to see the extent to which final year primary trainees use elements of the Quartet to plan lessons and choose examples. The analysis also gave scope for using this framework to support future cohorts of trainees. The Knowledge Quartet sets out to focus on the "classification of the situations in which mathematical knowledge for teaching surface in the classroom" (Rowland 2008).

The Study: Context and Methods

The study reported here began during the 2007/8 academic year in one ITT institution. Data was collected from two final year (3rd Year) cohorts of the B.Ed programme and included the following range:

- School placement data in terms of year groups taught
- GCSE and A-level grades in mathematics prior to starting the course
- Mathematics interview test data – item scores and totals
- Mathematics module assessments from each year on the course
- Diagnostic Numeracy Test scores from early in the 1st Year
- Results from 'Confidence Counts', an additional mathematics support module
- Trainees' self-audit of subject knowledge for mathematics

In Phase I (2007-08), 22 trainees brought a collection of plans to be used as data. This was around 400 separate lessons, covering a range of pupil year groups and mathematics topics. In Phase II (2008-09), 18 trainees brought around 300 separate lessons to be used as research data, covering a range of pupil year groups and mathematics topics. By separating the year groups and topics, it was possible to focus the analysis of examples to those year groups and topics which offered most data, by selecting as cases those trainees who had taught those topics and in those year groups. The sorting demonstrated that Year 3 and Year 4 lessons were most commonly represented in the primary range, followed by those from the Reception year. Trainees tended not to have placements in Year 6 classes so that school preparation for Standard Assessment Tasks (SATs) at age 11 is not interrupted. The most common topics were addition and subtraction, which accounted for 66 plans, followed by 52 on multiplication and 42 on 2D and 3D shape.

As well as lesson plans, resource materials such as worksheets were collected, both published and 'home-made' by either the trainees or their placement class teachers. The published resources tended to come from a limited range of materials, with only a small number of educational publishing companies being used. Amongst the material from publishers are mostly pupil textbooks, with pages allocated to different topics, demonstrating particular concepts by way of a range of examples. These examples form part of the analysis. Self-produced worksheets are generally variations of those found in published materials, and modified to suit the particular needs of a teacher and their pupils. The modifications are most often based on the notion of differentiation of tasks for the attainment range of the group or class for which the worksheet is designed.

By considering the range of data for each trainee, it was decided to give each trainee an overall grade in the range A, B, C to indicate whether they were likely to be of higher attainment, middle attainment or lower attainment relative to the cohort. This grading was largely the researcher's subjective decision and was not arrived at by trying to calculate an overall result by any formulaic process, but sought to allow the selection of the case study students to form a purposive sample which, as fairly as possible, represented the range of students from two cohorts in terms of their past achievements, which informed my assessment of their potential for teaching mathematics.

The decision to choose the trainees to be the seven case studies was therefore then taken based on the following criteria:

- Year 3 trainee in either of the 2005-08 or 2006-09 cohorts
- Completing final school placement in the spring term of Year 3
- School placement was in Key Stage 2 and one of the most common school year groups
- Willing to submit mathematics lesson plans from final placement
- Mathematics topics taught were amongst the most common represented in the collection of lesson plans
- Willing to be interviewed about their planning and examples
- A selection of trainees from each of the A-C overall grading
- Interviews produced good quality data in sufficient depth for analysis

From the data and the criteria, seven trainees were selected for analysis of interview responses and lesson plans, against the research questions and literature themes.

Ref. No.	Pseud.	School Year Group	Pre-course data (GCSE, A-level)	Interview test score	Y1	Y2	Y3	No. of Lesson plans given for use	Grade for case choices
ED06	Suzy	Y4/5	A,B	70	62	68	78	25	A
ED13	Dawn	Y4	B	50	65	57	80	22	B
ED18	Sharon	Y4	A*,A	90	71	67	68	33	A
ED21	Victor	Y3/4	C	15	53	55	48	8	C
ED24	Naomi	Y4	C	n/a	40	45	40	21	C
ED26	Rachael	Y3	B	71	60	58	65	19	B
ED37	Andy	Y5	B	42	48	57	62	20	B

Results and Discussion

This paper reports analysis of the data collected from Phases I and II in terms of obtaining an overview of the range and scope of data within the lesson plans and how the interview outcomes provide insight into the approaches trainees use when planning mathematics lessons and selecting examples. The findings from the study are given briefly here in relation to the research questions. With regard to the first research question 'What pedagogic considerations do a cohort of trainee primary teachers use when choosing mathematical examples in the classroom?', the findings suggest that trainees use a variety of approaches, these being broadly summed up as 'reliant on the Primary Strategy', 'reliant on other sources' and 'use of their own knowledge'.

The second research question was 'How do these pedagogic considerations fit within current theoretical frameworks in primary mathematics pedagogy?' This resulted in finding generally that primary trainees from the study are mostly able to recall aspects of theory from various module assignments during their course, but were collectively rather inconsistent in considering any theoretical frameworks specifically when planning to teach mathematics. The third question asked 'Is there a relationship between the cohort of primary trainees' level of mathematical subject knowledge and the types of examples they select?' In response to this question, each of the trainees identified with the idea that subject knowledge is related to choice of examples. However, their views on the range and scope of subject knowledge and their understanding of mathematical 'examples' led to a blurred interpretation of the relationship between subject knowledge and examples.

The higher attainers demonstrated greater understanding of the 'knowledge – examples' relationship and believed in each case that they chose better examples but also pitched their lessons at a level too difficult for pupils. The middle attainers had varying mathematical competence but firmly believed that better subject knowledge led to better examples being chosen. The two lower attainers had different views; one lacked confidence and needed more help with subject knowledge and choosing examples, which he did not regard as being related, whilst the other was confident but recognized she was still learning and felt that better knowledge of the pupils helped her choose better examples.

Amongst all the trainees, there was ample evidence in both interview discussions and from examples on lesson plans that all the case study trainees, regardless of attainment, used differentiation as a key feature of their examples. There were many instances of trainees talking about starting off with easier examples, so that every pupil could do them, and then making the examples progressively harder in order to challenge pupils of middle and higher attainment. This finding was not anticipated from the research questions but was drawn directly from the collected evidence and emerged during the analysis phase. Looking back through the interview data, it is also apparent that none of the trainees who were in the case study group mentioned the importance of assessment in choosing examples, even though assessment can be regarded as a theoretical perspective. It may have been interesting to explore this aspect, relating trainees' choices to their assessments of children's prior learning.

Summary

The data collected in this study has provided an insight into the subject knowledge and choice of examples by two groups of primary trainees. From a small, self-selected sample of final year B.Ed trainees, there is some evidence that awareness of theoretical influences is weak, subject knowledge continues to be a cause for anxiety and the process of selecting examples for teaching and learning mathematics is rather more random than pedagogically planned. In order to extend this study, one of the most beneficial approaches would be to combine data collection of lesson plans and interviews with other data such as video-taping of lessons, interviews before and after the lessons with the trainees, and interviews with other participants in the lessons, such as teaching assistants and the children. Video tapes from lessons can be analysed in their own right, but could also be used in video-stimulated reflection with the trainees to enable them to focus on aspects of their practice which might improve when they see their teaching from the perspective of observer.

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The Discursive Construction of Learning Mathematics

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The nature of mathematics, the nature of beliefs about mathematics and what it means to learn mathematics have long been discussion points in mathematics education (Thompson 1984; Boaler, Wiliam and Zevenbergen 2000). The research discussed in this paper focuses on what is said in the classroom during whole class teaching episodes. Using transcripts from two secondary mathematics teachers, we examine how the learning of mathematics is discursively constructed by the teacher and his/her pupils. This conversation analytic approach uses only the content of the interaction to describe the nature of mathematical activity in that interaction. We cannot directly access the beliefs of teachers and pupils, but an examination of how they talk about mathematics reveals how the learning of mathematics and classroom mathematics can be jointly constructed by a teacher and his/her class in quite different ways.

Conversation Analysis, classroom mathematics, discourse.

Introduction

What it means to learn mathematics is at the heart of mathematics education research. Different perspectives have evolved over the years and, with these perspectives, and the developments in technology, new research tools have developed. In this paper, we use a conversation analytic approach to examine the discursive construction of what it means to learn mathematics in two classrooms. The nature of mathematics, nature of beliefs about mathematics and the connections with what it means to learn mathematics have featured in the mathematics education literature for a long time (Thompson 1984; Boaler, Wiliam and Zevenbergen 2000). Even though we cannot directly access the beliefs of teachers or pupils, an examination of how they talk about mathematics and learning mathematics can reveal how the learning of mathematics and classroom mathematics is jointly constructed by a teacher and his/her class.

Conversation Analysis

In this paper, a conversation analysis approach is adopted for analysing two contrasting whole-class episodes. Conversation analysis is underpinned by ethnomethodological principles such as the belief that actions and interactions are socially ordered, and that this order is observable by the researcher and by the participants in any interaction (Garfinkel 1967). The origins of conversation analysis lie in the lectures of Harvey Sacks (1995) and it has since been developed and extended in many disciplines (Drew and Heritage 1992, Schegloff 2007). The key focus of any conversation analytic approach to naturally occurring interactions is how the participants in an interaction jointly create the meanings and activities of the interaction, and how it is that these participants orient to these meanings.

Turns at talk are resources that the teacher and pupils use to perform social acts (Seedhouse 2005) and conversation analysis examines the organisation of these social acts in interactions. Conversation analysis describes and analyses how the participants themselves interpret each other's actions through the construction and order of their turns.

The two extracts included in this paper cannot be adequately understood without reference to the interactional context in which they occur. The aim of a conversation analysis approach to analysing these interactions is to determine which features of the context are relevant to the interaction at a particular moment during the interaction. However, context in the sense used in this paper does not refer to the physical, social or cultural environment that the interactions took place in, but to the context that the participants themselves have made relevant through their interactions. This context is dynamic: what it means to learn mathematics as it is constructed by the interaction can alter and change through the ways in which the participants construct their turns. It is also entirely possible that what the participants describe in their words can differ or even contradict what they are doing in their turns, but this is beyond the scope of this paper.

It is the different possibilities for constructing classroom mathematics by the teachers and pupils that are the focus of this paper. Discussions around what the teachers believe about the nature of mathematics or how pupils learn mathematics are not considered. The perspective makes no attempt to either extend the analysis to the cognitive processes of the participants or to use these processes to explain the participants' use of turns in the interactions, or to generalise to other mathematics that the teachers and pupils do. I first offer a short extract to illustrate how the learning of mathematics and school mathematics is discursively constructed by the teacher and his pupils. A second extract is then offered to demonstrate a contrasting discursive construction of school mathematics. The two extracts considered are chosen because they illustrate the points being made in this paper and cannot be used to make generalisations about the mathematics that the teachers and pupils do in the respective classrooms.

