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Sheffield Hallam University, Saturday 16 June 2007

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We hope that members will use the proceedings to give feedback to the authors and that through discussion and debate we will develop an energetic and critical research community. We particularly welcome presentations and papers from new researchers.

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# **MATHEMATICAL VISUAL FORMS AND LEARNING GEOMETRY: TOWARDS A SYSTEMIC FUNCTIONAL ANALYSIS**

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*Mathematics is a multimodal discourse in which mathematical texts use, at least, three different semiotic systems: verbal language, algebraic notations and visual forms. Beside the research that has been done concerning the verbal components of mathematical texts, there is a need to develop tools to describe the non-verbal components. Based on Halliday's SF grammar, Morgan's linguistic approach and multimodality approach. I present a preliminary suggested descriptive framework for analysing geometrical visual forms. My intention is to use this framework in my PhD study which investigates the role of mathematical visual representations in the construction of mathematical meaning. In order to illustrate, aspects of two examples will be analysed using this framework.*

## **BACKGROUND:**

A general overview of the status of visual representations, i.e. diagrams, graphs, shapes, etc., in mathematical texts indicates that these representations are: a) limited in representing knowledge with possible misuse of diagrams (Shin, 1994 as mentioned in O'Halloran, 1999, 2005), b) of an 'informal and personal nature' so that the mathematical community will not accept such representations in a research paper even if they are motives and important for the researcher herself/himself (Misfeldt, 2007). One main reason for this view is that the main stream among mathematicians (or even among others) conceives mathematics as 'abstract, formal, impersonal and symbolic' (Morgan, 2001). At the best, mathematicians consider these representations have or own messages or meanings, even though these messages are limited, which students need to know how to 'grasp' or discover (Shuard & Rothery, 1984). In my prospective study, I consider visual representations as available resources for meaning-making and I intend to investigate what meanings students make when they interact with these representations while solving problems.

## **THEORETICAL FRAMEWORKS – A STATE OF THE ART:**

It has been argued that people communicate by using different modes from the resources available to them, for example spoken and written language, visual representations, gestures, music, etc. (e.g. Kress & van Leeuwen, 2006; Lemke, 1998; Morgan, 2006; O'Halloran, 1999). In order to achieve effective communication, people use what they think the 'best' mode to communicate- 'aptness' of mode (Kress & van Leeuwen, 2001). When people employ visual representations in their communication, visual representations then have a function in representing knowledge, just as language or any other mode. The need to understand and take these representations into consideration when analysing any text is, therefore, salient.

Halliday (1985) argues that any text fulfils three meanings: ideational, interpersonal, and textual. Our ideas about the world are represented in the ideational meaning, the interpersonal meaning is realised by the relationships constructed with others through communication. The textual meaning is realised as these representations get presented in a coherent way. This descriptive framework is called systemic functional linguistics (SFL) or grammar (SFG). Even though this framework was initially developed to account for verbal modes of communication only, it has been extended to include non-verbal modes too. The multimodal approach or the semiotics of visual representations developed by Kress & van Leeuwen (2006) is an example. They have developed a grammar to 'read' images using 'representation, interactive and compositional' corresponding to Hallidayan terms respectively. Other examples are: Lemke's studies in science education and language (e.g. 1998), the semiotics of art (O'Toole, 1990) and application of SFL in mathematics education by Morgan (1995; 1996a; 2006) and O'Halloran (1999; 2005).

Mathematics is a multimodal (or multisemiotic) discourse (Duval, 2000; Morgan, 1995, 1996a, 2006; O'Halloran, 2003) where three semiotic systems, at least, are used: verbal language, algebraic notations (or 'mathematical symbolism'), and visual forms (diagrams, shapes, graphs, etc.). As it has been observed by Morgan, 'the oral discourse of mathematical practices (...) has already been the subject of some research in educational contexts' (Morgan, 2003, p. 112). Furthermore, Morgan (1995; 1996a; 1996b; 2001; 2003; 2006) has opened mathematics discourse for Halliday's systemic functional grammar by adopting it as a framework and an analytic tool to analyse written mathematical texts, thus, developing a linguistic approach to mathematical text. In written mathematical texts, while the ideational meaning is realised by the writer's view to the nature of mathematics and the existence of human agent, the interpersonal meaning is realised through the relationship established between the producer of the text and its reader and the roles of both of them within the text. The textual meaning is –in turn– realised by the role that a coherent text plays, such as developing a mathematical argument, concept or proof (Morgan, 2006).

For non-verbal features of mathematical texts, O'Halloran (1999) develops SFL frameworks for both mathematical symbolism (or algebraic notations) and mathematical visual displays. She uses O'Toole's systemic functional framework to analyse mathematical visual representations. In analogy and accordance with the Halliday's systemic functional linguistics, O'Toole (1990) suggests that 'the semiotic codes of the visual arts (...) are realized through systems of representational, modal, and compositional choices' (p. 187). In turn, O'Halloran (1999; 2003; 2005) adopts this framework for analysis the meanings of mathematical graphs and diagrams. Nevertheless, O'Halloran's framework applies only to graphical forms and her work has not been directed towards geometry, which is the focus of my interest. It is doubtful whether her framework can be applied in a straightforward manner to geometry. I argue that a specialised framework for the grammar of geometrical visual diagrams is needed. Moreover, Morgan (1995; 2006) states that there is a need to

develop tools to describe the non-verbal components of mathematical texts from the systemic functional perspective.

Following the efforts of previous research (Chapman, 2003; Morgan, 1995; O'Halloran, 2003), I intend to investigate what meanings visual representations do offer. As a first step towards this aim, I present a 'first' draft of a preliminary suggested framework (annex 1) which needs more developing and thinking. This framework is mainly based on Morgan's linguistic approach (2006) and Kress & van Leeuwen (2006) framework as well.

## **TWO EXAMPLES:**

I will try in this part to 'apply' the suggested framework to two examples (annex 2). Because of the limited space available I will focus on one feature: representation of the image of mathematics (ideational meaning) in diagrams in two texts (Examples 1 & 2 in annex 2), one is English and the other is Arabic.

The representational (ideational) meaning in diagrams is realised by determining the nature of the diagram; whether it is a narrative structure or conceptual structure. The main feature is the presence of an action or not, that is, following Kress & Leeuwen (2006), the presence of a vector. Vectors might be a curved arrow, 'attenuated' vectors (dotted or solid line) or 'amplified' vectors. In both structures, we need to look at the types of processes and participants active in them. Based on Hallidayan functional grammar, Kress & Leeuwen state that in narrative structure, the type of processes is that of 'happening', 'doing' or 'going on' and the participants are active; they are carrying out the identified process. In mathematical discourse, these processes might be generalisation, measurement, naming, etc. In conceptual structures, no actions are being carried out; the participants are, thus, not active. There are three types of processes represent participants 'in terms of their class, structure or meaning' (Kress & van Leeuwen, 2006, p. 59): classificational, analytical and symbolic.

The diagram in example 1 is an example of a narrative structure. There are some processes occurring here such as generalisation since the diagram uses symbols rather than specific numbers. This process suggests that this is a general situation, not an example, to represent the equation of a circle where the centre is the origin. The dotted line (PN) needs to be drawn in order to prove the equation. This suggests that a human agent exists and, consequently, the image of mathematics is as a human practice rather than being impersonal. The labelling process also emphasises this image; different kinds of labels are presented: measurements ( $r$ ,  $O$ ,  $y$ ), names ( $y$ ,  $N$ ), variable ( $P(x, y)$ ), or property (the right angle symbol at  $N$ ). It is also significant to observe the position of the diagram as a feature of the compositional/textual meaning. The diagram stands in the middle of the upper section of the page, surrounded by white space. That suggests a certain theoretical or ideal situation; it constitutes a unity that stands on its own and invites for interaction.

In example 2, there are three shapes/figures. The upper rhombus and Venn diagram are, respectively, symbolic and classificatory (conceptual) structures. The upper rhombus's identity is clear since no names, symbols or measurements are on it. The Venn diagram is a classificatory structure presenting the relationships between rhombuses, parallelograms and quadrilaterals. The lower rhombus, on the other hand, is a narrative structure with dotted lines (which represent its diameters) that need to be formed in order to solve the problem. In this case, a human agency is clearly needed; therefore, the mathematical activity is portrayed as human-made.

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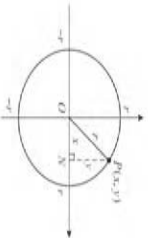
## A preliminary suggested framework

Based on Morgan (2006) and Kress & van Leeuwen (2006) frameworks

| <b>Representational/Ideational meaning</b><br>designing social actions & constructs  | <b>Interactive/Interpersonal meaning</b><br>'designing the position of the viewer'   | <b>Compositional/Textual meaning</b><br>Unity & Coherence   |
|--|--|---|
| <ul style="list-style-type: none"> <li>• <b>Nature/image of mathematics and mathematical activity</b></li> </ul> <p>The picture of mathematics might be represented through the examination of types of processes and participants acting in them. This meaning (ideational) is realised by determining the nature of the diagram; whether it is a narrative structure or conceptual structure:</p> <p>* <u>Narrative structures</u>: (designing social actions) 'goings-on' - 'doing', 'happening', 'sensing', 'meaning' (vector: action)</p> <p><i>Processes</i>: generalisations, measurements, naming, ...<br/><i>participants</i>: active</p> <p>* <u>Conceptual structures</u>: (designing social constructs)<br/><i>Processes</i>:</p> <ol style="list-style-type: none"> <li>1. Classificational: classify</li> <li>2. Analytical: part-whole</li> <li>3. Symbolic: meaning/identity of participants</li> </ol> <p><i>Participants</i>: not active</p> | <ul style="list-style-type: none"> <li>• <b>Roles and relationships between author/producer and viewer</b></li> </ul> <p>There are two kinds of participants in the (re)production of diagram, represented participants ('things' depicted) and interactive participants (real people, the producers and the viewers). Hence, there are three kinds of relations between these participants. These relations are realised by:</p> <p>* <u>Contact</u>: Does the diagram offer information not mentioned in the co-text? Is the diagram drawn 'differently' that demands attention?</p> <p>* (<u>Social</u>) <u>Distance</u>: personal, impersonal. (drawing the diagram neatly vs. roughly)</p> <p>* <u>Attitude/point of view</u>: involvement vs. detachment, relationships (power, equality). (specialty, certainty and authority)</p> <p>* <u>Modality</u> (design the reality/truth) (naturalistic vs. scientific modality). 'shared truths', 'imaginary we' – mathematical community</p> | <ul style="list-style-type: none"> <li>• <b>Unity &amp; Coherence</b></li> </ul> <p>The way that elements are presented/ placed in a text contributes to its meaning. This textual meaning relates the ideational and interpersonal meanings together into a 'meaningful whole' or a message by:</p> <p>* <u>Information value</u>: 'placement of the elements': left-right, top-bottom, Centre-Margin.</p> <p>* <u>Salience</u>: 'eye-catching' or 'attract the viewer's attention': colour, size, perspective (foreground, background, overlap, appearance of human figure)</p> <p>* <u>Framing</u>: separation such as frame lines, white space, colour, etc.</p> <p>What message(s) does the whole/integrated mathematical text present? Examples: 'instructions for a calculation, argument, new mathematical concept or procedure, proof or a solution to a problem, story', etc.</p> |

## Example 1: English text

We shall now take the radius of the circle to be  $r$ .



If we take any point  $P(x, y)$  on the circle, then  $OP = r$  is the radius of the circle. But  $OP$  is also the hypotenuse of the right-angled triangle  $OPN$ , formed when we drop a perpendicular from  $P$  to the  $x$ -axis. In the right-angled triangle,  $ON = x$  and  $NP = y$ . Thus, using the theorem of Pythagoras,

$$x^2 + y^2 = r^2.$$

and this is the equation of a circle of radius  $r$  whose centre is the origin  $O(0, 0)$ .



