The functions and effects of real world contextual framing in A/AS pure mathematics questions: developing an evaluative framework

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This paper reports on ongoing research into real world contextual framing (RWCF) in A/AS mathematics. After a review of research into real world contexts in mathematics, it discusses its possible effects, for example applicability, motivation, teaching modelling skills, and providing a mental scaffolding for mathematical concepts. The paper proposes a framework for evaluating RWCF, based on the notions accessibility, realism and authenticity, and presents some preliminary findings of an analysis using this framework on a sample of RWCF in A/AS pure mathematics questions.

Keywords: Assessment, post-16 mathematics, real world context, modelling

1 What is real world contextual framing?

Consider these two AS level mathematics questions.

An arithmetic progression has first term 7 and common difference 3.
(i) Which term of the progression equals 73?
(ii) Find the sum of the first 30 terms of the progression.

Fig. 1

Chris saves money regularly each week. In the first week, he saves £7. Each week after that, he saves £3 more than the previous week.
(i) In which week does he save £73?
(ii) Find his total savings after 30 weeks.

Fig. 2

Each question has the same mathematical solution. In the first version, the language used may be identified as from the mathematics register (Pimm, 1987), whereas in the second version it utilises an everyday context of money and saving. The same mathematical questions are framed in terms of a narrative involving 'Chris', 'saves', 'weeks'. We can therefore identify a 'real world' context, namely finance and savings, which is used to frame the same mathematical question. This is an example of what might be termed real world contextual framing (RWCF).

The use of the phrase 'real world' here presupposes a distinction can be made between this and a 'mathematical world'. The 'reality' of mathematical concepts is an epistemological problem which is beyond the scope of this paper to consider. It will suffice, in determining a meaning for the term 'real world contextual framing', to be able to classify questions according to whether it is present or absent.
Let us look at a further version of the example (see Fig. 3).

A spiral is formed with sides of lengths 7 cm, 10 cm, 13 cm, … which are in arithmetic progression.

(i) How many sides does the spiral have if its longest side is 73 cm?

(ii) Find the total length of the spiral with 30 sides.

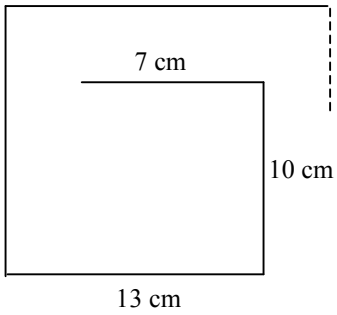

Fig. 3

Here, the mathematical content (arithmetic progressions) is expressed through a secondary context, that of spirals. Is this a 'real world' context? In a sense, the term 'spiral' may be defined as a geometrical (and therefore mathematical) object, similar to 'circle', 'square', 'straight line', etc. On the other hand, the context suggests a spatial 'realisation' of the algebraic concept (arithmetic progression). The narrative of the question is detached from the primary mathematical content, and requires a process of transfer from one context to another. For this reason, we classify 'pattern' contexts like this one as 'real world' contexts.

2. Real world contextual framing in UK public examinations

Real world contextual framing appears to have become a common feature of public examinations in mathematics in the UK. It is commonly used in GCSE questions, as well as in English and Welsh national tests at Key Stage 2 and 3. In A/AS applied mathematics questions in mechanics, statistics and decision mathematics, we would expect real world contextual modelling to be an intrinsic characteristic of questions. After all, how can we test applications of mathematics without reference to a context outside of the realm of pure mathematics?

However, the degree to which RWCF is used in A/AS pure mathematics papers varies considerably from specification to specification. In an analysis of 44 recent pure mathematics papers for the two current OCR A/AS specifications, 5.2% of the marks for syllabus A, as opposed 31.5% of marks for syllabus B (MEI) were allocated to questions which involved some degree of RWCF.