Methods

The two extracts included in this paper are taken from detailed transcripts of whole-class interactions. Video and audio recordings of 17 lessons in secondary mathematics classrooms in the UK were made. Four teachers with a class of 12-13 year old pupils were recorded over a period of 6 weeks, with between 3 and 6 lessons recorded for each teacher. The data collected are considered to be naturally occurring in that the teachers were given no guidance as to what or how to teach and were told only that the researchers were interested in whole-class interactions. These video and audio recordings were transcribed according to the Jefferson transcription system (2004), though for ease of reading, not all features are included in this presentation.

The choice of extracts was based on a need to offer contrasting examples of the discursive construction of mathematics. Both extracts begin with the teacher beginning a task that links to tasks that were completed in a previous lesson. In the first of the extracts these links are specifically mentioned in the transcript but in the second the links are not made until later in the interaction and are not included here. These extracts include instructions of how to complete the task and what is expected of the pupils. Whilst a conversation analytic approach would normally include longer sequences of interaction, restrictions on space make this difficult. The analysis of the extracts is undertaken within the wider interactional context of the extracts, but only the turns presented here will be the focus of the descriptions offered.

Findings

The first extract presented is a brief extract consisting of two turns of interaction between Simon and Charlie. A brief reading of the extract will soon make the reader aware of the institutional identities of the two participants. Simon is introducing the task, controlling the resources, and controlling who can take the next turn and what can be said in this turn. There are also significant pauses during Simon's turn, during which none of the other participants in the interaction select themselves as next speaker (McHoul 1978, Ingram 2010). Even when Simon has asked a question which requires an answer there is a pause of 0.9 seconds in line 25 where no one self-selects to speak, though a few participants have raised their hands to bid for the turn, and is followed by Simon nominating Charlie in line 26 to take the following turn and in line 27, give an answer to the question. The intonation of Charlie's answer is that of a question, a strategy often used by pupils (Rowland 2000, Ingram 2010) and Simon's turn that follows immediately latches onto Charlie's turn on its completion. Charlie does not nominate Simon as the next speaker.

```
Simon:   okay part two:. do you ↑know that little bit of paper I
         gave you yesterday with the table on and we filled in
         one side.
         (1.2)
         I'm going to a:sk you today to do some practice on this
         and before we do that (.) I just want to go through
         another example just to remind everyone
         (0.3)
         of um
         (1.5)
         of how it's done. so can we just li-, the other side
         that we haven't filled in.
         (7.2)
         it's this one here you should have one, ooops, you
         should have one that looks a little bit like this.
         (0.8)
         ok?
         (1.1)
         now I'm going to be honest with you, I was talking to um
         (1.8) ((teacher pulls down projector sheet and then up
         again))
         I was talking to Mrs Smith the other, yesterday and she
         thinks I'm being much too nice to you when I did this
         table. do you know why.
         (0.9)
         Charlie.
Charlie: 'cause you gave us the extra column?=
Simon:   =what instead of, ye- ye:s! because I gave you that
         extra column there.
         ((teacher points to the third column on the projected
         table))
         because (0.4) sometimes in the exam they won't give you
         that extra column they'll just give you these two, and
         they'll expect you to know (.) that it might be useful
         (.) to put this extra column on, do you know what I
         mean. and in a minute, when you do some practice from
         the text book it's the same thing. they just give you
         this bit of the table and they expect you to use your
```

initiative (.) to draw in the extra column to do it. ok.
well let's go through these then, the mo:de, the median,
the mean and the range. I think we'll leave the mean
till last because it's a bit like the mean one. um Alex
and (.) Chris, paying attention now specially, right any
offers anyone for telling me what, m-why of course we
always want to know why (.) what the mode, the median
the mean and the range are.
(1.7)
…

Extract 1 – Simon and Charlie

In this extract, Simon talks about doing "some practice", going "through an example", and filling in a table. Each of these activities is described more than once. He also talks about reminding his pupils of "how to do it", and 'knowing' is referred to again in Simon's second turn. Towards the end of the second turn, Simon constructs a question that begins to ask what the mean, mode, median and range. However, Simon corrects himself (initiates a self-repair) to say that "we always want to know why" before re-asking the question in the form of "what". In the turns that follow but are not included in this paper, each of the mean, mode, median and range are calculated by the pupils and are followed by a brief description of how it was calculated. For example, "three is the mode because it's the most common one." In this brief exchange of three turns, learning mathematics is constructed as going through examples, practicing examples, and knowing and remembering both what terms mean but also how to answer questions.

In this brief extract, Simon contextualises the tasks and activities within a wider context of school mathematics. He positions these tasks and activities within a specific time frame and plan that Simon has for this lesson. He begins by situating the task in respect to what has been done previously. In lines 1-3, this is done by explicitly referring to a previous lesson, whereas in lines 11-12, the reference is more implicit in that the sheet in question has been partially completed in that previous lesson. In lines 23-24 and 28-29 he refers to the preparations that he has made for today's main task, preparing the table which included the 'extra column'. Simon then contrasts what he has planned for the class to do later in the lesson (lines 5-6 and 36-37) to going "through another example" as the next planned task (lines 6-15), which begins at the end of Simon's second turn (lines 40-46).

Simon also positions the tasks and activities in the lesson within the wider mathematics community in the school and the wider examination system. In lines 19-31 Simon refers to a conversation with another mathematics teacher about today's task and then situates this conversation within the context of examinations in lines 32-40, indicating what 'they' (the examiners) will do and providing an insight into their thinking (line 34).

In this extract, the mathematical tasks and activities have been clearly constructed as something that can be described as school mathematics. There is a plan for what the class will do both across a series of lessons and during this individual lesson, and this plan is specifically discussed within the context of preparing and supporting pupils for their examinations. In other words, the tasks and activities are part of a school mathematics curriculum which are endorsed by other mathematics teachers in the department and the examiners.

In the second extract, Tim similarly constructs his turn in such a way that his institutional identity of teacher is clear and, like Simon, Tim is controlling the topic of interaction, the floor and the timing of the activities. There are several significant pauses in which no other participants self-select as next speaker. However, in contrast to the extract above, two other participants do self-select (lines 25 and 26), and overlap, to answer the question that Tim asks towards the end of his turn. This self-selection occurs after a pause of 0.8 seconds (line 24). However, Tim's reference to doing 'the first one together' in line 22 implicitly directs the question to the class as a whole, allowing self selection and therefore opening the floor to multiple answerers.

```
Tim:    Ok
        (0.6)
        your fir:st thing today, I've put a problem on the board,
        I will have a problem on the board in about
        (0.3)
        30 seconds, ok I want you to look at that. first question
        is quite an easy one, the second question we have to need
        to think about in terms of (.) what it actually means,
        (1.3)
        ok. and I want you to try your best and try and understand
        (.) how far you can get it done, ok. here is your problem.
        have a go at this. I've just inherited twelve thousand
        pounds, (0.4) ok and being the generous man that I am I
        want to donate (.) some of that to charity. but because
        I'm not totally generous,
        (1.2)
        ok. I'm going to donate one quarter of the twelve thousand
        pounds, then the following week I want to donate a quarter
        of that amount, following week a quarter of that amount.
        ok. how much will I donate in each of the first four
        weeks, the first few are obviously easy. how much will you
        donate in total. ok let's just do the first one together,
        in week one how much have I donated?
        (0.8)
Drew:   thre[e thousand]
Chris:      [three thou]sand
Tim:    three thousand pounds.
        (3.3) ((writes on whiteboards))
        wee:k two:, how much am I donating if I'm donating a
        quarter of that. (.) Harry?
```

Extract 2 - Tim, Drew and Chris

Tim describes the task as a 'problem' that his pupils need to "think about" and "try and understand". He asks his pupils to try their best and "have a go". He also twice describes particular aspects of the problem as 'easy' and contrasts this to aspects that his pupils will need to "think about". In particular, he describes finding a quarter of twelve thousand pounds as "obviously easy" and combining this with his direction of the question to the class as a whole, rather than nominating a specific pupil, he is indicating that the arithmetical calculation is not important and does not need to be thought about. By asking the question to the class as a whole, the likelihood that the answer three thousand pounds is given is significantly higher. It is the question of how much is given away in total that is interactionally constructed as the important question.

In this second extract, learning mathematics is discursively constructed as solving problems, thinking and understanding. There is no mention of practicing, though the pupils do need to do an arithmetical calculation in order to answer Tim's first question. It is also something that the class do "together", which is further reinforced by more than one pupil offering the answer three thousand. Tim uses the pronoun 'I' frequently in his first turn but as a way of personalising the problem. It is he that is giving away the money, not an artificially manufactured individual. Tim switches between describing the pupils as 'you' and including them in 'we', interestingly using the pronoun 'we' when describing thinking.

The extract from Tim's lesson focuses on the 'problem' being discussed. Whilst it is clear that Tim has planned the problem (lines 3-4), Tim makes no attempt to situate the problem in a school context. There are no references to previous or future tasks and activities, there are no shared objectives or outcomes for the problem, and no external agency (such as examiners) is invoked: the discourse is focused on working on the problem in its own right.

Whilst the extracts shared above are necessarily brief, they do illustrate how quickly learning mathematics can be constructed and some of the diversity of the constructions. The analysis above has focused on what Simon and Tim are doing with their turns at that particular moment in time, further analysis is being undertaken of the lengthier sequences of interaction to include a wider range of social acts that the participants perform and the relationships between these and the learning of mathematics.

Transcription Conventions

.	falling intonation contour
?	rising intonation contour
[ ]	onset and end of overlapping talk
(1.5)	pause, timed in seconds and tenths of a second
(( ))	additional information including non-verbal actions

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'Ability' ideology and its consequential practices in primary mathematics

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'Ability' is a powerful ideology in UK education, underscoring common practices such as setting. These have well documented impacts on pupils' attainment and attitude in mathematics, particularly at the secondary school level. Less well understood are the impacts in primary mathematics. Further, there are a number of consequential practices of an ability ideology which may inhibit pupils' learning. This paper uses data from one UK primary school drawn from my wider doctoral study to elucidate three such consequential practices. It examines why these issues arise and the impacts on pupils. The paper suggests that external pressures may bring practices previously seen in secondary mathematics into primary schools, where the environment intensifies the impacts on pupils.

Keywords: Ability, Primary Mathematics, Setting, Educational Triage

Introduction

This research, from my doctoral study, examines the unexpected and sometimes unnoticed consequences of ability-predicated practices such as setting. Three issues representing different ways unintended consequences may be enacted are discussed.