The equation of a circle of radius  $r$  and centre the origin is

$$x^2 + y^2 = r^2.$$

3

© mathcentre July 18, 2005

Source: Mathcenter. (2005). The geometry of a circle. Available at: <http://www.mathcentre.ac.uk/students.php/mathematics/geometry/circle/resources/3>. Retrieved 27/12/06.

## Example 2: Arabic text

حالات خاصة لمثلثي الاضلاع  
٤-١ ( المتعيق - والمستطيل - والمربع )

المتعيق :-

هو مثلثي الاضلاع فيه ضلعان متجاوران متساويان ويمثل يعني ان جميع اضلاع المتعيق متساوية.



الشكل الربيعي



وتبين شكل في المجاور ان المتعيق حالة خاصة من مثلثي الاضلاع، اذ ان الضلعيات هي مجموعة جزئية من مثلثيات الاضلاع.

عرفت في صفوف سابقة ان نظري المتعيق متساويان ويعتقد كل منهما الآخر، وسوف تعرف هنا على اثبات صحة هذه الخاصية.

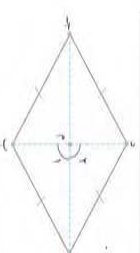
**نظرية** قطر المتعيق متساويان، ويعتقد كل منهما الآخر.

في الشكل المجاور أ ب ج د معين تقراء متساويان في م.

ثورة اثبات أن : (١) م = م د

(٢) م = م ا

(٣) ب د عمودي على أ ج



المبرهان :-

المتعيق حالة خاصة من مثلثي الاضلاع ولهذا فان نظري المتعيق ا ب ج د يعتقد كل منهما الآخر. وعندما اثبت الجزيئين الأول والثاني من المثلث ب د.

يعني علينا ان اثبت ان القطرين متساويان. لاحظ ان جميع اضلاع المتعيق متساوية. المثلث ا ب د متساوي الساقين وفيه م يصل من الرأس الى منتصف القاعدة.

اذن م عمودي على القاعدة ب د ( اذ ان  $\angle \text{خا} = 90^\circ$  -  $\angle \text{خب} = 90^\circ$  قائمة). اذ ان القطرين متساويان (المساواة).

٤ ٣

Source: Palestinian Ministry of Education and Higher Education (2004, in Arabic). Mathematics textbook for 8<sup>th</sup> grade-part 2. Available at: [http://www.pncde.edu.ps/textbooks/math\\_G8\\_p2.pdf](http://www.pncde.edu.ps/textbooks/math_G8_p2.pdf). Retrieved 26/12/06.

## **USING DYNAMIC GEOMETRY TO INTRODUCE CALCULUS CONCEPTS: CALGEO AND THE CASE OF DERIVATIVE**

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*CalGeo is a three-year project supported by EU programme Comenius 2.1. Amongst the objectives of this project is the design of an in-service teacher education programme which employs dynamic geometry tools for teaching Calculus in upper secondary education. In this paper we present the project, its main objectives and the produced material; an example of a learning environment/activity designed for the introduction to the notion of derivative at Year 12; and, some results of the application of this activity in a real classroom situation. In this activity we use the tangent line and the property of local straightness to introduce the formal definition of derivative. Several cases of differentiable and non-differentiable functions are discussed through their geometrical and symbolic representations.*

### **INTRODUCTION**

Calculus has a wide field of applications in other disciplines and constitutes a basic part of the mathematical curriculum of secondary education. Calculus knowledge is also necessary for the successful study of several subjects at university. Nevertheless research shows that the majority of students face serious problems in understanding basic Calculus concepts (for example see Harel, Selden & Selden, 2006).

The work presented in this paper originates in a three-year project called *CalGeo* (Teaching Calculus Using Dynamic Geometric Tools). The main project objectives are:

- i) the investigation and planning of a programme for secondary education which employs dynamic geometrical tools for the teaching of Calculus, and
- ii) the design of in-service training course for mathematics teachers based on the above programme.

The project focuses on the following topics: introduction to infinite processes, limit, continuity, derivative and integral. For each topic the training material includes documentation that raises mathematical, historical and didactical / pedagogical issues as well as a set of proposed activities. The produced material was tested in pilot teachers' training course as well as in real classroom situations in each of the participating countries.

The participants of this project are University of Athens (Greece), which is the coordinating institution; University of Crete (Greece); University of Southampton (United Kingdom); University of Cyprus; and, University of Sofia (Bulgaria).

In what follows we describe the rationale of the activities; the dynamic environment within which the activities were developed; and, we exemplify with an activity

concerning the concept of derivative, as well some results from its implication in real classroom conditions.

## **LEARNING ENVIRONMENT / ACTIVITIES**

The activities of the project were designed in order to be used towards the introduction of Calculus concepts at upper secondary education level (Year 11 and 12). They offer problem solving situations in which previous knowledge will turn out inadequate and the opportunity to explore alternative and generalisable aspects of an already known concept (e.g. the tangent line of the circle as the limiting position of secant lines).

The learning environments were designed in order to approach intuitively the corresponding mathematical notion(s) in ways that are consistent with formal mathematical theory (e.g. visual representation of the  $\varepsilon$ - $\delta$  definition of the limit) taking into account the students' previous knowledge and the topics which have proved to be a source of learning difficulties in calculus courses.

In this project we employed more than one dynamic geometry software (DGS). In Greece, we used a DGS called EucliDraw v.2.2.2. In addition to DGS facilities, this software offers a function editor / sketch environment as well as some tools appropriate for Calculus instruction. Indicatively, we refer to the 'magnification tool' that can magnify a specific region of any point on the screen in a separate window. This magnification can be repeated as many times as the user specifies through a magnification factor. Other useful, for Calculus, facilities are these that can partition an interval; construct the lower and upper rectangles covering the area defined by a graph and the  $x$ ' $x$  axis; control the number of the decimal numbers of calculations etc.

For more information about the project, its theoretical assumptions, the dynamic environment and the produced activities see (Biza, Diakoumopoulos and Souyoul 2007) and in the project website: [www.math.uoa.gr/calgeo](http://www.math.uoa.gr/calgeo).

## **ACTIVITY ON THE CONCEPT OF DERIVATIVE**

The aims of this activity are: the introduction to the definition of the derivative at a point; the introduction to the definition of the tangent line of a function graph as the limiting position of the secant lines as well as the linear approximation of the curve at this point; the reconstruction of students' previous knowledge about tangent line grounded to the Euclidean Geometry context in order to be applicable in general cases of curves; the connection of the symbolic and geometric representations of derivative at a point; and, the recognition by the students the property of the "smoothness" of a function curve at a point and its relationship to the differentiability of the function at this point.

According to Tall (2003), the *cognitive root* of the notion of derivative is the *local straightness*. The property of *local straightness* refers to the fact that, if we focus close enough on a point of a function curve (a point at which the function is

differentiable) then this curve looks like a straight line. Actually, this ‘straight line’ is the tangent line of the curve at this point. This property is valid in all cases of tangent lines and its understanding could be facilitated by the use of new technology with appropriately designed software (Tall, 2003; Giraldo & Calvalho, 2006). On the other hand the early experiences of the circle tangent contribute to the creation of a *generic tangent* as a line that touches the graph at one point only and does not cross it (Vinner 1991). Furthermore, students perceive not generally valid properties related to the number of common points or the relative position of the tangent line and the graph as defining conditions for a tangent line. Different combinations of these properties create intermediate models of a tangent line. This occurs through the assimilation of new information about graph tangents in the existing knowledge about circle tangent (Biza, 2007; Biza, Christou & Zachariades, 2006).

The activity starts with the notion of circle tangent in the context of Euclidean Geometry. The students are asked to sketch in the EucliDraw environment a circle; a point of it  $A$ ; and, a line vertical to the radius  $OA$ . The tasks of the worksheet intend to make the students observe that the tangent line is the limiting position of the secant lines  $AB$  as  $B$  approaches  $A$  and with the help of the ‘magnification tool’ to magnify the region around  $A$  and observe that the circle looks like its tangent as the magnification factor increases.

In the next step – through the investigation of the tangent line in the case of the semicircle as a function graph – students make the transition to the Calculus context. Thereafter, the students work in an already constructed environment of EucliDraw and they are introduced to the tangent line of function graph and through this to the definition of the derivative. In this environment the graph of  $f(x) = \sin(x)$  and a point  $A(x_0, f(x_0))$  of it are sketched. In the display of this environment in Figure 1 we can notice some other constructions as: the number  $h$ ; the points  $B(x_0+h, f(x_0+h))$  and  $C(x_0-h, f(x_0-h))$ ; the magnification window of a region of  $A$  related to a magnification factor equal to  $1/h$ ; the secant lines  $AB$  and  $AC$ ; and, the slopes of these lines.

Students, following the tasks of the worksheet: decrease the values of  $h$  and through this increase the magnification factor and move the points  $B$  and  $C$  closer to  $A$ ; observe what happens with the secant lines and their slope and express their slope symbolically.

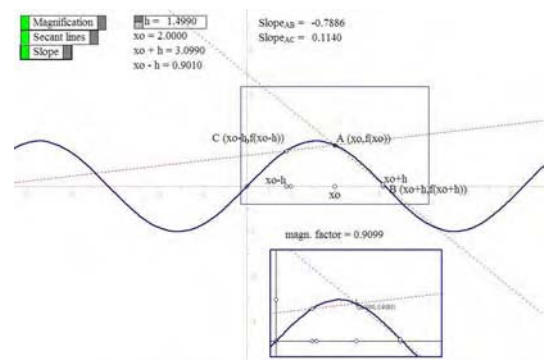


Figure 1: A EucliDraw environment for the introduction to the notion of

Several cases of differentiable and non-differentiable functions are discussed through their geometrical and symbolic representations. For example:  $f(x) = |\sin(x)|$  that is not differentiable at the points in which the graph intersects the  $x$  axis;  $f(x) = x^3$  in which the  $O(0,0)$  is an ‘inflection point’; and the  $f(x) = \sqrt{|x|}$  in which the  $O(0,0)$  is a cusp point.

## **APPLICATION OF THE LEARNING ENVIRONMENT**

We applied the teaching material described in the previous paragraph in a Greek Year 12 classroom of 17 students (8 girls). The mathematics teacher and the first author conducted the application. The teacher was familiar with different types of educational software and their applications. In addition, he had followed the training programme of the Calgeo project.

All students had mathematics as a major subject (they were candidates for science or polytechnic studies in the university admission examination that, in Greece, takes place at the end of that year) but had varying levels of performance. By the time the application took place (at the end of the first semester), the students had been taught functions, limit, continuity and they were just before the introduction of derivative. Most of these students had previous experience of DG environments in their Euclidean Geometry lessons but not in Calculus lessons.

This application lasted two sessions (one hour and two hours, respectively) and took place in the mathematics classroom replacing the traditional lesson. The students had been split in groups of three or four and each group used one of five portable PCs whereas the instructors used a sixth one plugged into an LCD projector.

The teaching material consisted of the electronic environment designed in EucliDraw software and the students' worksheets. The records collected across this application included: pre and post questionnaires; audiotapes of the lessons; students' work in their worksheets; and, classroom transcripts.

### **Observation on the classroom application**

In the first session dedicated to the application students became familiar with the novel instructional situation: the transformation of their classroom into a laboratory; the electronic environment; and the rules of collaboration and communication in the group and between the groups and the instructors. It was easy for those students to feel comfortable in the EucliDraw environment but very difficult for them to exchange ideas and work in the new classroom situation. The collaboration was established in the second session when the transition to Calculus context was beginning to be discussed. At that point and onwards several issues of discussion emerged.

Although sometimes the questions posed by the students appeared to be outside the main aims of the task, we tried to take advantage of these questions in ways that would help carry on with the activity. For example, questions like: "is the circle a function graph?" "what is the formula of the graph of the semi-circle?" proved very useful in terms of the transition from Euclidean Geometry to Calculus.