The historical roots of RWCF in our public examinations seem to date from the 'modern mathematics' movement of the 1960s, and the influence of projects such as the School Mathematics Project (SMP) and the Mathematics for Education and Industry Project (MEI). The growth from the 1960s in applications of mathematics such as statistics, decision mathematics and numerical methods, together with the development of computers and calculators, led to an increased demand for A-level mathematicians to acquire applied mathematical skills such as mathematical modelling, and this in turn has created A-level syllabuses which place an emphasis on real world application. Thus the current Qualifications and Curriculum Authority criteria for A/AS Mathematics include assessment objectives to recall, select and use standard mathematical models to represent situations in the real world, and comprehend translations of common realistic contexts into mathematics. However, the extent to which RWCF, as illustrated in Figures 1, 2 and 3, effectively meet the requirements of these assessment objectives is open to question.

3. Research on real world context in mathematics questions

What does existing research tell us about the function and effect of using real world context in mathematics problems? Two alternative, and somewhat contradictory, perspectives can be drawn from this. On the one hand, some research (Clausen-May, 2005, Vappula and Clausen-May, 2006, Nickson and Green, 1996) proposes that real world contexts can provide the solver with 'models to think with', or 'mental scaffolding'. On the other hand, there is a substantial body of research on 'word problems' (Verschaffel, L, 1994, Verschaffel L. et al, 1997, Silver, 1993, Greer, 1997) , which catalogues young children's inability to apply realistic considerations to mathematics questions framed in real world contexts. Others (Boaler, 1993, Boaler, 1994, Wiliam, 1997) have criticised real world contexts on the grounds of artificiality. Cooper and Dunne (2000), in a major study, found that real world contexts are differentially misinterpreted by children of different social classes. Research on its use on public examinations (Ahmed and Pollitt, 2007, Ahmed and Pollitt, 2000, Pollitt et al, 2000, Pollitt and Ahmed, 2001) suggests that real world contexts can threaten the validity of questions by introducing a construct-irrelevant variance (Wiliam, 2007).

Thus, it is clear that the framing of questions in real world contexts is not entirely uncontroversial. Most of this research has been directed at tasks in pre-16 school mathematics, and its relevance to A-level mathematics questions is therefore open to question.

4 The effects of RWCF in mathematics questions

We now consider the potential effects of RWCF of mathematics questions. The research into context in pre-16 questions outlined above suggests the following hypotheses. On the one hand, adding real world context may increase the difficulty of questions, by adding to the complexity and demands on information processing and comprehensibility. It may lead to misinterpretation, and a lack of recognition of the appropriate mathematical methods. On the other hand, RWCF may decrease the difficulty of the questions, by providing the solver with mental scaffolding for the mathematical ideas.

Real world context may motivate the task, by presenting the mathematics as a useful tool for solving real world problems through mathematical modelling. On the other hand, it may appear as artificial to solvers, by over-simplifying reality, making unjustified assumptions, and 'camouflaging' the mathematics. Analysis of RWCF in linear equation questions (Little, 2008) suggests that modelling has little utility other than in motivating general algebraic methods.

In an ongoing study, these hypotheses are being researched using a questionnaire of student opinion, and a test in which questions on arithmetic and geometric sequences are offered in alternative versions, some deploying contextual framing, and some without.

5 A model for evaluating RWCF in questions

Little and Jones (2007) propose an initial model for evaluating the role of RWCF in A/AS questions. Further analysis of questions has suggested refinements to this model, as follows.

Accessibility of script, match and language

a) Is the script implied by the real world context universally held, commonly held or not commonly held? Is it familiar to students, or novel?
b) How close is the match between the structure provided by the real world task and the structure/concepts provided by the mathematical model?
c) Does the language used in the question make implicit or explicit the connection between the real world context and the mathematical model? What additional comprehension demands are made of candidates in explaining the real world context? How many words are used? Is there any specialist language used which requires definition or is less commonly understood?

Realism of model

Are the data generated by the mathematical model realistic? Are the assumptions made when applying the mathematical model to the real world context appropriate?