Ability predicated practices have increased in primary schools, particularly in mathematics, following the implementation of the National Strategies (Hallam, Ireson, and Davies 2004). Successive governments have repeatedly called for an increase in ability-based grouping at both secondary and primary levels. These changes come despite our lack of understanding of the impacts of ability practices at the primary level. Our understanding of the impacts comes predominantly from the secondary mathematics literature. This was explicated in earlier work (Hodgen and Marks 2009, Marks 2011) and for brevity is not rehearsed here. Instead, findings within the three themes are discussed with respect to the key literature.

Research design

The wider doctoral research of which this paper is a part was a mixed-methods study taking the form of a multiple case study. Two diverse school environments were included, although only data from one school – Avenue Primary (a pseudonym, as are all names), with a strong philosophy of setting – are discussed.

Sample

The wider project involved 284 Key Stage Two (ages 7-11) pupils in two UK primary schools, one using a high-degree of setting for mathematics and one using limited setting. Avenue was a three-form entry primary school in Greater London. Pupils were set for mathematics into four sets in each year group from Year 2 (ages 6-7). Movement between sets was very limited.

The study involved Year 4 (ages 8-9) and Year 6 (ages 10-11) pupils. This gave access to a range of experiences, additionally allowing a focus on the impacts of the mathematics Standard Assessment Tests (SATs – the tests taken by pupils in Year 6 at the end of primary school) on ability practices. All pupils were involved in the quantitative elements of the study. For the qualitative elements, top and bottom sets in each year were selected as focal sets. Within each focal set, three focal pupils were chosen by the teacher to reflect the range of attainment within the set, totalling 12 focal pupils at each school. The focal set teachers were also included within the study.

Research methods

A variety of research methods were employed to gather data at different levels and to allow for data triangulation (Denzin 1997). Attainment tests developed at King's College London (Brown et al. 2008) were conducted with the full cohort in October 2007 and July 2008. These allowed the measurement, as maths ages, of the attainment gains made by each pupil over the academic year. Additionally, Nicholls et al.'s (1990) attitudinal questionnaire was conducted as pre- and post-tests. Quantitative data were collated in SPSS and descriptive and inferential statistics applied.

Over this same time period, 48 mathematics lessons involving 13 sets/classes were observed, and 48 interviews were conducted with the 24 focal pupils and 8 teachers to explore their experiences. The qualitative data were collated in NVivo and analysed using constructivist grounded theory (Charmaz 2006). Both quantitative and qualitative data are presented in this report, allowing for the elucidation of key data trends alongside rich accounts of events as experienced by the research subjects. Together these provide a fuller picture of the issues discussed, allowing for analytic theory and generalisations to be drawn from the data

Findings

Three key themes giving an overview of the issues arising from consequential practices are presented below. With each, data extracts are used to illustrate the findings discussed; these are selected as typical rather than extreme examples.

Educational triage

The notion of educational triage originates in Gillborn and Youdell's (2000) study into the allocation of educational resources. They describe it thus:

> In a medical emergency triage is the name used to describe attempts to direct attention to those people who might survive (with help), leaving other (less hopeful) cases to die. In school, educational triage is acting systematically to neglect certain pupils while directing additional resources to those deemed most likely to benefit (in terms of the externally judged standards). (134)

Their study referred to the practice of targeting resources at pupils attaining at the grade C/D borderline in the General Certificate of Secondary Education (GCSE) examinations taken at the end of compulsory schooling. The aim of such an intervention was to ensure these pupils attained a minimum of grade C, the benchmark used to construct school league tables and externally measure school effectiveness. A result of this was that pupils deemed unlikely to attain a C grade, even with intervention, were given the lowest priority and least support. A similar study was conducted by Booher-Jennings (2005) examining the impact of reading tests as gatekeepers to Grade 4 entry in Texas elementary schools. Her study showed teachers redirected resources to those pupils who would succeed with intervention whilst taking away support from those pupils unlikely to pass the reading test.

Educational triage was applied, knowingly, at Avenue Primary. Set 3 pupils were referred to by teachers as the Cusp Group. These pupils were identified as likely, with support, to achieve a Level 4 in the mathematics SATs at the end of Year 6 (the Government target and a measure of school success). The Cusp Group was allocated the "strongest teacher" (Mr Iverson, Year 4) and the teachers talked about using a different approach with these pupils:

> With the Cusp Group you have to, sort of, you know, push open those doors a bit and not be frightened and say right, what about these numbers … the idea is to push them up and get them moving. (Mrs Jerrett, Year 4)

Whilst the teachers saw this additional input as supportive of Cusp Group pupils, they seemed unaware of the consequential impacts on Set 4 pupils who were deemed unlikely to attain a Level 4. With the strongest teacher placed into the Cusp Group and the subsequent priority being Sets 1 and 2, Set 4 pupils in Year 6 were taught by supply teachers or Teaching Assistants. Additionally, lesson observations supported by teacher interviews suggested the Set 4 curriculum was bland in comparison to the Cusp Group. The impact of Cusp Group practices on attainment can be seen by comparing the pre- and post- attainment test scores for each set in Year 6 (Figure 1):

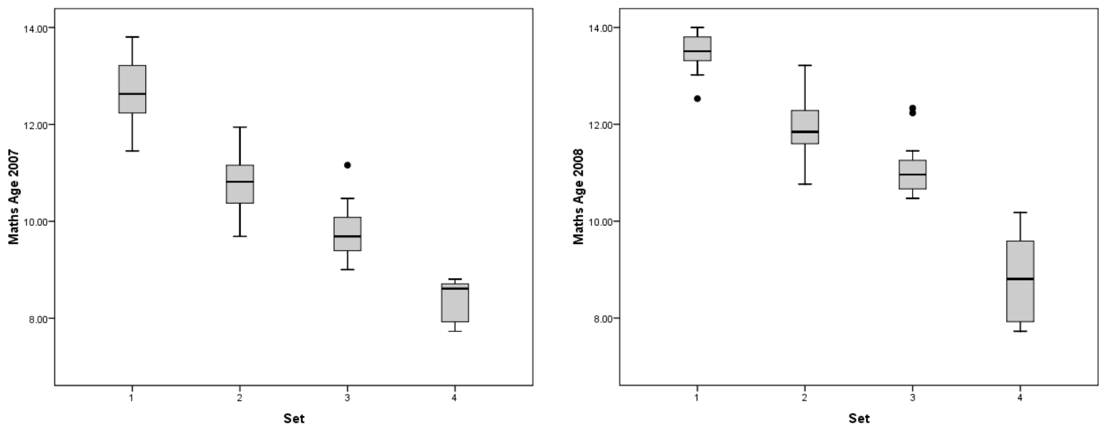

Figure 1: Boxplots showing maths ages for Year 6 pupils in each set as assessed in October 2007 and July 2008.

The first boxplot shows that in October 2007, towards the beginning of Year 6, there was a difference of approximately one year between the median maths ages of pupils in Sets 2 and 3 (Cusp Group) and between pupils in Sets 3 and 4. The difference in the maths ages between Sets 1 and 2 was slightly larger. If all pupils had received a similar educational input over the academic year, the between set differences would be expected to remain similar. However, as the second boxplot, showing maths ages at the end of Year 6 in July 2008, shows, Set 3 – the Cusp Group – has moved away from Set 4. Set 3 have made a median maths age gain of one year and 4 months, a gain of over a year more than pupils in Set 4 who made a gain of 3 months.

The gains difference between Set 3/Cusp Group and Set 4 pupils seems to suggest that the teachers' differential practices do have the intended impact of increasing Set 3 attainment, with these pupils on target to achieve the coveted Level 4 in the Year 6 SATs. However, this neglects to address the impact, both in terms of attainment and attitude, on Set 4 pupils. In their interviews, the pupils demonstrated an awareness of these differential practices:

> Samuel: The different groups get different things, I want Mr Quinton [Cusp Group], they have fun.
>
> Peter: He actually makes learning maths fun.
>
> Samuel: I would literally love to move to Mr Quinton's group. He's really really nice but I have Mr Leverton, in class he tells us to talk and then comes over and whacks the table and goes what are you doing?

Samuel, who was in Set 4, was aware he received a very different mathematical learning experience to Peter, who was in the Cusp Group. Data from further interviews with Samuel and other Set 4 pupils suggests they hold fairly negative attitudes towards mathematics and that these can be, in part, attributed to their experiences in Set 4. Whilst it is not possible to ascertain whether the Set 4 pupils would make more substantial gains if given the Set 3 experience it does appear that an ability ideology and associated beliefs allow teachers to justify such differential treatment. An unintended consequence of this is the development of self-perpetuating practices which trap Set 4 pupils into limited gains and hence a belief they have been correctly placed, resulting in the continuation of a remedial curriculum.

Restricted mathematical access

One of the commonly held ability beliefs which teachers use to justify practices such as educational triage is the association of learning styles with ability levels. Many teachers hold a view that pupils in the highest sets are auditory learners whilst those in the lowest sets are kinaesthetic learners requiring a more concrete approach. This view underlies some of the differential practices seen between sets, reflecting those in the secondary mathematics literature.

Within top sets, characterised by a fast-paced competitive environment and procedural learning, pupils are restricted in their mathematics learning due to the pupils' perceived need to strictly adhere to the taught algorithms rather than consider or develop an understanding of the underlying mathematics. Additionally, competitive practices have the potential to enhance pupils' self-interest, reducing peer support and discussion and hence restricting the pupils' mathematical experience.

In bottom sets, different practices apply, but the potential again exists for restricted mathematical experiences. Set 4 teachers at Avenue talked about caring for their pupils and wanting to ensure they were not frightened by the mathematics:

> We'll only go with numbers up to 500, we won't be going up to 5000, or 500000 … I think I'm a little bit sort of, oh, don't want to make it too hard, don't want to scare them off, keep it small … for fear of them all sort of panicking and freaking out. (Mrs Jerrett, Year 4)

With the intention of reducing pupils' fear, and drawing on beliefs that Set 4 pupils are kinaesthetic learners, Mrs Jerrett compelled the pupils in her set to use cubes for all calculations. This led to these pupils not being required to learn number bonds and relationships, and not having the opportunity to explore and use derived facts. This, as Gray and Tall (1994) assert, resulted in having to do more mathematics and at the same time restricted the possibility of richer mathematical experiences.

Educational spaces

A further consequential impact of ability practices in primary school mathematics concerns the allocation of learning spaces. Avenue Primary created more sets than classes (four sets in each year group from three classes), the rationale being that smaller set sizes particularly in the lower sets would be beneficial to pupils' learning. However, Avenue did not have the physical space in terms of empty classrooms to accommodate the extra sets, leaving Set 4 pupils without a stable base. This represents an area with limited coverage in the literature with Fisher (2004) noting very limited consideration of the impact of physical space on pupils' learning.