Conversation upon the image in the magnification window brought up questions like: "can we draw a tangent line at a vertex of a polygon?" or "how does the vertex look in the magnification window?" To clarify this, we made a parallelogram in EucliDraw and a line passing through a vertex of it. We magnified the region around

this vertex and we discussed the figure in the magnification window (the image did not look like a straight line at any region of the vertex). We could say that the representation of the *local straightness* proved very illustrative when the curve is not *smooth* and thus does not look like a straight line in the magnification window (e.g. when the point is vertex or when the function is not differentiable).

Some students following the instructions of their worksheet tried smaller and smaller values of  $h$  but not with smaller absolute value. This actually was a wrong statement in the worksheet and we grasped the opportunity to discuss the meaning of “the  $h$  tends to 0”

In the discussion about how can we define the tangent line in the case of function graphs, students used arguments based on both the context of Calculus and Euclidean Geometry context. For example, in the case of the  $f(x) = \sin(x)$  graph and its tangent line at point  $A(x_0, f(x_0))$  a student declared:

[S1]: The tangent line is a line that has one common point with the graph.

Then we moved the point  $A$  so that the constructed line (the limiting position of  $AB$  and  $AC$ ) to cut the curve in another point:

[I]: What do you say now?

[S2]: We could say that the tangent line is a line that has one common point at a neighbourhood of the tangency point and does not intersect the curve at this point.

We kept this statement written in the whiteboard and later on the discussion of the  $f(x) = x^3$  and the tangent at the point  $O(0,0)$  the same students said:

[S2]: It looks like it is [a tangent] but it cannot

[I]: Why?

[S2]: It cannot be a tangent because it cuts the graph ... how can I say that ... it intersects ... it splits it in two parts, on part on the one side and one on the other [of the line] ... on the other hand it looks to coincide with the curve near point  $A$  at the magnification [window]... I don't know ...

Through the discussion on several cases of graphs we tried to facilitate students' reconstruction of their previous knowledge about tangents. As the analysis of the post-application questionnaires revealed some students did make this reconstruction.

## CLOSING COMMENT

The application of this specific material raised some issues concerning the teaching material and its application. Some of these issues concerned the student's adaptation in the new for them classroom environment (e.g. collaboration, communication, familiarity with electronic environments etc). Some others were related to this specific instructional approach (e.g. different representations, students'

misunderstanding and the role of the several examples, accuracy of measurements in electronic environments, the visual perception of local straightness etc.)

These observations were demonstrative of the diverse situations that these environments created in the classroom community. Although further investigation and more systematic research are needed for more valid results we could say that this application proved very helpful for the development and elaboration of the material of our project.

### **Acknowledgments**

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# **MATHEMATICS EDUCATION FOR THE GIFTED IN EGYPT**

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*This article provides a description of the status of the education of gifted students in Egypt in general and especially the education of mathematically gifted students. It explores some differences between world trends and the status of Egyptian educational policy for mathematically gifted students.*

## **INTRODUCTION**

Gifted students – who will be the creators and the leaders of the next generation – are the hope of Egypt in its advancement in keeping pace with civilization and progress. Consequently gifted education has been gaining interest and support by the Egyptian government. Thus it raised a banner of “learning for excellence and excellence for all”. Mathematics is the gate of progress of nations and can be the gatekeeper for many areas of advanced study, accordingly the interest in mathematics is more pressing than ever. Mathematically gifted students are distinguished from their non-gifted peers by their mathematical reasoning, their capacity for learning and their mathematical orientation. The Egyptian policy for mathematically gifted students’ education has some strong and weak features concerning definition, identification, appropriate curriculum instruction, assessment and teacher preparation, which I will outline in this article

## **HISTORICAL DEVELOPMENT OF GIFTED EDUCATION IN EGYPT**

The interest in the education of gifted students in Egypt began at the beginning of the 19<sup>th</sup> century when king “Mohammed Ali” sent gifted students to Europe to study modern sciences and acquire advanced experiences in different sciences. In 1932 the Minister of Education established some special classes joined with a teacher training institute which was transformed later into a model school. Teaching in that school was based on “project method”. After the 1952 revolution interest in gifted education increased. In 1962 a ministerial decision established the special school for gifted students in Ain Shams “The Model School for Gifted” which is considered the only centre of gifted education in Egypt. In recent years the Egyptian government allowed to some civil society institutions to establish centres for developing giftedness and creativity but there are few links between these institutions and mathematically gifted students in schools.

## **A COMPARISON BETWEEN EGYPT AND ELSEWHERE**

I now look at various aspects of gifted education I have gained from the literature and, for each aspect, comment on the situation in Egypt.

## **Definitions of gifted students**

There are many definitions for giftedness. Ford and Grantham (2003) state that “most definitions of giftedness are IQ based and test driven” (p.218). The national curriculum in the UK defines gifted learners as those who have abilities in one or more subjects on the statutory school curriculum other than art and design, music and PE while ‘talented’ learners are those who have abilities in art and design, music, PE. In contrast a ‘gifted and talented’ student is defined by the State of Texas as one who “shows the potential for performing or performs at a remarkably high level of accomplishment when compared to other children of the same age, experience, or environment, and who exhibits high performance capability in an intellectual, creative, or artistic area, possesses an unusual capacity for leadership, or excels in a specific academic field.” (Texas Education Agency, 1996, 29.121).

**In Egypt** giftedness is defined as blessing from the creator to a few pupils which enables them to excel and perform better than their peers in special academic fields

## **Identification of the mathematically gifted students**

Identifying gifted students is just the first step towards helping them achieve their full potential and early identification is essential for them. Smulny (2000) refers to the most effective way to recognize them and identifies giftedness as the use of a variety of approaches over an extended period of time. Schwartz (1997) suggests that schools can use the following methods in concert to ensure that all students receive a fair consideration: standardized tests such as intelligence, creativity and mathematics achievement and aptitude tests; observation and/or recommendations from teachers, parents and/or classmates; self identification, e.g. biographical inventories; portfolios as repositories of students’ potential or achievements. To avoid bias in the selection process, identification procedures should include a wide variety of measures to identify the broadest number of both females and males from diverse cultural and socioeconomic backgrounds. Sheffield (1999) suggests that measures might include any or all of the following: self-selection, observation of students during the problem-solving process; teacher, parent or peer recommendation; standardized tests; measures of creativity and/or problem solving; grades in mathematics classes; performance in mathematics contests; tests of abstract reasoning and measures of special reasoning.

**In Egypt** Identification of gifted students is based on the following conditions: students who get an overall score above 90% in the final exam of the end of the general preparatory certificate (GPC); psychometric test; creative thinking test (Ministry of Education, 1996). This identification strategy deals with both general giftedness and mathematical giftedness but the roles of teachers are not considered and neither are those of parents or peers.

## **Grouping – distribution and the gifted students**

A critical issue in gifted students’ education is their distribution or grouping. Some views about this issue include questions of heterogeneous, homogenous or cluster

groupings. Allen (1991) uses the terms comprehensive, between-class, within-class ability grouping and specific subject areas. Stepank (1999) says that research on schools with inclusive classrooms shows that differentiated instruction is essential and is necessary in order to ensure equal opportunities for all students. Allen (1991) refers to the meta-analysis conducted by Slavin (1990) and Kulik & Kulik (1992) which show that ability grouping has essentially no effect on student achievement across ability levels and explains how gifted students show positive academic gains from some forms of homogenous groupings. Cluster grouping refers to a group of five to eight identified gifted students which are clustered in the classroom of one teacher and the other students in that class are a mixed ability cluster.

**In Egypt** programmes adopt a homogenous model where students who get the first order in the final exam of every area are grouped into the “Model Gifted School” in Ain Shams and when the sum of students who get an overall score above 90% in the final exam at the end of GPC is above 18, then the school puts them in a special class which is called “gifted class”. The inclusive model is not applied In Egypt.

### **Learning environments for gifted students**

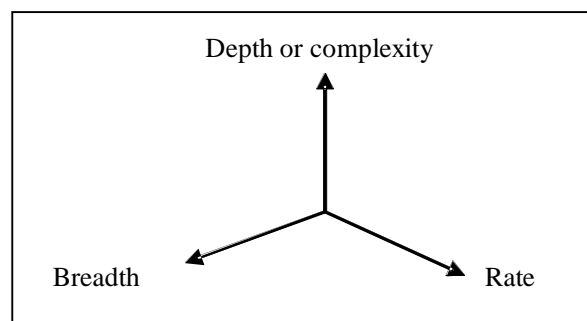
One of the first steps to consider when meeting the needs of gifted students is the classroom environment. According to Wheatley (1999) learning environments for gifted students should have the following characteristics: learner-centered rather than teacher or content-centred; independence rather than dependence emphasized; openness to new ideas, innovations and explorations; acceptance rather than judgement exercised; complexity rather than simplicity as a focus; varied groupings rather one grouping as a general organization; flexibility rather than rigid or chaotic structures; high rather than low mobility permitted and encouraged.

**In Egypt** there is a resource room, which contains computing facilities for all students. This can support gifted students learning but there is no special learning environment for mathematically gifted students.

### **An appropriate curriculum for the gifted**

Mathematically gifted students have needs that differ in nature from regular students. They require some differentiated instruction in the depth, breadth, pacing, levels of complexity of content and curriculum materials.

Sheffield (1999) states that the model for the mathematics curriculum must take into consideration the depth of mathematics that is being learnt and the need to look at into a curriculum that provides challenges along at least three dimensions of learning: breadth, depth and rate.



**In Egypt** the mathematics curriculum provided at the gifted school is not different from that of ordinary schools except that teacher notes for students contain some activities and problems which require a high level of thinking to solve. Traditionally

taught lessons are considered inappropriate. There is no specific mathematics curriculum for gifted students.

### **Teaching gifted students**

Teaching mathematics to gifted students differs from teaching it to regular students. Gifted students will respond to teaching strategies that focus on enquiry, problem solving and critical thinking.

**In Egypt** there is no guide for teaching mathematics to gifted students, in fact no guide exists for teaching mathematics to regular students.

### **Activities for the gifted**

Schools identify and use in- and out-of-school extracurricular activities to challenge and motivate gifted students. Activities should have clear goals and should aim to increase pupils' ability to analyse and solve problems, stimulate originality and encourage initiative and self-direction. The most important activities for gifted are considered below.

Enrichment activities that supplement the curriculum, are generally not specified in it, and are selected by the teacher and/or students in a given classroom. Enrichment opportunities for the gifted should involve students in interaction with new ideas and topics. Enrichment activities include elements that (1) broaden gifted students' understanding mathematics by introducing innovative topics that go beyond the standard curriculum and (2) encourage meaningful insights and offer opportunities for gifted students to become a part of mathematics community (Keynes et al 1999)

**In Egypt** school timetables include a weekly lesson known as an activity lesson in which students choose the type of activity (maths-science-art-computers.....) they want to do. This slot can be utilized for enrichment activities for gifted students. Enrichment activities commonly associated with teaching gifted students are workshops and speakers' tours. Summer schools do not exist.

Acceleration refers to moving students up to a level/grade of study that matches his/her aptitude and mastery level in one or more curricular areas. Accelerated progression is viewed positively since it is a readily available educational alternative which can provide a challenging and satisfying environment without disadvantaging the student educationally, emotionally or socially (Merrottsy, 2003).

**In Egypt** schooling is divided into three stages and twelve school grades (primary, 6 years; preparatory, 3 years; secondary, 3 years). The Egyptian educational system supports the use of acceleration (grade skipping). Accelerated progression is possible at the end of the primary stage by sitting sixth grade exams a year early.

Mathematics competitions, whose purpose is to increase interest in mathematics and to develop problem solving ability through friendly competition.

**In Egypt** the Ministry of Education provides opportunities for gifted students to participate in competitions. In secondary schools the competition covers all subjects

(not just mathematics). Schools compete as teams in solving challenging problems. Egypt has participated in TIMSS since 1999. There is competition in primary schools conducted through cooperation between the Ministry of Education and Nile Educational TV.

### **Teacher preparation for the gifted**

The teacher is the cornerstone in the educational system but there appears to be a lack in training teachers to deal with gifted students. Arguably all teachers who work with gifted students should have formal training. Such training should focus on testing and assessment, instructional strategies and models, social emotional needs and development and working with families (Ford & Grantham, 2003).

