

The role of attention in the learning of formal algebraic notation: the case of a mixed ability Year 5 using the software *Grid Algebra*

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The learning of formal algebraic notation is seen as a challenge for many students (Van Amerom, 2003). The act of symbolising is not so much a problem. Hughes (1990) showed that very young children are able to symbolise in order to record how many items were placed into a tin. The problem is more concerned with interpreting and using someone else's notation, in this case the socially agreed convention of formal algebraic notation. In activities where students are asked to find rules for pictorial patterns, they can often find rules but expressing those rules in formal notation is seen as difficult. An indication of this is seen in the levels of "patterning abilities" used by Ma (2008) and based upon Orton and Orton (1999) where the highest level of this scale is students' ability to express their rule in formal notation.

The difference between finding rules and expressing those rules in formal notation highlights a difference I see between these two aspects of mathematics. Spotting patterns and finding rules is algebraic in nature whereas how to express those rules is a matter of language and notation. This is an example of the arbitrary and necessary divide (Hewitt, 1999) where I call those things which are socially agreed, such as names and conventions, *arbitrary* as they can appear to feel so for a learner and are a matter of choice. Those things which are *necessary* concern properties and relationships and are not a matter of choice. For students to learn the arbitrary they need to be informed of these socially agreed names and conventions, whereas students can come to know the *necessary* through their own mathematical activity. This implies there are different pedagogic challenges for a teacher between the arbitrary and the necessary. The learning of algebraic notation is essentially about accepting and adopting a socially agreed convention and thus lies in the realm of the arbitrary. The finding out of rules is a different matter and lies in the realm of the necessary. Thus, the teaching of notation is pedagogically a different challenge to the teaching of algebra and requires me providing notation one way or another and helping students to accept, rather than question, notation and adopt it within their work.

The Software

1	1	2	3	4	5
2	2	4	6	8	10

Figure 1: the first two rows of the grid

The vehicle I used to provide notation is the software *Grid Algebra*³ which is based upon a multiplication grid (see Figure 1 where just the first two rows are shown). Movements between the numbers can be made through dragging a number from one cell to another cell. A move to the right would involve addition, to the left subtraction, down involves multiplication

³ Available from the Association of Teachers of Mathematics

and up division. These movements can be carried out with the software and result in formal notation as indicated in Figure 2.

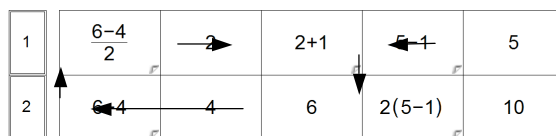


Figure 2: some movements on the grid

As movements were made students began to see the formal expressions as historical artefacts, representing journeys which had been made on the grid. At times the notation was the only evidence available for journeys which had taken place and thus there was a need for students to use this notation in order to engage with tasks such as trying to re-create the journey made. Brown and Coles (1999) have talked about setting up activities so that there is a need to use algebra and here activities were set up where there was a need to use the notation. Indeed the formal notation was subordinate (Hewitt, 1996) to some of the tasks as the notation was not a requirement in students' understanding the task yet the students were forced to go through the notation in order to carry out those tasks. Such activities involve students meaningfully practising interpretation of the formal notation even if that notation is relatively new to them.

The study

The study took place in an inner city multi-cultural primary school which has a greater percentage of children with free school meals than the national average and whose KS2 results are below the national average. There were 21 students in the group taught and these were the total number from two Year 5 classes who returned parental permission slips and who were personally happy to be part of the study. They were a mixed ability group ranging from teacher assessed National Curriculum levels two to five. The class was taught by myself on three consecutive days for one hour on the first day and one and half hours on the other two days, giving a total of four hours. Some of this time was spent with them working on pen and paper tasks and some of this time included two half hour sessions with them working individually or in pairs in a computer room. The rest of the time the class worked with a whole-class focus on the Interactive Whiteboard. Except for the pen and paper tasks, all the time was spent working with the *Grid Algebra* software, and even the pen and paper tasks were largely based upon the software, some of them being printout of sheets accompanying the software. Their class teachers reported that none of them had been taught any formal algebra, including use of a letter, or used formal algebraic notation.