Set 4 pupils at Avenue were taught in a variety of areas including infant classrooms, computer rooms and corridor spaces. At the beginning of every session there was uncertainty over where the lesson would be conducted and pupils sometimes had to move during lessons. Not having a base meant limited access to mathematical equipment. In both years 4 and 6 at Avenue, pupils in Sets 1 – 3 were taught in classrooms where they had access to supporting equipment, mathematical displays and aids such as number lines on the walls. Conversely, Set 4 pupils only had what they or the set teacher could carry, reducing the opportunity for spontaneous exploration of concepts not planned for. Additionally they were taught in areas where the displays related to other subjects, serving only as a distraction rather than a potential support for learning. As a result, Set 4 pupils were more limited in their mathematical learning opportunities due to the physical constraints imposed by setting, potentially increasing the attainment gap between them and other pupils. This limitation was raised by their set teacher during her interview:

> In our group we could have done more get up and do except in that computer room there isn't a lot of space and you know in the corridor you're a bit constrained and a bit public as well because everyone is walking through. (Mrs Jerrett, Year 4)

The issues created by a lack of physical space to meet the perceived need for practices predicated by an ability ideology suggest how widespread impacts of ability constructions are. Further, they suggest how many elements of the school day, beyond the mathematics teaching, are implicated in the reproducing ability discourses.

Discussion

This paper suggests how an ideology of 'ability', prevalent in UK mathematics education, may impact on primary school mathematics learning in many ways, some of which go unnoticed or with the impacts not fully considered. This paper has only considered three consequential practices, but with the finding of the wider study that an ideology of ability is pervasive in primary mathematics, is seems likely that there are further consequential practices. These practices, alongside more explicit ability practices, impact on very young pupils who are potentially being turned off mathematics at an increasingly young age.

It is important to stress that this paper does not blame the teachers concerned for engaging in these consequential practices. Some practices go unnoticed, yet many others are enacted from the position of care, for instance in protecting pupils from what is considered to be hard mathematics or in providing them with smaller classes and therefore, it is argued, greater teacher input. Other practices arise from external pressures which teachers feel trapped within. In order for this situation to change, teachers need the opportunity to engage with and understand these practices.

As things are currently, many of the issues arising in secondary mathematics are being seen in the primary mathematics classroom. In some ways these may be more detrimental in the primary context where the pupils' main classroom is not just a base as in secondary schools, but the centre of much of their education and their relationships with others for an entire year. This context may intensify the detrimental impacts of ability practices, affecting the mathematics learning of all pupils. As such, we need to look beyond the most explicit practices to more fully understand the impacts of ability in primary mathematics.

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Is progress good for mathematics/education?

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In this paper I raise questions about the role of progress within mathematics education. I look at how progress defines a linear and teleological relationship between the past, present and future. This idea then sets the parameters within which researchers, policymakers and practitioners work in mathematics education, constraining the questions that we ask, the answers that we give and the actions that we take in the present. I suggest some alternatives to the progressive narrative of past-present-future as perhaps the only way not to answer 'yes' to the question: does mathematics/education make things better?

Feeling backwards

About eight years ago, in my doctoral thesis (Mendick 2003), I argued that mathematics operates oppressively, and specifically contributes to the reproduction of gender inequalities. In the conclusion, I positioned my approach within the larger project of developing a sociology of mathematics and contemplated the implications:

> In thinking about what futures may follow from a social theory of mathematics, Connell's (1987) discussion of possible futures, which can be built on a social theory of gender, is helpful. He sees two possibilities: the abolition of gender or its reconstitution on new bases. The first is a deconstructive strategy that is powerful as a direction but is impractical as an immediate goal. However, beyond these considerations it raises questions about whether our current gender relations have any value.
>
> > What would be our loss if they went down the gurgle-hole of history?
> >
> > It has to be said that a great deal of our culture's energy and beauty, as well as its barbarism, has been created through and around gender relations. A gender-structured culture, and quite specifically sexist sensibilities, have given us *Othello*, the *Ring of the Nibelung* and Rubens portraits, to go no further. Much of the fine texture of everyday life, from the feel of our own bodies, through the lore of running a household, to popular songs and everyday humour, are predicated on gender. Our eroticism and our imagination seem to be both limited and fuelled by gender. To discard the whole pattern does seem to imply a way of life that would be seriously impoverished by comparison with the one we know. At best it would be so different from the world of our experience that we can hardly know whether it would be desirable or not. (p.288)
>
> Returning to mathematics, here too the abolition of mathematics is not only impractical but it is also questionable whether a mathematics-free world is desirable. My own view is that it is not. It is clear to me that the social and historical practices of mathematics have resulted in a great deal more than oppression and inequality. A mathematically structured culture, and quite specifically absolutist and sexist sensibilities, have given us the Internet, the central limit theorem, and the Mandelbrot set, to go no further. Just as masculinity is not all bad ... neither is mathematics.
>
> This leads to the second option, the reconstitution of mathematics on new bases ... I am attracted to this option not just because of the positive contributions that mathematics has made to society but also because of my positive relationship with the subject.

So, despite having spent three years and 80,000 words unpicking the role of mathematics within systems of domination, I resisted the idea that the world (and I) might be better without it. Writing at a distance, I wonder how much this resistance was due to my own investment in mathematics/education as, at least potentially, progressive? More generally, I wonder, how much are the theories we use, the questions we ask and the answers we give tied both to an assumed relationship between mathematics/education and human and individual progress and to assumptions about the value of progress itself?

Modernity, Progress and Mathematics/Education

Wendy Brown (2001, 5-6, original emphasis) identifies progress as a key narrative of modernity, along with right, sovereignty, free will, moral truth and reason: "The conviction that history has reason, purpose, and direction is fundamental to modernity … modernity is ... premised on the notion of emergence *from* darker times and places, it is also structured *within* by a notion of continual progress". Modernity's narrative of progress thus constructs particular relationships between past, present and future, and our own relationship to these temporalities. However, Brown continues by focusing on the fracturing of this narrative of progress in contemporary times. She argues that this narrative is breaking down as people now gaze "backwards to glimpse better times" (7); the conjuring of a Golden Age may be familiar, but for the first time, she suggests it comes detached from any progressive story. "Today ... it is a rare thinker, political leader, or ordinary citizen who straightforwardly invokes the premise of progress" (6).

However, my own and others' readings of mathematics/education policy, practice and research indicates the persistence here of the anchoring narrative of progress, as speaker after speaker "straightforwardly invokes the premise of progress". Indeed, neoliberal education policy derives its authority from discourses that relate individual and national progress (Archer et al. 2010). This is evident in the policy documents produced by the New Labour government in the UK (1997-2010). Within their policies, progress figures as an unproblematic good. For example, in *New Opportunities* (HM Government 2009), the first white paper focused on education to address the impact of the global economic downturn, the word progress/ive had 45 mentions (compared to only 42 for teacher/s). Two will suffice:

> This is also an economy in which the knowledge and skills of people are now the most important resource as well as our best chance of social progress. The countries which succeed will be those which make the most of the talents and potential of all their citizens. (3)
>
> We will not just manage the downturn fairly, but make of it the beginning of a new era for our nation – with an historic commitment to the greatest possible achievement of modern progressive politics as we lay the foundations of true social mobility and social justice in modern Britain. (2)

In the first extract, we see the conflation of national and individual progress, and the binding of these to discourses of economic competition and individual potential. In the second extract, there is an explicit linking of these discourses to a concept of history as linear and teleological: the "modern" and the "new" improves on what has gone before - "with an historic commitment to the greatest possible achievement" - and makes possible a utopian future - "lay[s] the foundations of true … social justice". This offer seems difficult to refuse. Teachers too (of course!) want to see progress in the children they teach.

It is not just education policymakers and practitioners who have intrinsically linked education and progress. Roger Dale (2001, 8) identifies that:

> Education has been seen both as the dominant symbol and the dominant strategy for that mastery of nature and of society through rationality that has characterized the project of modernity from its origins in the Enlightenment. ... it has been a keystone of attempts to extend the benefits of progress to whole populations, indeed to the whole of humanity. It has come to stand for the possibility of individual and collective improvement, individual and collective emancipation.

This emancipatory narrative, he argues, has been implicitly (or explicitly) taken on by sociology of education research, and has shaped its focus on removing barriers to educational progress. As researchers we see our project as improving, not eliminating, education. As Dale points out, this is very different from researchers in sociology of religion or the family where the research agenda operates independently of their personal views on the social roles of religion or family. In contrast, in the sociology of education, education is treated less an object of study than as a resource.

Mathematics, and science and technology, are implicated in narratives of progress through discourses that position them as the subjects of the future and vital to our national competitiveness and more generally to human progress. The compulsory status of mathematics in the school curriculum attests to this and to its centrality to the 'progressive' project of compulsory education (Jivaji 2011), while quantitative methods provide the apparatus for measuring progress. Within our neoliberal moment, progress in mathematics education policy (and in much practice and research) stands for improvements in national test results: for example, the UK government publication *Making Good Progress in Mathematics* (DCSF 2008) focuses on how to 'convert' good results in the infant school mathematics tests into good results in the junior school tests. The idea of human progress and development in operation is normative. Some lives and histories lie outside its boundaries and thus as Other to what counts as 'normal' and 'human' (Butler 2004). We can see this in recurrent references to school failures using animal imagery for example in the newspaper headline: "Feral youths: How a generation of violent, illiterate young men are living outside the boundaries of civilised society"[1].

One possible (and common) response is to argue that progress stands for the wrong things. So instead of identifying progress with improvements in results, we could argue for identifying it with more equitable results or more 'authentic' learning. Such a response risks contradiction (even unintelligibility) as versions of progress are labelled unprogressive/regressive. To avoid this pitfall we can play with the language, as in the distinction between 'traditional' - chalk-and-talk, results-oriented - pedagogy and 'progressive' - investigative, student-centred - pedagogy (Boaler 1997). We could look to Marxism (or other radical philosophies) to underwrite this strategy so that the direction of history is determined by class struggle, with the revolution of the proletariat bringing an inevitable end to history (Bowles and Gintis 1976).