**In Egypt**, although teacher training courses now provide specialist preparation for teaching mathematics to gifted students, graduates of these courses have, in general, not been able to find work in teaching. In-service teachers continue to receive training in teaching strategies for how to help students solve questions in TIMSS competition but their experience and training regarding mathematically gifted students is lacking.

### **Assessing the gifted student**

Assessment is necessary for gifted students because it provides them with an agenda for progress and enables teachers to plan future work to meet individual needs and to set appropriate targets. Gifted students benefit from a mixture of teacher and peer assessments.

**In Egypt** gifted students sit the same examinations as other students except in the first grade of secondary stage where they have an additional question in the final examination. This is usually marked for gifted students and does not count towards the final examination result.

## **CONCLUSION**

The Egyptian educational program for gifted students has some strong features but it is seriously lacking in many aspects. On the strengths side there is a resurgent interest in preservice teacher preparation that deals with gifted students. Moreover academic research into giftedness has received considerable impetus in recent years. The adaptation of acceleration and enrichment is a noteworthy feature in the educational program as well as the encouragement to take part in national and international competitions.

With regard to weaknesses, gifted students are not systematically identified because no communication channel exists between parents, teachers and counsellors. Moreover no guide for meeting the needs of mathematically gifted student is available. Finally, there is very little link between schools and universities regarding the education of gifted students and the overall educational programme for gifted student is still in need of development, evaluation and refinement with most

mathematics teachers having limited knowledge of resources and limited time to provide challenging experiences for mathematically gifted students.

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# PAIRED INTERVIEWS IN MATHEMATICS EDUCATION

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*We consider the advantages and disadvantages of carrying out interviews with pairs of children. Although the Mathematics Education literature contains examples of this method, there is relatively little detailed discussion of the rationale for its use, nor of its consequences. We draw on examples from the literature and from our own task-based interviews with pairs of ten and eleven year-olds. We develop a simple typology of this type of interview, and we propose that children respond differently in the three different contexts. Researchers therefore need to differentiate carefully between them, and to consider their findings in the light of the exact type of paired interview used.*

## INTRODUCTION

In this paper we discuss issues associated with the use of paired interviews in mathematics education. Our interest arises from use of interviews with pairs of children in the course of our own research. This raised our awareness of some of the advantages and disadvantages associated with this method and posed more fundamental questions about the nature and purpose of paired interviews.

Our decision to interview children arose following a project in which we analysed the answers given by ten and eleven year-olds to written questions which we felt offered potential for algebraic thinking. In common with others (eg. Watson 1999), we felt that we would learn more about children's responses by interviewing them, and we chose to conduct interviews both on questions the children had previously completed in writing and on a question they had not seen before. We hoped that the interviews would enable us to explore possible misconceptions as well as seeing whether children could extend the given question. The children were interviewed in pairs in the hope that this would provide opportunities for interaction and discussion as well as putting the children at ease. We also had in mind that the balance of interviewees to interviewer would be better and that there are advantages in outside researchers using this method given current child protection concerns.

## METHODS: BACKGROUND

### Individual, group and paired interviews

For the purpose of this article we take paired interviews to mean that there are two interviewees and one interviewer. This is also called joint interviewing, for example by Arksey and Knight (1999) who advocate this method partly because they feel it is easier to establish an atmosphere of confidence with two interviewees and also because the interviewees may 'fill in gaps' for each other and their interaction may also be of interest. In general, discussion of paired interviews is sparse in the literature, much of which assumes that interviews are normally conducted one-to-one

(e.g. Kvale 1996). There is more discussion, however, of group interviews where several interviewees are involved. For example, Lewis (1992), in a detailed consideration of group interviews with children, suggests they may be useful for exploring consensus views and may generate richer responses by allowing participants to challenge one another's views. It has also been pointed out (Watts and Ebbutt 1987) that group interviewing changes the interviewer's role, which may become more that of a facilitator or moderator. The possible domination of individuals is considered to be a possible disadvantage of this method (Breakwell 1990) and it is also acknowledged that the method raises practical issues such as difficulties with transcription (Ebbutt 1987). Our position in this paper will be that although paired interviews may carry some of the advantages and disadvantages of group interviews, they are essentially different from both individual and group interviews.

### **Interviews in mathematics education**

Many of the writers mentioned above are concerned with interviews in the field of education, but they are not writing specifically with mathematics education in mind. Interviews in mathematics education can differ from more general educational interviews. We consider there to be two broad types of interview in mathematics education. The first of these, the 'affect-related' interview, concerns the attitudes, approaches and beliefs of interviewees in relation to mathematics. Usually such interviews are discussion-based, though occasionally interviewees are asked to do something such as draw a mathematician or describe an ideal mathematics lesson. The second type of interview is 'task-based' where the focus is on carrying out a mathematical task or tasks. Such interviews vary in type, as discussed by Ginsburg et al (1983). One method known as 'talking aloud' (Newell and Simon 1972), involves a description of the technique used with minimum intervention from the researcher. In contrast, clinical interviewing involves flexible intervention by the researcher. It is acknowledged that it is possible to use a mixture of talking aloud and clinical interviewing.

### **Related research methods in mathematics education**

Also relevant is work which involves two or more students working together on a mathematical problem. Some studies either interview students while they carry out a mathematical task together or question them about a task they have previously completed. Although this research is strictly speaking not paired interviewing, there are clear similarities. Much of this work stems from Schoenfeld (1985) on mathematical problem solving, which aims to capture part of the problem-solving process by listening to the reasons that students give to each other. Categories proposed by Schoenfeld for analysing the problem-solving process have since been used and adapted in more recent work analysing problem-solving amongst pairs and small groups (Artz and Armour-Thomas 1992, Goos, Galbraith and Renshaw 2002).

## **EXAMPLES FROM THE LITERATURE**

Despite the scarcity of articles discussing paired interviews as a research method, they are not uncommon in mathematics education research. Accounts of such interviews vary in the detail of justification given for working with pairs of students and the information about how the pairs were selected. Sometimes such information is completely omitted.

Some paired interviews are affect-related, for example interviews by Boaler (1997) with pairs of secondary school students about their experiences of learning mathematics. Because our interest is more in task-based interviews, we will look in a little more detail at some examples of paired task-based interviews. Bills, Ainley and Wilson (2003) conducted interviews with 12 year-old pupils paired by their teacher to make compatible pairs of similar attainment. The pupils were given questions in written form at the start of the interview and questions were also read aloud to them. In a study with pairs of eleven to fourteen year-old students, Cooper (2003) gave them a problem in the interview which they had previously solved and asked them to read it aloud and explain to each other how to solve it. The technique of interviewing pupils about a task they had previously solved was also used by Fujii (2003) who started his study with a written task given to junior high school pupils. As a result of students' responses to the task, pairs were selected for interview so that pairs of students held 'inconsistent conceptions'. This was done in order to create a conflict in the interview context in the hope that students would express their ideas explicitly to each other.

The examples above suggest that task-based interviews can be carried out in two ways, which we will call 'seen' and 'unseen'. Seen interviews involve asking children to discuss the solution of a task they had worked on previously. In unseen task-based interviews, the task is presented for the first time during the interview.

## **OUR RESEARCH**

Our interviews were carried out with nineteen pairs of ten to eleven year-old children who had previously completed a short written paper. The children were drawn from three schools and pairings were selected in a variety of ways which included careful choice by teachers based on their knowledge of the children as well as friendship pairs chosen by children and pragmatic decisions based on who was available.

The interviews started with invitations to them to discuss questions on the paper and they had their written answers with them to assist with this, though they were not told whether the answers were correct. One question in particular gave rise to discussion by the majority of pairs of children. This involved a sum and difference problem which we knew from our previous research on written answers often gave rise to an incorrect answer (Evens and Houssart 2004). Although we had not intended to pair children on the basis of their written answer, it happened that many pairs included one child who had answered the sum and difference question correctly and one who had answered incorrectly. This raises the issue of comparison with the work of Fujii

(2003), who aimed to set up cognitive conflict in paired interviews by pairing children with different answers. In our interviews, it was comparatively rare for children to change their incorrect answer as a result of being with a child who explained the correct answer and an appropriate method. Some were more inclined to try to justify their own approach rather than consider that of their partner and the justification was sometimes based on non-mathematical reasons. A related issue is the degree of interaction between pairs of children and, although some pairs interacted and discussed their work with each other, some did not really do so. For some children, the seen part of paired interviews consisted mainly of turn-taking where one child explained their answer to the interviewer and this was then followed by the other child giving an explanation that was not always influenced by what they had just heard. We plan to carry out further analysis of this part of our work and one area of interest would be to consider which factors encourage children to engage with each other's ideas and possibly reconsider their own answers.

The second part of the interview was unseen in that children were shown a question they had not previously worked on. This question concerned what happened as a result of a game involving spinners. Although we had previously given this question as part of a written test (Houssart and Evens 2005) we considered it particularly suitable as an interview question as it included potential for making general statements and justifying answers. Part of the question was open, allowing a range of answers and we hoped that the interviews would help us explore whether children were aware of the range of options available. Our preliminary analysis of the data indicates that this part of the interview also resulted in differing levels of interaction within the pairs. Some children started by taking turns to explain the first part of their answer to the interviewer. Notable here was the fact that in some pairs, even though they abided by this turn-taking, the second child to speak made reference to the first answer, for example by saying in which way their work was similar. In some pairs, however, interaction between the two children started very early on in the interview. In one case, this occurred because one boy had misread the question, leading to a discussion between him and his partner that resulted in the mistake being acknowledged and corrected. For some pairs of children, this part of the interview was also used to make and explore suggestions and to extend the original question. There was also clarification of misunderstandings, often between the pairs of children. It is possible that children were more willing to consider and even change their answer in this unseen part of the interview than in the seen part, where some of them seemed to feel the need to defend their previous written answer.

One of the initial reasons for carrying out the interviews was the hope that it would tell us more about how the children approached the questions. This was usually the case, especially in the second part of the interviews. However, there are clear dangers in saying that the interviews simply tell us more about the answers of the individual, as this ignores the presence of their interview partner. This is just part of the problem about how to proceed which we were faced with, when moving on to more detailed analysis of the interviews. It no longer seemed appropriate to classify each child's

answer individually, as the interaction between children now assumed more importance. For the seen interviews, we were also interested in whether or not children were prepared to change their mind during the interview and this offered another possible classification. For the unseen interviews, a possible way to proceed was to treat the interview as a paired problem-solving situation and hence use protocols developed for that purpose. Overall, the interviews challenged our original, somewhat naïve assumption that they would give us more information about what individuals ‘could do.’

## **SUMMARY AND FURTHER QUESTIONS**

Our research and brief examination of the literature have raised many questions about paired interviews in mathematics education. We have tentatively identified three types of paired interview: affect-based, those based on seen tasks and finally unseen tasks. The first category can be seen as having some similarity with other interviews in educational research. The final category is perhaps more closely related to paired or group problem-solving. The middle category is particularly interesting and our experience suggests that for some pairs they were closer to affect-related interviews while for others they were closer to task-based. Children appear to respond differently according to the type of interview used. The method that has been used in any given research needs to be specified and the results considered accordingly.

There are many other questions about paired interviews, for example how the pairings might be chosen, what the role of the interviewer is and what types of question are particularly suited to this method. Ways of carrying out analysis were a further problem, and an underlying question is what the interviews are actually telling us. Our data suggests that, inter alia, interviews offer complex information on children’s mathematics, their interaction and their confidence. A key question is whether it is possible, or even desirable to try to separate these factors.