Task Authenticity

Does the task set pose questions which are valid, interesting and meaningful, not just in testing the mathematics, but in the real world context? Do they provide meaningful and useful insights within this extra-mathematical context, or are the tasks designed purely to test mathematical concepts and techniques?

Fig. 8 shows how this evaluative tool may be applied to a sample AS level question. In ongoing research, a sample of A/AS pure questions have been analysed in this way, in order to clarify what function real world context serves in questions, and to propose criteria for evaluating their quality. Some preliminary findings are presented below.

There is overwhelming evidence that questions with RWCF are more wordy, and therefore impose greater tests of comprehension. This might well have a bearing on their accessibility.

The structure dictated by the context may present different levels of modelling complexity, which may affect accessibility. In the case where the match is isomorphic, the transfer from context to mathematical model is relatively straightforward. In other cases, the solver may be required to take account of, and discount, features in the context when mapping the context to the mathematical model.

Some contexts, for example radioactivity for exponential growth and decay, may be classified as natural in cases where extra-mathematical justification exists for modelling them with the mathematical content being tested, or synthetic, in cases where the context is chosen and manipulated to fit the mathematical content.

Authentic contexts furnish questions with a sense of purpose which is absent from the pure mathematical questions. However, artificially constructed contexts may have a negative effect on the perceived utility value of mathematics to candidates.

Some of the contexts used may contribute to the understanding of the mathematics, by requiring students to think of the mathematical ideas in novel and unexpected ways.

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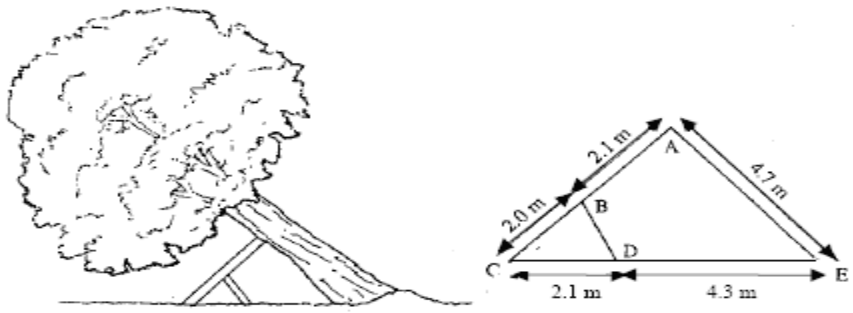

In the gales last year, a tree started to lean and needed to be supported by struts that were wedged as shown above. There is also a simplified diagram giving dimensions.
Calculate the angle the tree makes with the vertical, giving your answer to the nearest degree. [5]

Real world context (script)	Mathematical model
A leaning tree.	A triangle ACE with a line BD, B on AC and D on CE
Angle which tree makes with vertical.	Angle AEC, calculated using trigonometry in the triangle ACE.

1. Accessibility	
(a) Script	Easily recognisable context.
(b) Match	Does the 'tree' have a uniquely defined angle with the ground? The 'struts' contribute little to the task set, except to complicate the diagram.
(c) Language	The 'tree' is a 'McGuffin' – providing an 'excuse' for solving the triangle. 47 words, 5 marks, 9.4 words per mark. No difficult vocabulary.
2. Realism	The measurements are reasonable, although the position of 'E' is ill defined, and might not justify distances to the nearest 10 cm.
3. Task Authenticity	In practice, trigonometry would not be an appropriate method for estimating the angle of the tree, and there is not reason why we should want to calculate this information.

Fig. 4 Evaluation of RWCF in a 'tree' question (source: OCR)

It appears that the negative effects of context reported by some researchers who have questioned its validity and value need to be weighed carefully against some of the potential benefits proposed above. More research is clearly needed to ascertain whether evidence of some of these effects, derived from detailed analysis of a sample of questions, can verified through students' responses and attitudes to questions involving RWCF.

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