These are important interventions. However, while they aim to appropriate and reinscribe progress, they leave intact the narrative of progress with its linear, teleological history in which "the past represents the logic for the present, and the future represents the fruition of this logic" (Halberstam 2005, 11). We can see this clearly in Noam Chomsky's use of history in debate with Michel Foucault:

1 http://www.dailymail.co.uk/debate/article-1214549/Feral-youths-How-generation-violent-illiterate-young-men-living-outside-boundaries-civilised-society.html

> I think it is perfectly possible to go back to earlier stages of scientific thinking on the basis of our present understanding, and to perceive how great thinkers were, within the limitations of their time, groping towards concepts and ideas and insights that they themselves could not be clearly aware of. … I think that as a matter of biological and anthropological fact, the nature of human intelligence certainly has not changed in any substantial way, at least since the seventeenth century, or probably since Cro-Magnon man … and that if you took a man from five thousand or maybe twenty thousand years ago, and placed him as a child within today's society, he would learn what everyone else learns, and he would be a genius or a fool or something else, but he wouldn't be fundamentally different. (Chomsky and Foucault 1971)

Here past ideas are continuous with, or "groping towards", present ideas; past people are essentially identical with present people, "not changed in any substantial way". Within this framework, the future too is imagined as a natural and inevitable unfolding of history, culminating in a society that allows the flowering of the "human intelligence" that we share with Cro-Magnon man. This future is a melancholic attachment to the past, a form of "temporal colonisation" (Cooper 2011).

I argue that we need to work with a different relationship between past, present and future. I do this because progress is a means of ordering and de/valuing difference; because research shows that mathematics/education increases inequality; because most mathematics is now developed for military or economic purposes; and because perhaps this is the only way not to answer 'yes' to the question: does mathematics/education make things better?

Alternatives to progress

> A Klee painting named 'Angelus Novus' shows an angel looking as though he is about to move away from something he is fixedly contemplating. His eyes are staring, his mouth is open, his wings are spread. This is how one pictures the angel of history. His face is turned toward the past. We perceive a chain of events, he sees one single catastrophe which keeps piling wreckage and hurls it in front of his feet. The angel would like to stay, awaken the dead, and make whole what has been smashed. But a storm is blowing in from Paradise; it has got caught in his wings with such a violence that the angel can no longer close them. The storm irresistibly propels him into the future to which his back is turned, while the pile of debris before him grows skyward. This storm is what we call progress. (Benjamin 1940)

Here Walter Benjamin is writing as a Marxist but against Marxist ideas of progressive history. His disillusion with progress is apparent in his reading of the Klee painting (www.wordglitch.com/?p=43) which hung on his wall. He rejects "left melancholia", in which people look to the political failures of the past and become immobilised by their sense of opportunities lost. For him the purpose of doing history is the "blasting apart of historical continuity which allows the historical object to constitute itself". In this way we create possibilities for action in the present. Although not sharing Benjamin's historical materialism, I think Michel Foucault's (1971) genealogy most fully realises this desire to "blast apart historical continuity".

Genealogical approaches to the past differ from progressive history by looking not for continuity but dissonance, not for identity but difference, not for inevitability but contingency. The resulting histories show not so much progress as transformation. Their purpose is to disturb what we take for truth: "Truth is undoubtedly the sort of error that cannot be refuted because it was hardened into an unalterable form in the long baking process of history" (Foucault 1971, 79). Thus we need to examine history, and to do so with a particular sensibility, in order to find ways of cracking such congealed truths. Genealogy sets out to:

> identify the accidents, the minute deviations - or conversely, the complete reversals - the errors, the false appraisals, and the faulty calculations that gave birth to those things that continue to exist and have value for us; it is to discover that truth or being does not lie at the root of what we know and what we are, but the exteriority of accidents. (Foucault 1971, 81)

Genealogy attempts to induce a sense of "vertigo" (Brown 2001, 97), to induce instability in the ground on which we (think we) stand. Foucault (1971, 88) talks about history as being "effective" only "to the degree that it introduces discontinuity into our very being-as it divides our emotions, dramatizes our instincts, multiplies our body and sets it against itself". The self is no longer secured by its relationship to the past or the future for it will not "permit itself to be transported by a voiceless obstinacy toward a millennial ending. It will uproot its traditional foundations and relentlessly disrupt its pretended continuity".

So Foucault, like Benjamin, has political intent. By looking for "seriality, repetition, absurdity, and anomaly" (Halberstam 2005, 140), "genealogy articulates politically exploitable fissures and fractures in the present; it produces openings and interstices as sites of political agitation or alternatives" (Brown 2001, 113). Thus, we no longer need progressive history, with its promise of revolution, as a ground for political action. Indeed when challenged in debate with Chomsky to speak about the future, Foucault refused. As he explained, all the concepts on which we seek to ground such speculation, be they human nature or justice, are formed within our own society and "one can't, however regrettable it may be, put forward these notions to describe or justify a fight which should - and shall in principle - overthrow the very fundaments of our society. This is an extrapolation for which I can't find the historical justification" (Chomsky and Foucault 1971).

I now want to turn to some work in queer theory which develops Foucault's refusal of the future. Queer theory is characterised by an:

> appropriately perverse refusal ... of every substantialization of identity, which is always oppositionally defined, and, by extension, of history as linear narrative (the poor man's teleology) in which meaning succeeds in revealing itself - *as itself* - through time. Far from partaking of this narrative movement toward a viable political future, far from perpetuating the fantasy of meaning's eventual realization, the queer comes to figure the bar to every realization of futurity, the resistance, internal to the social, to every social structure or form. (Edelman 2004, 4, original emphasis)

As Lee Edelman (2004) states, queers occupy a position outside the *heteronormative reproductive politics of futurity*. This is a politics secured by the symbolic figure of the Child. For all politics, and perhaps educational politics most of all, find their self-evident justification in their claim to be fighting for the future of our children. We can argue about what politics might be 'best for the children' but we cannot reject the criteria itself.

Edelman uses Lacanian psychoanalysis and readings of popular cultural texts to argue that, instead of (just) seeking incorporation into the reproductive order (for example, through struggles for gay marriage and parenting), queers should embrace the negativity of their position:

> by saying explicitly what Law and the Pope and the whole of the Symbolic order for which they stand hear anyway in each and every expression of manifestation of queer sexuality: Fuck the social order and the Child in whose name we're collectively terorized; fuck Annie; fuck the waif from *Les Mis*; fuck the poor,

> innocent kid on the Net; fuck Laws both with capital ls and with small; fuck the whole network of Symbolic relations and the future that serves as its prop. (29)

While provocative, it is difficult to think what politics might follow from Edelman's celebration of the death drive, his insistence that "the future is mere repetition and just as lethal as the past ... [and must] stop here" (p. 31) particularly give its location within Lacan's structural psychoanalysis (Butler 2004).

Judith Halberstam (2005, 2) rejects Edelman's attempts to make "community in relation to risk, disease, infection, and death". More positively she suggests that the "constantly diminishing future creates a new emphasis on the here, the present, the now, and while the threat of no future hovers overhead like a storm cloud, the urgency of being also expands the potential of the moment and ... squeezes new possibilities out of the time at hand". As Foucault and Benjamin approach the past in a way that multiplies the possibilities for action in the present and lends an urgency to such action, Halberstam's approach to the future works similarly. It is here that I recover a sense of hope from the ruins of modernity.

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Paperless classrooms: a networked Tablet PC in front of every child

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i-Pads in front of the children in networked classrooms have the potential to transform learning. In mathematics particularly, interaction by screen-touch using fingers or stylus seems preferable to keyboard and mouse. Their portability and reliability, so that children can take them home, and their potentially low price, are other attractions. It is proposed that, to maximize their potential to improve learning, the Tablets should be configured so that they emulate workbooks - combining textbook, exercise book, test-paper and progress record - and be embedded in a school-wide managed learning environment that combines effective learning management support for class teachers with safe-keeping of students work and records.

Keywords: Networked classroom, emulated workbook, mathematics learning

Introduction

In an earlier paper (Osmon 2011) I predicted the imminent popularity of iPad-style Tablets among schoolchildren and their potential for organised use in classroom learning. Since then iPad sales have been booming and other well-known IT brands are now competing with Apple, and I have found descriptions, in the form of patent applications, of relevant learning systems.

Networked classrooms with i-Pads in front of the children have the potential to transform learning. In mathematics particularly screen-touch interaction using fingers or stylus seems preferable to keyboard and mouse. Portability and reliability, so that children can take them home, and their potentially low price, are other attractions. It is proposed that, to maximize their potential to improve learning, the Tablets should be configured so that they emulate workbooks- combining textbook, exercise book, test-paper and progress record- and embedded in a school-wide managed learning environment that combines effective learning management support for class teachers with safe-keeping of students work and records.

Workbooks and e-Workbooks

Traditionally, in the UK, school-children write their work in exercise books: ruled with lines or, in the case of mathematics, ruled with a grid of squares and a textbook, alongside the exercise book, provides instruction and specifies exercises to be done. Besides serving as a place of *interaction* with work in progress, the exercise book *records* work done. For reasons of economy, photocopied exercise sheets, or exercises copied from the board, are sometimes substituted for textbooks. Workbooks- cheaply printed combinations of text and exercise book- are a more reliable alternative to supplementing exercise books with loose leaf material. They are more common in the United States than in this country. Workbook pages formatted with printed questions with space for answers have something in common with test papers.

Emulations of workbooks or "electronic workbooks", or e-Workbooks as I shall call them in this paper, potentially offer some advantages over pencil-and-paper workbooks. For example they are paperless and hence may provide more reliable and convenient recording of work done than paper workbooks and they can reduce the administrative burden on class teachers since individual learner data (work attempted, dates, and marks) held in electronic form in individual workbooks is readily copied automatically into class records, particularly if the workbooks are *networked*.

Currently, the closest approximation in most UK schools to networked e-Workbooks is the PC-lab. However, for *cost* reasons, classes typically have limited timetabled access and learners work in pairs. In these circumstances benefits are modest.

There are other reasons, besides cost, why e-Workbooks have not been adopted more extensively. Thus, paper workbooks are highly *portable*, they can be read anywhere, any time, without requiring access to a PC, and perhaps a broadband network connection. This means all learners can take them home to continue working, not just those with broadband at home. And, for many purposes, pen or pencil is more appropriate for *input* than a keyboard and mouse.

Paperless classrooms

There are published proposals/accounts from the US of more extensive use of e-Workbooks. Two of these (Dockterman 2006, Stuppy 2007) describe highly controlled and very specialised, paperless e-Workbook learning environments: what we would might call 'crammers', attended by students for intensive remedial courses in basic arithmetic. All the inputs of students, including their working, are recorded so as to provide a detailed account of progress. Additionally the networking allows a teacher to *monitor* in detail, by viewing on her screen, the work of individual learners in her class, in real-time, so that she may intervene promptly with assistance when necessary. Interestingly, based on initial assessments, the students may be assigned *individualised learning* programmes, or initial "student profiles". The networking allows their marks to be extracted and recorded and compared with expectations in their profiles, which may then be modified, and future learning tasks allocated accordingly.