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# **‘MATHS IN MY WAY’: CARIBBEAN STUDENTS’ PERSPECTIVES ON THE SOCIAL ROLE OF MATHEMATICS**

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*This paper looks at social views of mathematics expressed by Caribbean students. The students appear to say in their own words points made by Gates & Vistro-Yu’s (2003) in their work ‘Is Mathematics for All?’ Encapsulated in the students’ views are issues to do with gender, ability grouping practices and social class, issues made possible in part by the ways in which ‘school’ becomes enacted, and how mathematics is done in those schools. In seeking understandings of the students’ views, questions are raised such as who is mathematics for, what type of mathematics is for whom, and for which group(s) of students does mathematics get in the way.*

## **INTRODUCTION, BACKGROUND AND CONTEXT**

This paper is based on a smaller aspect of a PhD study. That study examined Caribbean students’ views of mathematics, how they may have come to have those views, and the influence of those views on their mathematics attainment. This paper will look specifically at an aspect of the first of these, students’ views of the social role of mathematics, with some insights into how they may have come to those views. The rationale for the PhD study was born out of concerns within the Caribbean that students were ‘underachieving’ in mathematics. Assessments of ‘underachievement’ were usually based on students’ attainment in mathematics in the Caribbean Examinations Council (CXC) Secondary Education Certification (CSEC) examinations, taken by students in most of the English-speaking Caribbean countries at the end of secondary schooling. These examinations replaced the former British-based GCE O’ level examinations in the early 1980s. A case study of secondary school students’ views of mathematics in two of these countries, Antigua & Barbuda (A&B) and St. Kitts-Nevis was conducted in order to gain insights into possible issues that may be influencing student achievement in mathematics.

At this point more will be said about the CSEC examinations, as in excerpts to be presented students make reference to the structure of these examinations. CXC offers two levels which it calls proficiencies at which the CSEC examinations can be written in some subject areas. These levels are called the Basic and General Proficiencies. Whilst the content of the Basic Proficiency is a proper subset of the General Proficiency, the grading/assessment of the examinations is not, and they are treated as two separate examinations. A candidate is deemed to be successful as defined by CXC at both proficiency levels if they receive Grades I-III on a 6-point grading scale. However, society has essentially defined success otherwise, in that in order for students to gain access to tertiary education and also the desired jobs upon leaving school they would have had to be successful at the General Proficiency of the examinations. This has particular implications in mathematics as more students sit the Basic Proficiency examinations in this subject than in any other.

Gates & Vistro-Yu (2003, p49) in their work *Is Mathematics for All?* noted the following about a social ‘organising’ role of mathematics in part in reference to ability grouping practices in this subject area:

...*school mathematics* plays a significant role in organising the segregation of our society... keeping the powerless in their place and the strong in positions of power... an accusation that you ‘can’t do maths’... is a positioning strategy... It locates you as unsuccessful and lacking in intellectual capability; it locates you on the edge of the employment and labour market... Mathematics education thus serves as a “badge of eligibility for the privileges of society” (p49, emphasis in the original)

When compared, it could be considered that Caribbean students expressed, in their own words, similar views of mathematics. As such, this paper will report on and discuss five views students expressed that may be considered ‘social’. These views see mathematics as (1) gendered, (2) important, (3) according respect, (4) a means of protection, and (5) in the way.

## **METHODOLOGY**

The over-all PhD study was conceived as a multi-site case study employing a mixed methods approach (Creswell, 2003) to data collection and analysis. As with Hammersley (1992, p172), the choice of this methodological approach was based on ‘the purposes and circumstances of the research, rather than being derived from methodological or philosophical commitments’. In this paper I will present findings primarily from student interview with some results of questionnaire survey data. Fourth form students (Year 10 in a British context) from 12 schools in A&B and St. Kitts-Nevis were the study’s main participants. Student group interviews were conducted in both territories, but questionnaire data were obtained from students in A&B only. Data were collected over the period September 2004-January 2005. The questionnaire sample of students in A&B (286 students, approximately  $\frac{1}{4}$  of the all 4<sup>th</sup> form students in A&B) had a mean age of 15 $\frac{3}{4}$  years.

## **FINDINGS AND DISCUSSION: WHAT THE STUDENTS HAD TO SAY**

Student views of the social role of mathematics will be discussed with reference to five perspectives. Excerpts from group interviews will be given, followed by interpretations.

1. *As gendered.* Some students’ responses in interviews suggested this view, rather than its being explicitly stated. Such may be interpreted in the following excerpt:

B2: My mother would say that ‘am, you used to be good at maths in school when you were small, how come you can’t do it now, [...] what kind of man...

B1: My mother would say, right, she keep saying she ain’t good on maths, ‘cause she always used to fail it at high school and primary school right, but yet she would still tell me that she’ll be doing good, right, and when I ask her for help she can’t help me. My father good on maths but I don’t really see him ...

G2: Well, it depends. If I do good in maths my mother ain't goin' say nothing but if I do bad she going, she going say. (2b+2g) [1]

The excerpt points to a source of these students gendered view of mathematics. In the excerpt, B2 alluded to the way in which his mother (and arguably society) viewed who it is who *ought* to be good at mathematics, when he recounted an exchange with his mother due to his falling grades in mathematics. Although he does not continue the line of response, his mother's comment of 'what kind of man...' signals that there was an implicitly held view, perhaps even belief, that a characteristic of a *man* was that he could do (well in) mathematics. B1's response also reflects gendered views about mathematics; his mother was not good at mathematics and had said so, nor could she help him with his mathematics schoolwork. B1 did not say whether his father had described himself as good in mathematics, but B1 did seem to *know* that his father was good at mathematics even though he was not able to get help with mathematics from his father. The implication is that all *men* should be able to do mathematics. The students did not suggest that women could not be good at mathematics, but this, presumably, was not an inherent characteristic of a woman. None of these students (nor most of those interviewed) espoused any *explicit* views of mathematics as gendered. However it is arguably the case that messages students were getting in part from parents about who is or ought to be good in mathematics become embodied into dispositions that may unwittingly influence students in their learning of mathematics at school, these messages becoming part of the habitus (Bourdieu, 1990, p77) they bring with them to school.

2. *As important.* This view had to do with mathematics as important for *getting* a 'good job', less so for mathematics being particularly useful *in* the job. In the students' views, a qualification in mathematics guaranteed a person a 'good job', as suggested in the following excerpts:

G1: Anybody who has a good brain in maths, it's like, they know they're gonna get a good job and, you know... (3g)

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B1: Cause when you go college, you need it. [...]

Int: Where.. why do you need it?

B1: You need it... 'am, without maths...

G1: Maths is in everything.

B1: ... you can't get jobs.

B2: You need maths certification to get a good job in the future. (2b+2g)

This student view of mathematics could be compared to Gates & Vistro-Yu's assessment of being unsuccessful in mathematics bringing a person to 'the edge of employment'. This does not discount that students also saw mathematics as being useful in everyday life, and as important for getting into higher education (HE). However, related to the view of mathematics as important was also that of

mathematics as *necessary* because of where it could take you, the access to places and spaces it provided, and much less so for any intrinsic worth of its own. Students had bought into the view that success in mathematics (i.e. a qualification therein) was related to the quality of life a person could have after leaving school.

3. *As according respect.* The two interview excerpts following came from students in separate single-sex schools. In this country, single-sex schools are an indication of parental socio-economic status, having proportionately more students of middle-class parentage than students in mixed schools. There was a practice in the two girls' single-sex schools of 'ability grouping' students at the beginning of the 4<sup>th</sup> form for mathematics teaching based on the levels of the CSEC examinations (i.e. General or Basic), a practice which did not occur in any other subject area. Then, approximately two years later, the girls would be entered for the mathematics examinations based on these levels. These 'practices' did not occur in the two boys' single-sex schools, and boys at the school leaving stage sat only the General proficiency examinations.

B2: When they [employers] look at your report or look at maths they see it's good, respect, they respect you.' (3b)

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G3: 'Am, also, the students that do General maths, they look down on us.

G2: Not all.

G3: Not all of them, but some of them. Some of them was with us, and just went over.

G1: I find that the Antiguan society is like, if you're a maid, it's like people don't care nothing, if you're a doctor, people will more see, respect you... (3g)

The boy's comment in the first excerpt here could be seen as an example of mathematics serving as a "badge of eligibility". It is also instructive that the comment comes from a *boy* in essentially a *middle-class* school. This is not to say that students in other schools did not have similar views of mathematics. However, the nature of this boy's comment, which was agreed to by the other boys in this interview, suggested that for him mathematics was his insurance in the world of work outside school, and there was no sense of mathematics being 'in his way'. This contrasts with the perspective of the girls in the single-sex school given in the second excerpt here. These girls had been grouped into a Basic proficiency class for mathematics. For these girls, mathematics had become a positioning device. The analogy G1 used in a discussion primarily surrounding those of her classmates who were now in a General mathematics class is notable. This discussion of General/Basic mathematics and the social consequences of the division as was already being played out in the school – was arguably a 'class' division even in this single-sex (i.e. middle-class) school, delineating the 'intellectual capability' of students, being made possible by how the school chose to 'do' mathematics. As this practice occurred only in mathematics, mathematics thus becomes a resource of power, and these girls come to 'realise their conditions within a system of class division *through their mathematics education experience*' (Valero, 2003, p14, my

emphasis). Thus it seems that at least in relation to mathematics, students who would otherwise be seen as advantaged because of their social class, become disadvantaged because of their gender.

4. *As protection.* This view of mathematics is connected to students' views of mathematics as important. Much of the locus of this view surrounded students' impression of mathematics as keeping them safe, and further, preventing them from being susceptible to the wiles of other perhaps more powerful people. The following are the views of the girls in the single-sex school (from no. 3 previous), and that of another girl in a mixed school.

Int: So the disadvantage to doing...

G1: Basic maths, you're not sure about the future.

G3: Exactly.

G2: I do not believe that.

G1: That is my fear. I am not sure about the future because I'm... bare. (3g)

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G1: Well, yeah, I don't like maths, but [...] the disadvantage [to not doing well in mathematics] is [...] me having my own business, I won't be able to count and knowing how to do this and that to get, so people can rip me from my money and so on, so I would make, instead of making a profit, I would make a loss... (2g)

In the first excerpt here the girls spoke as if they had already failed mathematics, although the interview took place approximately 1½ years prior to their actually taking the CSEC examinations. Basic mathematics was equated with being unsuccessful in mathematics, and, from the reference to being 'bare' to no mathematics at all. In the second excerpt, the student, a girl in a mixed school, expressed a view of mathematics which was more specific and also more common amongst students in mixed schools, that specifically of mathematics offering protection from being robbed of money. The view expressed by the girls in the single-sex school was a more generalised view of mathematics offering protection in an after-school life. In both these views however, the students appear to see mathematics as (a part of the) armour they will need outside of school. Again, there is a sense here of the power that resides in mathematics, and of access to this mathematics providing a share in that power. The essence of these students' statements compares to Gates & Vistro-Yu's assessment of mathematics as 'keeping the powerless in their place and the strong in positions of power'.

5. *As in the way.* The following excerpt is a continuation of the perspective of the three girls given in nos. 3 and 4 previously.

G3: And I want to be an accountant and I see Basic maths in my way.

G1: Exactly. (3g)

There is an inherent tension here; whilst these girls may not have had 'good' mathematics grades prior to the ability grouping process, mathematics had now

effectively been placed in their way, blocking access to hoped for career-paths on leaving school.

Although said in relation to the Basic proficiency of the CSEC mathematics, the girl's comment in the last excerpt has wider meaning. Society has given mathematics, and in the Caribbean, the General Proficiency mathematics, an importance that becomes by its absence an obstacle in the pathway to what some persons hoped to be. Other data from the case study in A&B do suggest certain types of persons for whom mathematics may more consistently get in the way. One such type comes from evidence of out-of-school persons taking the CSEC mathematics, where it is for females more so than males that mathematics appears to be 'in the way', as more females write the examination than males, and further they do so in proportions higher than the in-school male-female ratio. A second person-type for whom mathematics becomes an obstacle is students in mixed schools, i.e. those from working class backgrounds, this based on the results of the proportions of students who are successful in mathematics. For example, the 2006 CSEC results of the study's questionnaire sample of students in A&B showed that whereas 53% [2] of the students in mixed schools had been successful in five or more General Proficiency subjects, with 61% of them being successful in English, only 23% of them had been successful in mathematics. Similar results for students in single-sex schools were 90%, 90% and 86% respectively. Thus, just as mathematics may empower persons who are successful in it, it perhaps also more poignantly dis-empowers persons who have not enjoyed such success.