Some key aspects of the way in which I worked with the students included a general absence of anything being explained by myself. Instead there was use of questioning and an expectation that students would notice and abstract rules concerning how notation was written along with the mathematical processes required to solve linear equations.

There were a number of stages in the foci of the teaching sessions, which were: students meeting notation for multiplication (brackets) and division (division line) and how addition and subtraction fit in with those in larger expressions (placing of an addition/subtraction sign following a division); order of operations within an expression; introduction of letters; substitution; multiplying out brackets; inverse operations and solving equations.

Several of the tasks involved students re-creating journeys given final expressions, or finding where a journey began given only the final expression. These involved students

having to work through the notation as it was the only information provided and in this way the notation was subordinate to the task in that reading the notation, and order within the notation, was a necessary aspect of carrying out the task. This work I am not reporting here but I will comment that all students at the end of the lessons were writing their work in correct algebraic notation. What I do report here is a small window of time when the students first met and began to work on writing expressions involving division.

Attention

Attention has been discussed as part of the teaching and learning dynamic in different ways. Mason (1989) has talked of a shift of students' attention which is indicative of a learning process whilst Ainley and Luntley (2007) have discussed the "attentional" skills which experienced teachers exhibit. Wilson (2009) has discussed the relationship between teacher and student in relation to attention. She talks of the notion of alignment between teachers' practices and students' focus of attention. My interest in attention concerns pedagogic decisions about where a teacher might wish students' attention to be placed at particular moments in a lesson. This will involve deliberate teaching acts which attempt to direct students' attention onto particular aspects of what is being discussed. My interest is in catching such teaching moments but also following through and examining evidence of where students' attention might actually be placed over time following such teaching acts.

Methodology

Lessons were video recorded and written work was collected. The later viewing of these I describe through the metaphor of glasses. The viewing of anything is never carried out neutrally. In viewing something I bring with me my experience and the particular interests which are currently present for me. In this case my viewing glasses contained lenses which represented my particular interest in the placement of attention. Thus the viewing of video and students' work was carried out through these lenses. The act of noticing (Mason, 2002) particular incidents or aspects of writing is indicative of links made between those artefacts and my particular interest. What follows is a description of some of those artefacts along with my accounting for their significance to me in relation to my interest in the placement of attention. In particular, this paper will focus the notation associated with the division line.

The notation for division: attention on particularities of expressions

The notation of addition and subtraction was familiar to the students and so the focus up to this point in time was on seeing an expression as an object rather than a process to be carried out. For example, seeing '2+1' as how it is rather than wanting to carry out that addition and say *three*. However, the notation of division was new to these students and attention was now focused onto particular aspects of the expression which were novel for the students.

Having made a downwards movement to create a multiplication I asked them what was the opposite of doing this. They responded almost in unison with "divide" and after me asking "...by what?" they said "Divide by two". This continued as follows:

DH: Divide by two? OK. So when I go from 8 to 4 I've got to do eight divided by two. Is that right?

Students: *Yes*.

DH: OK, and here it is [movement made resulting in ,8-2.]

Students: *Oh...* [A few seconds of quiet]... *Fraction* [several voices] / *It's a fraction but it stands for divide* [one voice].

DH: So, say this to me [pointing at the 8].

Students: *Eight...* [DH points to the divide line] ... *divide/over/add* [DH points to the 2] ... *two*.

DH: OK. Just say divide. OK? Ready? [Points to the 8 then the divide line and then 2].

Students: [In time with the pointing] *Eight... divided by... two*.

DH: OK? Eight divided by two.

For many students this notation appeared to be a surprise. There were also several voices saying that ,8-2. was a fraction and I would conjecture that this form of notation had previously only been used under the topic of fractions, whereas the notation $8 \div 2$ had been used for division. Here attention was brought to the individual parts of ,8-2. in order to learn to read how this expression was going to be said within these lessons, as a division rather than as a fraction. This required taking something which the students viewed as an object, a fraction, and seeing it as a process, a division. This was the opposite shift of attention which was involved earlier when I wanted to help students see an expression as an object rather than a process.