Another interesting aspect of networking in these accounts of paperless classrooms is reference to the use of copyright learning materials and automatic debiting of students' accounts when their workbooks receive *downloaded instructional text*. In a paperless world textbook publishers will doubtless seek such new ways of maintaining their revenue streams.

In these accounts there is nothing especially innovative about the learning materials presented to students. What is interesting is how the details of their interactions are visible to the teacher, and may be timed and recorded. Also, learning is evidently a more intensive experience for the learners than in a traditional classroom. And, *support for the teacher's role*: real-time access to students' efforts, semi-automatic management of individualised learning programmes, automatic recording of students' progress and record updating, ensures nearly all of her class time is available for actual teaching.

These accounts relate to highly controlled and specialised learning environments and, at least while the networked e-Workbook technology available in our schools is the familiar PC-lab, paperless classrooms, with their potential for supporting teachers, are unlikely to become the norm across the wide range of subjects in the curriculum

Tablets and e-Workbooks

In principle, the adoption of networked classrooms by schools, with an e-Workbook in front of every child, could offer a range of educational benefits arising partly from the interactivity of the workbooks and partly from the communication opportunities presented by networking and partly from the ability of the technology to record learners' progress.

However, several barriers to adoption of PCs as all-purpose platforms for implementing e-Workbooks are apparent: cost, keyboard-and-mouse as the principal data-input means, and lack of portability. Tablet PCs (i-Pad clones) are preferable as e-Workbook platforms under all of these headings. In the rest of this section I address the cost issue.

When the market is highly competitive and supply can match demand, price tends towards manufacturing cost. These conditions are not yet met for iPads and their clones but because of their relative simplicity of design, compared with netbook PCs (most obviously the absence of rotating storage, keyboards and multiple connector sockets), we can expect unit prices to fall with a year or two- from around £400 towards £100- when production ramps up to meet or exceed demand and there is more competition, greatly undercutting PC prices. But, cost-of-ownership is not just purchase price. Product lifetime- which depends on reliability- is also a factor, and ease of use reduces the need for technical support. iPads appear to be superior in these respects too.

Overall costs for a school going paperless depend on the system architecture and this includes: wireless networking, school server, broadband connection to the internet, and of course the system software to integrate these. Many schools will have wireless networking in their classrooms already along with the other elements of the system. However, it seems unlikely that the current generation of "integrated learning environments" will be adequate to support paperless schools with a Tablet in front of every child. A later section discusses such architectural issues and how they may be resolved.

e-Workbooks for learning in various subjects

Tablets are multimedia devices- aural input and output, equipped with cameras and with the ability to show video clips- and not merely a paperless alternative to pencils and exercise books and printed textbooks. However, multimedia learning materials for Tablets are unlikely to appear overnight, and certainly not meeting needs across the whole curriculum. It seems likely therefore that the first e-Workbook "Apps" will be derived from existing paper-based learning materials, and at least one company (Inkling 2011) is already offering education publishers a service. According to chatter on the Internet, McGraw-Hill are already clients and Pearson are negotiating.

Different subjects are likely to use the multimedia capability of e-Workbooks in different ways. And these subject-characteristic ways will take time to develop, as teachers and the authors and publishers of learning materials become more aware of the potential of Tablets. For example, language learning would surely benefit from their multimedia capabilities: we can readily envisage a language class of children, with each child immersed in dialogue in her personal language lab. A project at Tufts University (Tufts 2009) emphasises the learning possibilities of real-time interaction with the physical world. Engineering applications are described but applications in science would also be feasible. The potential of Tablets for supporting learning across the curriculum will surely hasten their adoption.

e-Workbooks for learning mathematics

Touch input (fingers and stylus) is better for doing mathematics than keyboard and mouse. This is because mathematical language employs a range of mathematical symbols, including oversize characters, not readily accessible from a keyboard and because of the non-linear syntax of mathematics language. Doing mathematics also involves drawing shapes, building tables, making charts, and plotting graphs, all of which are potentially easier with touch input. Certainly touch input with a stylus is more like drawing with a pencil on paper so that Tablets make a better bridge to traditional mathematics classroom working than PCs with keyboard and mouse.

The traditional pencil-and-paper tool for learning mathematics is a *grid-of-squares exercise book*, not just in Britain but in many countries worldwide. There are good reasons for the ubiquity of the grid-of-squares, including its support for mathematical language (learning the symbols and syntax) and facilitating orderly presentation of mathematical expressions, and mathematical working.

We can envisage a Tablet screen displaying a grid of squares- and looking like a page from a traditional mathematics exercise book. Emulating this traditional platform should give comfort to parents and teachers who may struggle in a new world of paperless learning! Besides its traditional merits, the grid of squares is potentially a rich base for interactive mathematics learning: we can suppose the grid indicates a basis set of data structures: *arrays* of: *cells* containing symbols (e.g. characters, tiles carrying icons), *line elements* (horizontal and vertical), and *points*- where grid lines intersect.

Then, touching a screen location points to a particular element of one of these arrays so that, for example, by touching the cells a child can push tiles around to assemble shapes- blocks of tiles- on the grid of squares, to use for developing counting, etc. Tablet touch-screens are a medium that offers a more flexible and intense learning experience than working either with physical tiles or pencil and paper and working with tiles like this seems to have great potential for developing primary mathematics knowledge, whether in obviously mathematics learning contexts like counting and adding and multiplying exercises or by using tiles on a grid of squares as the basis for constructing a whole range of board games and puzzles on the screen. A computer science master's student at King's is aiming to demonstrate this potential in her project.

We can envisage the range of operations available to learners increasing as they progress: writing symbols (aided by symbol recognition) and pasting them into a mathematical expression then syntax checking the expression and then evaluating it, constructing tables from data semi-automatically and then displaying graphically using different degrees of zoom, and so on. The multimedia capability of Tablets has less obvious potential in mathematics than some other subjects. In all subjects there may be advantages to specifying tasks using voice instead of or as well as text. Demonstration examples of procedures, and proofs, feature strongly in traditional mathematics teaching. Tablets can do better than present them statically as on the printed page: recordings, with commentary, that can be replayed step-by-step may prove a helpful way to learn.

System architecture

Putting Tablets, with learning Apps installed, in front of children may well not be sufficient for them to gain significant learning benefit. Preferably the Tablets should be configured so that they emulate workbooks- combining textbook, exercise book, test-paper and progress record- and are embedded in a school-wide systems architecture that stores students work and serves three kinds of user:

(a) Learners (the children) who will need to:
Read (including observe or listen to) learning material stored in their workbooks; *Interact* with this learning material (interactions recorded); *Read* their earlier work and teacher's feedback.

(b) Class-teachers who will need to:
Distribute learning materials to the children's Tablets, either individually, to groups, or to the whole class; *Monitor* the work of individual children, in real-time and also periodically for marking and progress review; *Record* the awarded marks and other progress review data in each child's workbook and also in a whole-class record.

(c) Director (maybe head of department or head teacher) who will need to:
Monitor the progress of individual teachers and learners.

Besides these internal users there will be external ones: authors and publishers of learning materials, and also examination boards. Processes will be needed so that the school can select and install learning materials and examinations can be conducted via the school server and the children's workbooks.

i-Pads are probably more reliable than PCs, but not perfectly reliable, and inevitably children will occasionally lose or break them. i-Pads have quite large, but not unlimited storage. The systems architecture can overcome these limitations and ensure learners' records are *complete* and *secure*. To this end three layers of stored data are envisaged: Individual workbooks, School Server, Secure Data Repository. The storage role for each layer is as follows. Individual's workbooks store the data in recent and current use, including a record of the user's interactions, and also newly downloaded data intended to be used soon. The School Server has storage capacity greater than all the Tablets in the school and contains a copy of the data currently stored in every Tablet, and data likely to be needed soon- such as next chapters from textbooks. The Repository stores a copy of the data currently held in the server as well as all earlier data.

The three layer storage mechanism works as follows. Copies of data move down on demand from users, and newly entered data moves up, after a while, if it is not being used. During the school day, the record of work done during that day, or done previously at home, gets automatically copied from learners' and teachers' workbooks to the School Server, and from time-to time relatively old data is moved up to make space for new- but old data is always accessible on demand, albeit with some delay. (NB no data is ever destroyed or altered- though amendments may be appended- this is feasible because secure bulk storage is so cheap.)

The Tablet market is divided among at least four operating systems: Apple (iPad), Google (Honeycomb), HP (Touchpad), and Microsoft (Windows 7). A common platform for workbook emulation would be very helpful.

Challenge of e-Workbooks for mathematics examiners and researchers

Paperless classrooms make paperless examination rooms feasible. In principle, mathematics tests can be downloaded into the School Server over the Internet by the

Examination Board, and then downloaded into individual learner's workbooks and only become visible at the time the examination is due to start and, at the end of the allotted time, be whisked away for marking.

Teaching and learning for the test has been a baleful influence on mathematics education. If learners' work is indeed recorded securely and in detail in their e-Workbooks, as proposed in the model systems architecture above, then reviewing these records would be a form of coursework assessment and might be a better kind of testing.

The proposed systems architecture automatically records the work of whole classes in great detail. This means cost-free collection of data where the work is part of a research investigation- surely a challenge to mathematics education researchers to devise materials for the children to work with that have potential to illuminate their difficulties with mathematics.

Conclusions

Reports of paperless classrooms using workbooks in highly controlled situations suggest they have potential to offer a more intensive learning experience, with teachers spending more class time actually teaching. e-Workbooks based on Tablets can have many of the positive characteristics of pencil and paper for mathematics learning plus benefits arising from interactivity and communication. And it seems major educational publishers are preparing tablet-compatible learning materials. I have proposed embedding networked Tablets in a school-wide systems architecture to maximise their potential to improve learning across the curriculum and also to protect users from any shortcomings. Progress may be slowed by absence of a common platform for e-workbooks across tablet operating systems. Much of its potential is waiting to be explored but it already seems that the Tablet may become the principal learning tool in mathematics classrooms.

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The English assessment regime: how consistency and standards stifle innovation and improved validity for the assessment of mathematics

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This paper describes the national assessment regime for mathematics in England for 5 to 16 year olds which is the basis of school accountability. Most of these assessments comprise timed written tests or exams that are designed to assess the statutory national curriculum programmes of study. For pre-16 learners the assessments are developed nationally and teacher assessment is reported alongside test outcomes. There is considerable evidence that teachers are over-reliant on the tests and adjust their assessment to match that of test outcomes. At age 16 independent commercial organisations (awarding organisations) develop public examinations (GCSEs) in a regulated market place. There is fierce competition between awarding organisations to gain and maintain market share. The regulatory system for the development of tests and exams and maintenance of standards is rigorous but restricts innovation and improvements in validity.