## NOTES

1. This (2b+2g) refers to the make-up of the interviewed group of students.
2. Percentages are given as a proportion of the no. of students sitting that proficiency of the examinations

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# ARITHMETICAL NOTATING AS A DIAGRAMMATIC ACTIVITY

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*Qualitative data is presented from the trialling of a software-based arithmetical notating task designed to foster engagement with the structure of equality statements. The design rationale is “diagrammatic activity” (Dörfler, 2006) where arithmetic inscriptions onscreen are observed and manipulated according to operational rules. The data suggest that the children’s readings of arithmetical notation were transformed from computation to pattern awareness and substitution making. This afforded the emergence of commutative and partitional meaning making for  $a + b = b + a$  and  $c = a + b$  syntaxes respectively.*

It is widely reported in the literature that children attend more readily to computational than structural readings of arithmetical equivalence statements (Behr et al., 1976; Jones, 2006; Kieran, 1981; Molina, 2006). These computational readings are deeply entrenched and can impact negatively when formal algebraic notation is encountered at the start of secondary schooling (McNeil & Alibali, 2005). In this paper I report on a trial of an arithmetical notating task designed to foster meaningful engagement with the partitional and commutative properties of equality statements.

## TASK DESIGN

The design rationale for the task is Dörfler’s (2006) interpretation of Charles Sanders Peirce’s “diagrammatic reasoning” as applied to mathematics learning. According to Dörfler’s view, the numerals, operator signs and other inscriptions that comprise arithmetic and algebra can be considered as the objects of mathematical activity, rather than as representational referents to abstract mathematical objects. An inscription can be considered a “diagram”, in the Peircean sense, when it can be observed and manipulated according operation rules. For example, the inscription  $30 + 41$  can be transformed into  $70 + 1$  by substituting 41 with  $40 + 1$  and substituting  $30 + 40$  with 70. The power of carefully designed diagrammatic tasks is that learning mathematics becomes an empirical activity involving making discoveries through experimentation with physical inscriptions on a page or computer screen.

The remainder of this section describes a computer program and task designed to afford learners diagrammatic engagement with arithmetical notation<sup>1</sup>. The software, called *Sum Puzzles*, was set up to present a sequence of 11 puzzles. Each puzzle presented a term, or “sum” lacking an equals sign, such as  $8 + 7 + 1$ , and quality statements that can be viewed as tools for acting on the sum (see Figure 1).

The software supports two basic functionalities: selecting equality statements and substituting terms. A statement is selected by clicking on the equals sign which

causes the statement to be highlighted ( $8 + 7 = 15$  has been selected in Figure 1). In this sense

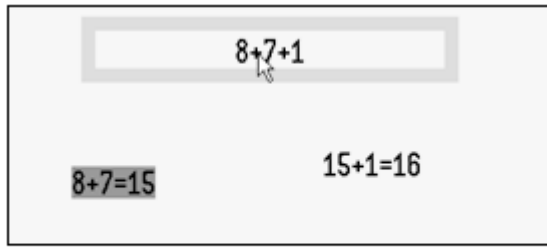


Figure 1

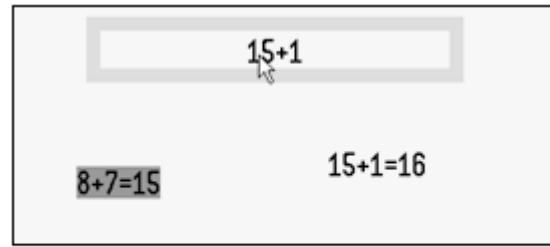


Figure 2

the inscription = might be thought to act as a handle for taking hold of equality statements. A term is substituted by clicking on it, and the substitution (if any) is determined by the currently highlighted equality statement. For example, clicking the inscription  $8 + 7$  in the sum in Figure 1 transforms it into 15, as shown in Figure 2. Substitutions are reversible. If the inscription 15 in the sum in Figure 2 were to be clicked again whilst the statement  $8 + 7 = 15$  is still selected the situation would return to that shown in Figure 1. Substitutions can also be made on terms within equality statements. Given the highlighted statement  $8 + 7 = 15$ , if the inscription 15 in the statement  $15 + 1 = 16$  were clicked it would become  $8 + 7 + 1 = 16$ . The observational process of finding like terms in order to make substitutions (“iconic matching”) can be considered as a potentialised diagrammatic activity embedded within the software. It was conjectured that the dual activities of selecting statements and clicking terms would transform children’s reading of notation from that of computation to noticing iconic matches and experimenting with substitutions. This in turn, it was conjectured, would afford them opportunities to construct commutative and partitional readings of  $a + b = b + a$  and  $c = a + b$  statements respectively.

## METHODS

The trial reported here involved a pair of Year 5 boys, T and A, deemed mathematically able by their class teacher, working together with the software and task for a period of about 40 minutes. My role as researcher was to show children how to operate the software and to set challenges. Other than this I occasionally prompted for verbal elaborations (as in “Why do you think that didn’t work?”) and offered encouragement and praise. Data was captured as screen-movies of the children’s interactions along with an audio track of their discussion. This data was transcribed using the qualitative analysis software package *Transana* and evidence for non-computational readings of equality statements sought.

## DATA AND DISCUSSION

Engagement was high during the trial, as illustrated by the boys’ responses afterwards:

1. A: It's quite challenging [T: Yeah] It's kind of addictive in a way. You don't want to stop. [T: Yeah] Kind of like, really determined.
2. T: Bit fiddly. And it can sometimes get really annoying but it's still fun.

During the first two puzzles, which contain only compositional statements, the boys' readings were computational, as would be expected from a reading of the literature:

3. T: I think it might be this one. Yeah, [A: Yeah] 'cause 7 add 1 equals 8, 8 add 8 is 16. There... then if you click on... click on the equals...
4. A: 7 add 1 is...
5. T: Yeah. Then, like
6. A: Yeah, 'cause 7 plus 7 is 14, [T: And then...] isn't it?

Early evidence for a shift towards iconic readings occurred during Puzzle 3 (see figure 3), which contains the first commutative statement. T highlighted  $1 + 9 = 10$  and attempted to make a substitution on the term  $9 + 1$  (R is researcher):

7. R: Why do you think that wasn't working?
8. T: [highlights  $1 + 9 = 9 + 1$ ] Maybe because... 1 and 9 is...
9. A: Oh, because it hasn't got that sum in it.
10. R: What do you mean?
11. A: Well 'cause that's [ $1 + 9 = 10$ ] got 1 add 9 but then the end of that's [ $1 + 9 = 9 + 1$ ] got 9 add 1.
12. R: Okay, I see.
13. T: [clicks on the sum] There. 1 add 9.
14. A: Now try that one T, and...
15. T: Yeah.
16. A: Yes!

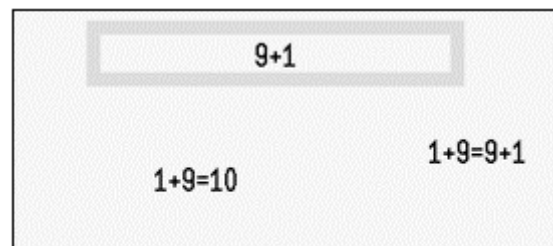


Figure 3

Following this both T and A came to use the verb “swap” when discussing and highlighting commutative statements throughout the trial. When they came to Puzzle 6 (Figure 4), which contains the first partitional statement, the boys initially used a computational reading to correctly eliminate the final statement ( $70 + 1 = 71$ ) in the puzzle solving sequence from their choice of three statements (lines 17 to 24). Then T had an insight: despite not having encountered or used a partitional statement previously in the software, he inferred a “splitting” reading of  $41 = 40 + 1$  (lines 25 to 30).

17. T: Er... 30 add 41. 30 add 40 equals 70.
18. A: Nah, maybe it's that one. Nah, it's that one. I think it's, maybe... that one looks like the closest to it. That one goes with that one, because that one's the sum you do at the end isn't it?
19. R: Which one's the sum you do at the end?
20. T: That one [indicates  $70 + 1 = 71$ ].
21. A: Because 70 plus 1 because you get seven- that must be the same.

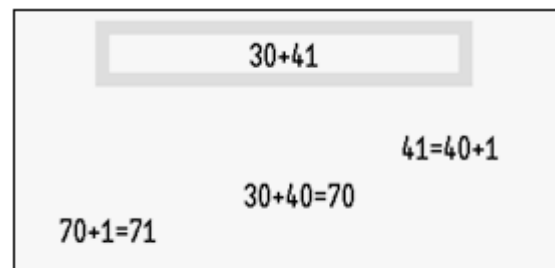


Figure 4

22. R: How do you know that's the one you do at the end?  
 23. A: Well it's already got half the answer. Because 30 add 40...  
 24. R: Oh I see, yeah.  
 25. T: Oh! That's [  $41 = 40 + 1$  ] the one that you do first! That has to be.  
 26. R: Why?  
 27. T: Because it's splitting up the 40 and the 1.  
 28. R: Ok.  
 29. T: So maybe if you... click on there now. Click on 41. Try that. Yeah.  
 30. A: Yes.

Over the duration of the trial the boys came to use the verbs “swap” and “split” when using commutative and partitional statements and developed an effective strategy for making sense of increasingly complicated puzzles. This involved identifying partitional statements in order to break sums up into constituent parts, then swapping and composing these parts into a final number. The following excerpt demonstrates the power of this approach, when T efficiently solved Puzzle 10 (Figure 5), which contains eight statements, and is significantly more complicated than any of Puzzles 1 to 9, which contain a maximum of five statements. Note, however, that despite an emerging grasp of the diagrammatic operation rules, T still repeatedly attempted to make iconically mismatched substitutions using  $5 + 7 = 7 + 5$ .

31. T: Okay! 65 add 87, any spli... 8... Yeah. That one. That splits it up. [highlights  $87 = 80 + 7$  and clicks 87 in  $65 + 87$ ; highlights  $65 = 5 + 60$  and clicks 65 in  $65 + 80 + 7$ ] Erm... 50... 5 add 60... 80

32. A: Yeah.

33. T: [highlights  $80 + 60 = 140$  and clicks centre of  $5 + 60 + 80 + 7$ ] No. Oh you need to move 'em, you need to swap 'em round.

34. A: Hm.

35. T: [highlights  $7 + 5 = 12$  and clicks 5 then 7 in  $5 + 60 + 80 + 7$ ] No. Those need to be swapped round as well. ... There. [highlights  $5 + 7 = 7 + 5$  and clicks 5 then 7 in  $5 + 60 + 80 + 7$ ] Swap round!