Following the above sequence, students were asked to write down what it would look like if I dragged the '6' in row two up to the cell above, then (having already hidden the ,8-2.) when the '8' would look like dragged up (so a repeat of what we had done above) and then, having dragged the '8' in row two to the left to produce '8-4', what it would look like if '8-4' was dragged up to the cell above. For each of these, students wrote down what they thought it would look like and then the action was carried out on the grid to reveal what it did actually look like. Out of the 18 booklets which were finally collected in on the last day, 12 of them wrote down ,6-3. , two of them wrote ,3-6. , another wrote $6 \div 3$ and the remainder wrote ,6-2. . My conjecture is that the attention of many of the students was on the grid and the start and finish cells of this little journey. It started with the number '6' and the cell which it would end on had '3' in it. What was not visually present on the grid was the fact that the operation was division by *two*. Instead many students have used numbers which were visible. In particular the '3' is visually above the '6' and this might be a factor why two of them wrote ,3-6. . What was adopted by all but one student was the use of a horizontal line for division.

The challenge of '8-4' divided by two was new to them as they had only seen single numbers being divided. This was successfully written by 11 of the 18 students. However, since the finishing cell of this movement contained the number '2' it was not clear whether these students wrote dividing by two because they were aware of the mathematical operation or whether the '2' came from them attending to the final cell. So, in retrospect, this was not a good example to have chosen.

Four of the students who wrote something different seemed to carry out the calculation of $8-4$ and so wrote either ,4-2. or ,2-4. . Two other students wrote:

$$8-4 \frac{4}{2}$$

This appeared to indicate the two stage nature of what was happening. The first operation of subtraction was carried out and the second operation of division shown with that answer. This reminded me of the common misuse of the equals sign when someone writes $8-4=4 \div 2=2$, only this time without the final division calculation taking place. One other person wrote:

$$\begin{array}{r} 8-4 \\ \hline 2 \end{array}$$

The fact that previous examples had a division line underneath a single number might have been generalised to this example. Since notation is arbitrary, there would be no way of

someone knowing how the case for single numbers is generalised to cases for expressions, and so this seemed a perfectly reasonable way to generalise the use of the division line.

Division not only brings with it the issue of the division line but also adds a notational issue when it is followed by addition or subtraction. Shortly after the sequence above I said I would drag $8-4+6-2$. one cell to the left (where this would produce $8-4+6-2.-1$). Of the 13 who attempted this, six wrote down the conventionally correct answer with five writing $8-4+6-1-2$. This is again a novel situation for them and so how would they be expected to know what is the conventional way of writing this? When I moved the expression across to produce the result there were a number of students who said “Yessss!” followed by a small gap of a couple of seconds followed by several voices saying “Uh?” with one student saying “Why’d the one have to go there?” I did not respond to this and I am not convinced the question was actually directed at me. It appeared more an expression of surprise. I did, however, want to focus attention, in a factual way rather than an explanatory way, as to this feature of where the subtraction sign was written. In doing so I offered a visual and aural image; I pointed the pen at the left hand end of the division sign and moved to the right along the line saying “Just notice, zzzzzipp,…” and then continued along this line taking my pen off briefly before putting it back down again along the subtraction line and continued by saying “... ping!” This aural image was heard being repeated by a number of the students as I then went about clearing the grid and beginning to start the next challenge. None of the students asked again for any explanation of why the notation was how it was.

In the expressions the students wrote down for the above, there was only one which involved a calculation: $8-4+6-1=4-2$. The ‘=4’ is where we finished up after this movement and so this equation could be considered mathematically correct and just not written conventionally. This whole sequence of the lesson from when the first division was carried out lasted under seven minutes and in this time there was a significant shift towards writing expressions conventionally; the division line was adopted and there were nearly no calculations being carried out, indicating a shift back from process to object.

Final remarks

Coming to accept and adopt formal notation involved a development where attention was shifting from process to object, and occasionally vice versa. At times attention appeared to be on what was visible within the grid rather than the operations which were visually implicit even if verbally explicit. Pedagogic techniques were occasionally used to draw attention onto particular detail and at other times to ignore detail and treat an expression as an object upon which further operations were being carried out. I note that there was a general sense of acceptance of the notation, rather than questioning why it appeared how it did. Even when this did happen, with the initial creating of a division and then with a subtraction following a division, any comments did not appear to be directed towards me. I conjecture that this was because I was not doing the writing of those expressions. Instead the computer created them and students are quite used to accepting all sorts of arbitrary conventions which form part of interactions with technology.

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