Assessment; accountability; regulation; measures of performance

Introduction

The extent of national testing increased considerably in England following the introduction of the National Curriculum (NC) in 1988 for 5 to 16 year olds. More recently it has begun to decline but there is still concern from the Office for Standards in Education (Ofsted 2008) that 'teaching to the test' dominates much classroom practice and teachers are over reliant on testing as opposed to other forms of assessment.

In this paper I describe the current system of assessment in England for 5 to 16 year olds and how it came about. I explain the processes in place to maintain consistency of standards over time and discuss the implications of these for innovation that might help to improve the validity of these assessments.

Background

The original NC included a ten level scale of attainment which was intended for use across the entire age range. Children's levels of attainment in the core subjects of English, mathematics and science were expected to be reported at the age of 7, 11 and 14 (known as the end of key stages 1, 2 and 3 (KS1, KS2, KS3) respectively). The level of attainment was to be determined by teachers using their knowledge of children's achievements on the basis of 'best fit' to the level descriptions in the NC. The levels were designed in such a way that the 'average child' might be expected to make one level of progress in two years. Assessment of KS4 was through GCSEs which are graded A (level 9) to G (level 4) with grade C (level 7) acting as the gatekeeper for progression to many vocational, professional and academic courses.

Subsequent revisions of the NC in the early 1990s replaced levels 9 and 10 with exceptional performance (EP), and restricted the use of levels up to age 14.

Key Stage	Age	Expected level of attainment for the 'average' child at the end of the key stage
1	5-7	2
2	7-11	4
3	11-14	5-6
4	14-16	7 (GCSE grade C/D)

Table 1: English NC expected levels of attainment.

National exemplification of standards materials were provided by the Schools Examination and Assessment Council (SEAC) to support teachers in making these end of key stage assessments (SEAC 1992). Teachers were encouraged to work within and across schools and phases to moderate their judgements and develop a robust understanding of what attainment at different levels looked like.

No funding was provided for teacher moderation of level judgements and concerns about consistency and rigour led to the introduction of national testing. In the early 1990s, national tests were initially introduced for KS3 mathematics, then KS3 English and science and KS2 mathematics, science and English. The first tests were criterion referenced and marked by teachers. Test results were expected to be reported alongside teacher assessment. However, no additional funding was made available to teachers to undertake test marking and teacher union action resulted in the external marking of KS2 and KS3 tests. The only concession won by one teacher union was that scripts would be returned to schools.

However, the time taken for external marking of tests meant that the potential of the tests to inform teaching and learning was lost, scripts were often returned after schools had finished for the summer and in the following September children would either transfer to a new school (for the start of KS3) or to a new teacher (for the start of KS4). Teachers tended to wait until test results were received before reporting their assessments, as if the teacher assessment didn't match the test outcomes they were criticised for over or underestimating children's level of attainment (Dainton et al. 1999).

KS1 English (reading and writing) and mathematics tests and tasks (for level 1) were marked by teachers and monitored by local education authorities. Since 2005 the use of reading, writing and mathematics tests and tasks remains statutory but the outcomes now inform teacher assessment which is reported for English and mathematics. Local authority monitoring has switched from the accuracy of test marking to the effectiveness of teacher assessment arrangements, and the system is nationally monitored by the Qualifications and Curriculum Authority (QCA, which became the Qualifications and Curriculum Development Agency (QCDA) in 2009).

National testing came at a time of increasing accountability for schools and led to the high stakes performative culture that now dominates English education (Ball 2003). This includes the use of national, local, school and individual targets and publication of results in 'league tables'. The school inspection regime focuses on school results relative to national norms and expectations. Schools that fail to meet government targets, regardless of the nature of their cohort, are named and shamed and given 'notice to improve' or put in 'special measures': failure to improve ultimately leads to closure.

In 2008 KS3 tests were abandoned, as were national targets and 'league tables' for 14 year olds. Teacher assessment is still collected nationally and a system of national sampling is under consideration. In 2009, following a review of national assessment arrangements (DCSF 2009), KS2 science tests were scrapped in favour of national sampling. The national sampling process for science is still in development but in 2010 a sample of KS2 children completed science tests, resulting in a drop of attainment by seven percentage points (81% at level four and above from 88% in 2009, DfE 2010). The mathematics community, led by the Advisory Committee for Mathematics Education (ACME), argues strongly against national tests for KS2 mathematics (ACME 2008). Many primary schools abandon teaching any new mathematics and begin test preparation as early as January because of the high stakes nature of the KS2 tests that are currently taken in May. However, a recent review of KS2 assessment and accountability stated:

> We have not received any evidence to suggest that there are significant issues with an externally-marked mathematics test. We recognise that it is relatively straightforward to create a valid and reliable test of mathematics, and we feel that the current mathematics tests achieve this. We believe that it is legitimate to use a test to establish how well a pupil can perform a range of mathematical operations within a finite period of time. We recommend that mathematics should continue to be subject to externally-marked testing. (Bew 2011, 63)

Assessment at Age 16 - GCSE

In 1988 the GCSE was introduced as a single qualification for the vast majority of 16 year olds. It replaced GCE O levels[2] and CSEs[3]. GCSE was expected to maintain standards by ensuring that grades C and above corresponded to O level passes and grades D to G corresponded to CSE passes (grades 2 to 5). There is evidence to suggest that grades were inflated in the new GCSEs (Hodgen et al. 2010). When first introduced GCSE could comprise up to 100% coursework (teacher assessment) and consortia of schools could work with an exam board to develop their own syllabi.

The examinations available to 16 year olds were originally through exam boards, many of which had started within universities that had devised entrance examinations for undergraduate courses. Over the years the number of exam boards has decreased as smaller organisations have merged, and there are now just three main awarding organisations in England who provide GCSEs within a ‘market place’. These high stakes qualifications are regulated by the Office of Qualifications and Examinations Regulation (Ofqual). The system for developing curriculum, assessment and qualifications in England and its increasing centralisation and narrowness is described by Isaacs (2010). A GCSE is intended to be a two year course and, for NC subjects, should assess the entire KS4 programme of study (Ofqual 2004).

Ofqual is responsible for the maintenance of standards of live qualifications. A code of practice describes the processes awarding organisations must have in place to ensure consistency of standards in both exam paper development and awarding the qualification. The regulator monitors awarding organisation processes and undertakes ‘standards over time’ scrutinies looking at exams and candidate scripts to see how different awarding organisations compare with one another.

GCSEs tend to be written by a small number of examiners approximately a year in advance and their draft papers are subjected to various quality assurance processes including a Question Paper Evaluation Committee (QPEC) meeting where each paper is subject to expert scrutiny (*shredding*) to ensure that it will be accessible and is consistent with the demand of previous papers. This means that novel items

[2] GCE O levels were introduced in 1951 as subject-based assessments for the top 20% of 16 year olds.
[3] CSEs were introduced in 1965 to enable a broader population to gain a qualification at age 16, the highest grade being notionally equivalent to an O level pass.

(e.g. those which expect candidates to make decisions about what mathematics, information and strategy to use) are altered to include more scaffolding and require less thinking.

National curriculum tests

Ofqual is also responsible for quality assuring national assessments. NC tests have always been developed by a single contractor and for a number of years they were developed 'in house' at QCA. There is a regulatory framework (Ofqual 2009a) which describes precisely the nature of the tests including how many marks for each level and each aspect of the curriculum. There is a general expectation that the threshold mark for a particular level on a particular suite of papers does not change by more than one or two marks year on year. The development process for NC tests is significantly different to that for GCSEs and is much more expensive. There is a lead time of at least two years. Initially 200% of material is generated and, after scrutiny by subject experts and practicing teachers, it is trialled with a representative sample of students (known as pre-test one). This provides a facility (the proportion of students that are successful) and discrimination value (how students working at different levels perform) for each item. The outcomes of pre-test one form the basis for developing a suite of papers for a further larger scale pre-test. All changes to items and mark schemes after pre-test one are minimal so the statistics can be used. Novel items which expect learners to make decisions about what mathematics, information and strategy to use are trialled with students preparing for the current assessments. As responses are not as statistically robust as more traditional items, few survive pre-test one. Where they do survive items are likely to have become shorter and more structured.

The implications of regulation

The purpose of regulation is to ensure that tests and qualifications are valid, reliable, manageable, comparable over time and across awarding organisations, and minimise bias. The regulators are responsible for ensuring standards are maintained and there is public confidence in regulated assessments. However, the rigorous processes in place to ensure consistency of standards actually conspire to stifle innovation (Wolf 2009). The 'market place' for GCSEs means that awarding organisations will not want to do anything that might jeopardise market share. For mathematics this means that questions tend to be highly structured and require little initiative from candidates. Where examiners attempt more innovative questions these will inevitably be remodelled through the QPEC process. Similarly, as national curriculum test questions are trialled with children who are being taught for the current tests the outcomes on novel items tend to be less predictable and consequently less statistically robust, which means that novel items rarely survive the pre-test process. Given the requirement for minimum variation in threshold marks for each level the tests have become increasingly predictable over time.

The assessment of mathematics process skills

The NC for mathematics is principally constructed around content: number and algebra; geometry and measures and statistics and probability. Mathematical process skills are articulated in the curriculum as 'using and applying mathematics' (UAM) which includes problem-solving, reasoning and mathematical communication. At

GCSE UAM was assessed through coursework until 2009, when following a national consultation coursework was removed and assessment of UAM was to be incorporated into exams. Few teachers noticed any changes in the exams for summer 2009 awards, although there was a notable decline in girls' performance.

In NC tests UAM was not included until 2003 when a small number of marks (approximately 10%) were allocated to UAM. Items that assess UAM are designed to assess mathematical content first and foremost but include an element of problem-solving or require an explanation.

The high stakes nature of the assessments (national curriculum tests and GCSEs) means that they tend to comprise items for which the outcomes are relatively easy to mark – there is one right answer and the method is relatively obvious. Teachers tend to believe that the assessments are the curriculum and certainly all that needs to be taught (Dainton et al. 1999, Wiliam 2001). Despite incorporating UAM into GCSE and NC tests there is little evidence that this has actually impacted on classroom practice. Many teachers think that UAM is an add-on to the curriculum rather than something to be integrated (Ofsted 2008).

In the latest revision of the NC for KS3 and KS4 UAM has been strengthened as Key Processes (QCA 2007). Key Processes draw on the mathematical process skills set out in the 2003 PISA framework for mathematics (OECD 2003) and include representing, analysing, interpreting, communicating and evaluating.