36. R: Why wouldn't it swap them round?

37. T: Because there's something in between?

38. R: Okay.

39. T: There that one. [highlights  $80 + 60 = 60 + 80$  and clicks centre of  $5 + 60 + 80 + 7$ ] There. Now you can do that one. [highlights  $80 + 60 = 140$  and clicks centre of  $5 + 80 + 60 + 7$ ] 140. Hopefully that will do it now. [highlights  $5 + 7 = 7 + 5$  and clicks 5 then 7 in  $5 + 140 + 7$ ] No. [pause] Ah! [highlights  $140 + 5 = 5 + 140$  and clicks 140 in  $5 + 140 + 7$ ] That swapped it round. ... There. [transforms  $140 + 5 + 7$  into 152 using  $5 + 7 = 7 + 5$ , then  $7 + 5 = 12$ , then  $140 + 12 = 152$ ] And... Yes!

|              |              |               |
|--------------|--------------|---------------|
| <b>65+87</b> |              |               |
| $87=80+7$    | $7+5=12$     | $5+7=7+5$     |
| $65=5+60$    | $80+60=140$  | $140+5=5+140$ |
|              | $140+12=152$ | $80+60=60+80$ |

Figure 5

Following this success, I reset Puzzle 10 and posed an alternative challenge: modify the puzzle such that the sum  $65 + 87$  can be changed into its answer in a single click. In the case of Puzzle 10, this involves transforming the statement  $140 + 12 = 152$  into  $65 + 87 = 152$ , and then using it to substitute the sum for 152 in a single click (the boys were not told this explicitly). This subtle change in task goal, without alteration to the diagram or its underlying operational rules, impacted significantly on the boys puzzle solving strategies (compare the T's confident competence in lines 31 to 39 with the uncertainty in lines 40 to 52). Initially A was drawn to compositional statements (line 40). When T initiated their emerging and successful strategy of starting with highlighting a partitional statement (line 43) they appeared unsure of where exactly to make their substitution, and tried iconic mismatches:

40. A: So, you usually use these things [indicates  $7 + 5 = 12$ ].  
41. T: Well that's [indicates  $87 = 80 + 7$ ] what we started with wasn't it?  
42. A: Hm.  
43. T: So, erm, select that maybe. [A highlights  $87 = 80 + 7$ ] Now click on...  
44. A: Maybe 60 or... [indicates  $65 = 5 + 60$ ]  
45. T: Does 87 use... 8... 80.  
46. A: [clicks 80 in  $80 + 60 = 140$ ] No it doesn't do anything. [highlights  $80 + 60 = 140$ ]  
47. T: Erm, try that one.  
48. A: [clicks 80 in  $87 = 80 + 7$ ] Nope.  
49. A: Which one's... [highlights  $5 + 7 = 7 + 5$ ]  
50. T: Erm, try and click... like that one. Now try 7 add 5 or something.  
51. A: [clicks + in  $7 + 5 = 12$ ] Yes.  
52. T: Yeah, 5 add 7!

The boys continued in a similar vein for several minutes, taking a total of 8 minutes to complete this challenge, in contrast to the original challenge immediately prior to this which had taken them only 1 minute for the same puzzle. At times of impasse and frustration, they abandoned iconic matching entirely in favour of blind experimentation, though not with much expectation:

53. A: Try 12 equals 7 plus 5.  
54. T: Maybe you have to do this or something. [clicks every numeral in  $140 + 5 = 5 + 140$  and  $12 = 7 + 5$ ]  
55. A: Try all the numbers.  
56. T: [laughs] Yeah, that's what I'm doing. [clicks every numeral in every other statement]  
57. A: They're not going to work though.  
58. T: Yeah, don't think it will.

Nonetheless, they made gradual progress in a two-steps-forward-one-step-back manner whenever they returned to iconic matching, until they were close enough to see the rest of the way and solve the puzzle.

## CONCLUSION AND FURTHER WORK

The data demonstrate a shift from computation to the diagrammatic activities of iconic matching and substitution making over the duration of the trial. This in turn afforded commutative and partitional readings of  $a + b = b + a$  and  $c = a + b$  statements respectively. The boys developed sustained competence with diagrammatic activity, although still frequently attempted to substitute iconic mismatches. A slight alteration of the task goal impacted dramatically on the boys' emergent (and successful) puzzle solving strategies and resulted in iconic matching being entirely abandoned at times.

The data reported here is from the first trial of the software and task after its development. Seven more trials with seven different pairs of Year 5 children of varied mathematical ability (as deemed by their class teachers) have since been carried out, with minor modifications to the puzzles and research focus each time. Two of these trials were abandoned after about fifteen minutes because the children were not comfortable talking to one another and no verbal data was being generated. In the remaining five trials similar results were obtained to those reported here, notably the emergence of diagrammatic activity in place of computation over the duration of the trials. Emergent commutative readings are clearly evident across all five trials, and emergent partitional readings evident to a greater or lesser degree across three of the trials.

## NOTES

1. The software and task was developed by the author in *Imagine Logo* (Kalas & Blaho, 2003).

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## INTRODUCING MORE PROOF INTO A HIGH STAKES EXAMINATION – TOWARDS A RESEARCH AGENDA

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*We raise issues in examining what ‘proof’ is in high stakes examinations and what research issues may be usefully explored in considering ‘proof in high stakes examinations’. Our considerations arise from practical 14-19 curriculum and assessment development work. This article is not a standard academic article but, rather, a collection of issues and ideas.*

### INTRODUCTION

If a student has gone through his mathematics classes without having really understood a few proofs like the foregoing one<sup>i</sup>, he is entitled to address a scorching reproach to his school and to his teachers. Polya (1945, 216)

The context within which we consider ‘proof in high stakes examinations’ is outlined in this section. In subsequent sections we consider: what is proof in high stakes examinations; problems in setting proof questions; research issues and opportunities in *14-19 Pathways* work; and ‘players and artefacts’ in high stakes examinations.

The influential Smith Report (Smith, 2004) made a number of recommendations. Two, of relevance to this article, are: a call for the commission of curriculum and assessment development studies for 14-19 mathematics (ibid., Recommendation 4.11); a call for (academic stream) students to engage more fully with proof (ibid., Recommendation 4.5). We were part of a University of Leeds team who developed a curriculum and assessment study (14-19 Pathways, phase 1) and are now, with the examination board AQA (14-19 Pathways, phase 2), trialling<sup>ii</sup> and preparing to pilot curricula and assessment for courses of study and examination from 2011.

In our phase 1 recommendations on proof at GCE (A-level) we noted that proof is not currently a significant feature in A-level. Syllabii referred to ‘knowing’ specific proofs, e.g. ‘knowing’ the proof to obtain the sum of an AP and GP and proving trigonometrical identities. In the examination papers the words ‘proof’ or ‘prove’ were often a call for symbolic manipulation, e.g.

Given that  $x = \tan \frac{1}{2} y$ , prove that  $\frac{dy}{dx} = \frac{2}{1+x^2}$ .

Our phase 1 report<sup>iii</sup> to QCA regarding proof at GCE recommended greater emphasis be given to proof and that this be reflected in more questions being set that involve proof and ‘proof-theoretic ideas’ in examinations equivalent to current C3/C4 examinations. This recommendation was incorporated into current phase 2 work with AQA and we have trialled (informal development and testing) proof questions with students and are preparing to pilot (full curriculum with examinations leading to a formal qualification) from September 2007.

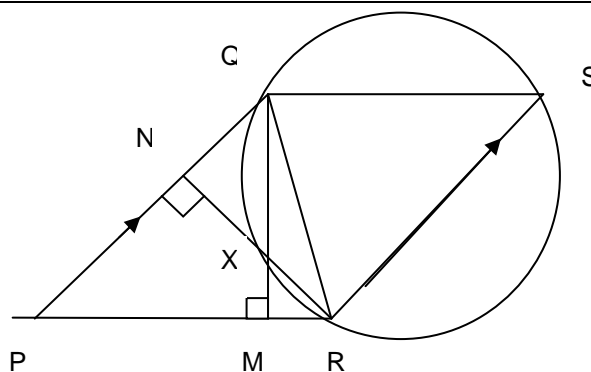
## WHAT IS PROOF IN HIGH STAKES EXAMINATIONS

The inclusion of proof in high stakes examinations is not new. It has been done before and it is worth considering what proof looked like in public examinations in the past. Whilst the curriculum under discussion is A-level, questions such as that below appeared in O-level (Joint Matriculation Board Syllabus B, November 1979 Paper II B9).

In the acute-angled triangle PQR, the point M is the foot of the perpendicular from Q to PR. The perpendicular from R to PQ cuts QM at X and meets PQ at N. A circle is drawn through Q, X and R. The line through R parallel to PQ meets this circle at S.

Prove that:

- (i) quadrilateral PNXM is cyclic,
- (ii)  $\angle QPR = \angle QSR$ ,
- (iii)  $\angle PRQ = \angle RQS$ ,
- (iv) PQRS is a parallelogram.



In this particular question the candidate is required to marshal his/her material and set it out in the form of a logical argument structured by a traditional format.

Proof at A-level was sometimes not dissimilar to what we find today:

The first and last terms of an arithmetic progression are  $a$  and  $\ell$ , respectively. If the progression has  $n$  terms, prove from first principles that its sum is  $\frac{1}{2}n(a + \ell)$

This example (JMB Syllabus A June 1973, Paper 1 S1 (part)) would be standard bookwork of the kind still done in the classroom today; it would have been rehearsed via past papers with the aim of perfect reproduction in the examination.

A second example (JMB Syllabus A June 1973 Paper 1 S8 (part)) however is:

Given that  $u$  and  $v$  are functions of  $x$ , prove from first principles that

$$\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}.$$

It is difficult to see exactly what kind of proof might have been expected for this example, perhaps the kind of demonstration given in text books and accompanied by a great deal of hand-waving by the teacher. Further the instruction “prove” can be seen to possess at least two synonyms in relation to the same piece of mathematics. Thus it is not difficult to find questions which say:

Prove that the equation of the tangent to the curve.... at the point...

Show that the equation of the tangent to the curve.... at the point....

Calculate the equation of the tangent to the curve.... at the point....

Are prove, show and calculate really all the same? In many respects the answer must be ‘yes’ since much of what we prove in mathematics is to enable the automatic and

safe calculation of certain quantities. For example, relevant theorems of analysis combine to tell us that it is safe to write that the derivative of  $y=x^2$  is  $dy/dx=2x$ ,  $x \in \mathcal{R}$ . If, then, in doing a calculation such as finding the tangent to a curve we assume the relevant theorems of analysis, then the calculation is, in one sense, a proof of a particular property of the curve; yet we do not ordinarily think of calculation as proof. What is a proof and what is not, and the role that proof has played in mathematics, is not without dispute, “there long have been and still are conflicting opinions on the role of proof in mathematics and in particular on what makes a proof acceptable.” (Hanna, 2000, p.5)

## **PROBLEMS IN SETTING PROOF QUESTIONS**

Notwithstanding the problems of characterising what a proof is, including proofs in high stakes examinations raises further problems. Perhaps the most important issue, simply noted in this article, is that proof may not be suited to high stakes examinations where students are under time and other pressures to ‘show what they have learnt’. This important issue does not appear, in our reading of the mathematics education literature on proof, to be considered. The issue does not appear to be a straightforward case of ‘you cannot produce an adequate proof in a timed examination’ because, as Buxton (1981, p.89) notes, “... time pressures are not automatically bad. Some of us work better under time pressures ...”.

An issue in setting proof questions is ‘who sets them?’ In our phase 2 work questions were set by a one of us (a mathematics education researcher) and the AQA Chair of Examiners. This was a respectful collaboration but there is an argument that neither ‘player’ in this activity was entirely suited to the task. In terms of their past histories and current briefs towards the trialling the mathematics education researcher was inexperienced in setting examination questions and the Chair of Examiners was influenced by matters other than just proof, e.g. developing workable markschemes, syllabus coverage and setting questions that are accessible to students at various expected grade levels. We return to such matters in the section ‘players and artefacts’ below but, for now, note that there is a sense in which the trial proof questions developed were a ‘compromise’ of the objectives of these two players.

In the BSRLM presentation we presented four ideas which had accompanying problems: use trigonometric identities; use definitions; first principle calculus ideas; use ‘easy’ content knowledge. Space does not allow for a consideration of all of these in this article and so we focus on one, ‘easy’ content knowledge.

In our work *14-19 Pathways* work on GCSE problem solving and on Functional Mathematics the Leeds team developed a rubric of *stepping back in terms of content*, i.e. using prior, possibly easier (to the student in terms of their development at the time of assessment), topic knowledge. Our evidence, not reported in this article, was that this was a largely (but not wholly) successful strategy in setting questions. It does not, however, appear to be as successful in setting proof questions. Consider the following:

Given that  $f(x) = \frac{x}{x-1}$ ,  $x \neq 1$ , prove that  $f^{2n+1}(x) = f(x)$ ,  $n$  a whole number.

In terms of content knowledge this does not require anything beyond Higher level GCSE but it was, for students, a difficult question. One reason for this difficulty is that the algebraic manipulation required is ‘fiddly’. But there were students who did not appear to have problems with the algebraic manipulation but who could not produce a proof. We cannot state with certainty reasons for this. It may be that students had little experience in producing proofs. It may be, however, and we put this forward as a roughly formulated hypothesis, that *stepping back in terms of content* is not an effective strategy in producing proof questions.