KS3 tests and GCSEs needed to change to reflect these curriculum changes. However KS3 tests were abandoned in 2008 so no changes were realised, although new optional tests include some items which require students to think for themselves and make choices about what mathematics to use and how to tackle a problem (QCDA 2010). Changes to GCSE were piloted but the independent evaluation of the pilot found that whilst there was some evidence of change it was limited and many items continued to lack authenticity, complexity and genuine opportunities for problem solving and reasoning (Noyes et al. 2008).

The new GCSE introduced from September 2010 (first awards in 2012) has a much greater emphasis on mathematical thinking and problem solving. Compared to the current GCSE which has just 20% of the marks allocated for UAM, 50% of the marks are allocated for selecting and applying mathematics in context and devising strategies to solve problems (Ofqual 2009b). Whilst the accredited materials look different, it remains to be seen to what extent the live assessments are different to the current GCSE. Awarding organisations will be expected to maintain year on year standards whilst working to the new subject criteria, which represent a significant and sudden change.

Conclusion

The rigorous systems in place for developing and regulating national assessments for 5 to 16 year olds in England whilst ensuring comparability and reliability are unable to accommodate innovation. Adjusting assessments to improve validity, even when the curriculum undergoes substantial change or inspection evidence identifies a significant weakness in the system, as is the case for mathematics, is highly problematic.

Despite substantial pilots of changes to GCSE the potential for change was barely realised and awarding organisations, constrained by market forces and the need for year on year comparability, are likely to make as little change as they can get away with as the new GCSEs are rolled out. Whilst national tests could undergo a gradual process of change, the development process makes it difficult for novel items to make it through to the tests.

Consequently, the assessment of mathematics which drives teaching and learning is unable to change significantly. Narrow, predictable assessments enable 'teaching to the test' in England's high stakes performative education culture.

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Disposition towards engagement in mathematics

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This is the first part of a project to explore the factors influencing life-long engagement in mathematical activity. In preparation for the first phase, involving semi-structured interviews, I propose a draft version of a construct for 'disposition' with four components; beliefs / values / identities; affect / emotion; behavioural intent / motivation; needs. I discuss this in the following brief account, which also draws upon the discussion from the conference session which highlighted the need for clarity and specificity in the use of terms such as 'attitude', 'disposition' and 'motivation'.

Keywords: affect, attitude, beliefs, construct, disposition, engagement, identity, needs, self-efficacy, values.

Introduction

The importance of promoting a mathematically engaged society has long been recognised and is reflected in prominent educational initiatives to increase attainment in school examinations. Less attention has been given to the factors which may influence the degree of an individual's engagement in mathematical activity. My interest is in exploring the factors which have had a bearing upon the extent of adults' engagement in mathematical activity, in order to cast some light on how our approaches to teaching mathematics at school may affect this life-long engagement. I have considered a number of factors, including attitude, affect, motivation, disposition, self-efficacy, beliefs, values etc. with a view to developing an operational construct which can, as far as possible, be related to our approaches to teaching mathematics in school.

Attitudes to mathematics – a changing view

Models of attitude to mathematics have evolved steadily during the 20th century, from a single dimensional construct, a "simplistic like-dislike model" (Ruffell, Mason and Allen 1998, 13), to a multi-dimensional construct, one where attitude is seen as "a constellation of impulses vying for cognitive attention and triggering physiological and hence emotional responses" (ibid., 13). There have been significant developments since the 1970s. Although not addressing mathematics specifically, Fishbein and Ajzen's (1975 and 1980) Theory of Reasoned Action addressed belief, attitude, intention and behaviour. Mandler (1989) and McLeod's (1989) cognitive-constructivist model of emotional experience is described by Zan et al. (2006): this model has some parallels with cognitive conflict. They identify three elements (ibid., 115):

- a discrepancy between events and expectations, causing
- physiological arousal and an evaluation resulting in the construction of an emotional response and

- a “reduction in conscious capacity available for problem-solving”.

McLeod (1989, 1992, 1994) explored factors underlying affect in mathematics. In particular he addressed beliefs, attitudes and emotions, “ranged along a dimension of increasing stability and decreasing intensity - with emotions as the most intense / least stable, beliefs as most stable / least intense and attitude in between” (Zan et al. 2006 115-116). Ruffell, Mason and Allen. (1998, 13) departed from a single-dimensional model and posited a “multi-dimensional construct with three interwoven components”:

- ‘cognitive’: to do with beliefs,
- ‘affective’: to do with feelings and
- ‘conative’: to do with behavioural intent.

Affective / behavioural / cognitive models of attitude as posited at the end of the 20th century might be broadly described as psychological in origin. In recent years, social-scientific approaches have led to related factors such as beliefs, values and identities being explored (DeBellis and Goldin 2006, DiMartino and Zan 2001, Goldin 2002, Leder, Pehnkonen and Törner 2002, Leder and Forgasz 2006, Maaß and Schlöglmann 2009, Mendick 2006). Bandura’s (1997) construct of self-efficacy (an individual’s belief in their own capacity to be successful) has been adopted by social-scientists. Akinsola (2009) has explored how teachers’ beliefs about self-efficacy contribute to pupils’ affective responses to mathematics.

Zan et al. (2006) note two approaches to investigating attitude. The constructs described above were addressed theoretically, however much of the research on attitude in the last forty years has been statistical. During the 1970s and 1980s there was a growing awareness of issues relating to gender in society The Fennema-Sherman Attitude Scale (FS scale) (1976) was developed with a focus on exploring research into gender differences in mathematical engagement and consists of nine sub-scales

- Attitudes towards success in mathematics
- Mathematics as a male domain
- Confidence in learning mathematics
- Effectance motivation in mathematics
- Usefulness of mathematics
- Father (concerning the father’s perceived opinions / beliefs / attitudes)
- Mother (concerning the mother’s perceived opinions / beliefs / attitudes)
- Teacher (concerning the teacher’s perceived opinions / beliefs / attitudes)
- Mathematics anxiety.

Although this scale is now more than thirty years old, it still has significant influence, many of the attitudes items in current use are very similar to those in the FS scale.

Large scale international research also took account of the affective domain. The First and Second International Mathematics Studies conducted by the International Association for the Evaluation of Educational Achievement in the 1960s and 1970s respectively (IAE 2011a, 2011b) included items addressing students’ attitudes to mathematics. The IAE’s third international survey, Trends In Mathematics and Science Study (TIMSS), first run by the IEA in 1995 (IAE 2011c), is repeated every four years (the last completed survey was in 2007). The 2007 survey had two clusters of attitudinal items addressing liking mathematics (seven items) and valuing mathematics (five items). As indicated above, the items used in TIMSS 2007 are similar to items used in the FS scale. It is interesting to note that the earlier versions of TIMSS contained more attitudinal items, distributed over more clusters. For example, in TIMSS 1995 (Keys, Harris and Fernandes 1997) there were 18 items in the following clusters:

- Students' perceived ability in mathematics
- Qualities required to do well in mathematics
- The importance of doing well in mathematics
- Reasons for doing well in mathematics.

More recently, the Programme for International Student Assessment (PISA) has reported on international attitudes to mathematics (Organisation for Economic Cooperation and Development, OECD 2004), having used similar items to those in the FS scale in the following clusters:

- Interest and enjoyment in mathematics
- Instrumental motivation in mathematics
- Anxiety in mathematics
- Self-confidence in mathematics
- Self-efficacy in mathematics

A construct for disposition

I have considered a number of names for the combination of factors that promote or militate against individuals' engagement in mathematical activity throughout their lives, including attitude, affective response, behavioural intent, disposition, motivation, self-efficacy etc. Ruffell, Mason and Allen (1998) have identified the confusion associated with different understandings of notions of attitude and affect, contrasting popular notions of 'mental orientation' with the "cognitive, affective and enactive aspects of the psyche" (ibid., 2). All of the possible names I have considered have similar potential for confusion, this was one of the themes in the discussion during the conference session. I finally chose to use disposition, as its common usage as tendency to behave in a particular way was consistent with my interest in engagement in mathematical activity (despite the possibility of confusion with popular notions of pre-disposition).

The reports from PISA 2003 (OECD 2004) noted that only 38% of students did mathematics because they enjoyed it (ibid., 115), "students' interest and enjoyment of mathematics have on average no clear association with performance" (ibid., 148) and that good results can be matched by low enjoyment (ibid., 119), In their analysis of TIMSS 2007, Sturman, Ruddock and Burge (2008, 10) observe "England's profile, of high performance but relatively low enjoyment, was common in other high scoring countries and that "pupils generally valued their learning in mathematics, despite their relative lack of enjoyment of it; they clearly recognise that it can be useful to them". This suggests that individuals may be disposed to engage in mathematics, despite attitudes / affective responses that would appear to militate against this engagement. I have used Maslow's (1970) hierarchy of needs (a hierarchy of human needs starting with the most fundamental, concerning survival and welfare, progressing through social and emotional needs, with the highest levels concerned with expressions of one's own individuality) to attempt to address this contributory factor influencing disposition.

In the longer term, I wish to identify how a disposition to life-long engagement in mathematics can be promoted more positively in school. This is my initial attempt at developing a construct for such a disposition. I have tried to use components that can be broken down further in a way that may help to identify how (and whether) they may be addressed. For example, an exclusive notion of the identity of a mathematician may be a deterrent to some pupils and will be redressed differently from a belief that mathematics is not relevant to an individual. I am currently using the following components.

- Beliefs / values / identities
- Affect / emotions
- Behavioural intent / motivation
- Needs: relating to Maslow's hierarchy

Review

The discussion during the conference session raised two issues that I have referred to in this account. The first was related to terminology and how terms such as attitude and motivation need to be used carefully. A word may be in common use, but this does not necessarily guarantee that, when used in a specific context, its meaning will also be shared commonly. The use of the term disposition to describe the construct was queried, and other terms such as motivation, resilience (and, unsurprisingly, attitude) were offered as better alternatives. I have reviewed the description of the components of the construct to reflect this, however I have retained the term disposition to describe the construct (as I describe above). The second issue related to the need for such a construct at all. It was suggested that the adaptation and use of existing attitude scales (especially those based on affective / behavioural / cognitive models) was adequate and preferable for addressing the notion of disposition of adults. Whilst accepting the value of established constructs and scales, I hope to develop a model of enquiry that allows general inferences to be drawn about how schooling affects adult actions. Many of the established scales (such as the FS scale and those used in TIMSS and PISA) use items that are designed specifically for school children, rather than adults. Although those aspects of disposition that I address through the components drawn from a social, rather than psychological, perspective (beliefs, values, identities and needs) may be addressed within an affective / behavioural / cognitive model, I find the latter approach less specific. For example, one individual's motivation to develop a high level of mathematical skills in order to support career choices could be clearly identified under the needs component of the proposed construct, whereas its situation within the affective / behavioural / cognitive model is less clear (cognitive was suggested).

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