## ISSUES IN INTRODUCING MORE PROOF INTO EXAMINATIONS

Of the many issues concerned with introducing more proof into an examination we raise: the preparation of students; ethics; trialling in the present for a future examination; questions about teaching.

An important issue to us is what might be called ‘the curricular preparedness’ of A-level students for proof. This has at least two related dimensions – geometry (an important area of mathematics with regard to proof) and induction vs deduction. Most English students study GCSE mathematics prior to their A-level studies. Since the introduction of the National Curriculum in 1988, GCSE mathematics has included little deductive geometry and a considerable focus, in general, on inductive reasoning (see also Healy & Hoyles’ (1999) comments on proof, on geometry/algebra and on induction/deduction). A reasonable question, then, is it ‘fair’ to impose more proof on English A-level students?

This is an ethical question and a further ethical issue is carrying out pilot work in the context of high stakes examinations where students must, for their future careers, perform to the best of their abilities. All parties we work with on the development of this project regard students’ grades as paramount but this, in turn, generates a certain ‘conservativeness’ with regard to the introduction of more proof into examinations. NB Schools/students following the pilot A-level can, if they want take both the old and the new examinations if they are concerned about grades.

Another issue which raises ethical, as well as practical problems, concerns trialling examinations questions for a future course of study on students not following this course of study. It is essential that proof questions developed by the AQA-Leeds team are tested on students but we expect that these students are going to do less well on these questions than future students following the pilot course because future students should be more prepared for proof questions in their studies. With regard to ethics we risk making students feel bad about their mathematical prowess if they cannot make a reasonable attempt at the question and, with regard to practical matters, we are left guessing what a certain correct response rate now might mean for correct response rates of future students.

There are a number of issues/questions concerning teachers/teaching and proof. How does one 'teach' proof? What approaches will teachers on pilot courses adopt? What CPD can/should be provided for these teachers? How can we evaluate these questions? As difficult as these questions are to address it is crucial that they are addressed so that information can inform future CPD for all teachers if the pilot course becomes the new GCE.

### **'PLAYERS AND ARTEFACTS' IN HIGH STAKES EXAMINATIONS**

In the section on setting proof questions above we mentioned an issue concerned with two 'players' (a mathematics education researcher and the AQA Chair of Examiners) working together. This was not an issue concerned with these two players 'not getting on' (they both felt they worked well together) but of the players coming from different communities of practice with different objectives and and rules (one may consider these issues within discourse on 'communities of practice' or 'activity systems' or 'actor network theory' but we are not concerned with this level of detail at this stage in our thoughts). We now take a wider consideration of players and artefacts.

There are a number of players and artefacts involved in the production of a high stakes examination (in this case a pilot GCE which includes proof questions). Some of these are:

- ◆ communities, e.g. LMS, IMA, ACME, AQA, DfES, QCA;
- ◆ individuals with specific responsibilities within these communities;
- ◆ artefacts including reports from communities and/or individuals, e.g. the Smith Report, and GCE 'specifications' including aims, objectives, schemes of assessment and subject content.

There are also relationships between these players and artefacts, e.g. it is clear to us that the DfES has a strong influence on QCA (but no reverse influence is apparent), that QCA demands artefacts such as specifications from examination bodies such as AQA and that AQA regards the production of the specification as a major activity (and these specifications are 'boundary objects' between different communities).

What is clear to us is that the introduction of more proof into GCE mathematics is not the product of the work of the mathematics education researcher and the AQA Chair of Examiners but the product of a network of players and artefacts. Researching the interrelations in this network in the development of this new GCE, which includes more proof questions in its examination, would be a difficult undertaking but one that could shed important light on how 'proof in a high stakes examination' is created.

### **POSTSCRIPT**

Many of the issues highlighted above would not have occurred prior to 1988, the introduction of the National Curriculum and its associated testing regime. Prior to 1988 the introduction of more questions associated with proof would have been at the

whim of the examination boards, perhaps prompted by a local or national curriculum development initiative or the predilections its chief examiner. However, whilst a highly regulated and high stakes system such as exists in England offers a degree of openness and a clear entitlement, curriculum development is then fraught with difficulty since the knock-on effects throughout the system of even the smallest change have to be considered. The introduction of more questions focusing on proof might appear to be relatively straight forward; such questions surely reflect the nature of the subject? Clearly they do, but an important question is, can such a development be made to fit within the system without challenging any part of the system? There appears to be no simple answer to this question.

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## NOTES

<sup>i</sup> The “foregoing one” being *that the sum of the three angles of a triangle is equal to two right angles*.

<sup>ii</sup> ‘Trialling’ is UK education-speak for developmental testing of materials and ‘piloting’ refers to the development of a full curriculum and assessment together with an award.

<sup>iii</sup> This does not, unfortunately, appear to be in the public domain.

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## CAN MATHS IN A TEST BE ‘FUNCTIONAL’?

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*This paper argues that the assessment of functional mathematics as currently being developed at GCSE level should go beyond testing mathematical skills and using word problems. Drawing on the literature about mathematics in the workplace, criteria for functional mathematics assessments are offered, and the constraints of tests considered, before answering the question, with: “Yes, but it won’t be easy”.*

### 1. WHAT IS FUNCTIONAL MATHEMATICS?

Functional mathematics is about mathematics being useful, and what can occur in school that enables that, but the scope of what is meant by the phrase varies considerably. Some authors use it to refer to a re-conceptualisation of the mathematics curriculum, a complete rethink about what is done in school to make the mathematics that is learned more useful, responding especially to the changes in the workplace occasioned by advances in technology (e.g. Forman and Steen, 2000).

A narrower view of functionality, and the one that is the focus of this paper, is the set of skills that enables a person to cope with everyday life. In this vein, functional mathematics is defined in the White Paper “14-19 Education and Skills” as the “maths that people need to participate effectively in everyday life, including in the workplace” (DfES, 2005, page 35). This functional mathematics is going to be examined both as part of GCSE, and separately as a series of stand-alone qualifications. Standards for these assessments can be found on the QCA website: [http://www.qca.org.uk/downloads/QCA-06-2932\\_Functional\\_skills\\_standards\\_maths.pdf](http://www.qca.org.uk/downloads/QCA-06-2932_Functional_skills_standards_maths.pdf)

However, there is ambiguity in the meaning of “the maths that people need”. It is possible to interpret that as the ‘core skills’ of basic mathematical competences that are applied when mathematics is functional. This interpretation sees ‘functional mathematics’ as the subset of mathematics knowledge that is useful (e.g. arithmetic and interpreting graphs), contrasted with the mathematics that is learned as a step towards more mathematics (e.g. algebraic manipulation and Pythagoras’ theorem).

While acknowledging that knowing some mathematics is essential to being functional, a more helpful interpretation of “the maths that people need” is a broader view of mathematics that includes the processes that are involved in actually applying mathematics. This interpretation of functional mathematics encompasses more of what a person does when they operate successfully with mathematics in real settings.

While less easy to pin down than a list of mathematics ‘content’, it seems important that an assessment of functional mathematics – and the teaching that precedes it – should engage with this broader view of functional mathematics.

## **2. WHAT IS INVOLVED IN BEING FUNCTIONAL WITH MATHEMATICS?**

A review of research on mathematics at home and at work (e.g. Lave, 1988; Nunes et al, 1993; Pozzi et al, 1998, Resnick, 1987) reveals a number of ways in which out-of-school mathematics differs from what is usually done with mathematics in school.

### **Purpose and meaning in the activity**

What is done at home and at work is almost always done with a purpose, which adds meaning to the mathematics that is deployed. In school, of course, the purpose is to learn, and the mathematics is done just as an exercise towards learning it.

### **Understanding and reasoning linked to the context**

In school there is an expectation that whatever understanding accompanies mathematical activity it is mathematical in nature, and mathematical reasoning is about drawing mathematical conclusions from mathematical premises. In situations in which mathematics is being used, however, the understanding and reasoning is derived from features of the context, with the mathematics subordinated to that.

### **Tool use**

Although school does to an extent permit the use of tools like calculators in restricted circumstances, the use of tools in work is much more pervasive, with tables and charts to support calculations commonplace. This no doubt reflects the contrasting purposes in the two settings, which in school are about learning the mathematics, and at work are about getting the right result using whatever means are available.

### **Situation specific competence**

The emphasis of much school mathematics is in developing general purpose competences. However, the competences of work and at home are usually specific to the context, again reflecting the purposes of mathematical activity out of school.

### **Error-free performance**

There is no benefit and some cost to getting an incorrect outcome in out of school settings. For that reason, circumstances evolve that minimise that risk, and people's performance out of school is for the most part without error. For example, in most work settings, people are matched to the level at which they will operate successfully, so some individuals are not permitted to undertake certain tasks. In school, however, much is made of the benefits of making a mistake, and everyone is asked to undertake tasks that are beyond them, which of course is because of the purpose – promoting learning.

## **3. CRITERIA FOR THE ASSESSMENT OF FUNCTIONAL MATHEMATICS**

A consideration of mathematics out-of-school suggests that assessment of the 'functional' ability to deal with contexts mathematically should require that students engage with problems-in-context in a way that anticipates real-life engagement, and:

- include contexts and questions that are realistic enough to invoke a sense of purpose, even though that purpose cannot be real;
- have what is in effect an assessment of process that does not include any attention to the actual process used;
- avoid unreasonable obstacles to success in the mathematical aspects of the assessment.

#### 4. WORD PROBLEMS IN THE ASSESSMENT OF FUNCTIONAL MATHS

Word problems are conventionally seen as the place where mathematics and the real world meet. They are the ‘obvious’ first candidate for assessment of functional mathematics, particularly in a test context, and so need to be considered in relation to these criteria. Will this kind of thing do?

“An electrician earns a basic rate of £11.20 per hour for a 35 hour week. For each hour worked over 35 hours, he earns 1½ times the basic rate. One week he works for 40 hours. How much does he earn?”

(Key Skills Application of Number, Level 2, 15 March 2004)

“I wish to paint the outside walls of my house. A tin of paint covers  $25\text{m}^2$  correct to the nearest  $5\text{m}^2$ . The outside walls of my house have an area of  $320\text{m}^2$ , correct to the nearest  $10\text{m}^2$ . Calculate the maximum number of tins of paint I may have to buy”

(OCR GCSE Specimen Assessment Materials, Mathematics A)

The first criterion for a functional mathematics assessment is that it brings a sense of purpose, which is contingent on some kind of engagement with the context that is described. However, Reusser and Stebler (1997) say that there is:

“ample evidence that many students in mathematics lessons ‘understand’ and ‘solve’ mathematical word problems without considering the factual relationship between real-world situations and mathematical operations.”

Students commonly consider word problems as a kind of mathematical exercise, and work with the numbers without thinking about the context much at all. For example, little of the context needs to be thought about in the first example above to arrive at  $11.2 \times 35$  plus  $(40 - 35) \times 11.2 \times 1.5$ .

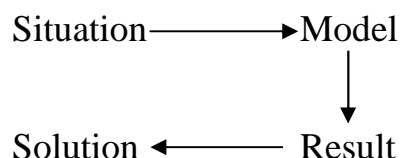
Wyndham and Säljö (1997) do not feel that students’ unwillingness to engage with contexts in word problems represent a lack of ability to do so.

“The student ... is not necessarily lacking in skills or competencies. Rather s/he is operating within a communicative contract of how to interpret problems, which in all likelihood has developed in response to how schools ‘do’ mathematics.”

As a result, whether word problems are answered correctly or not does not reflect functionality with mathematics, and they are not a good way to assess it.

## 5. MODELLING AND FUNCTIONAL MATHEMATICS

A different perspective on what it means to be functional is found in the notion of ‘modelling’. In dealing with a word problem, it is intended that the mathematical features of the described context are identified and abstracted from the situation, and the mathematical relationships worked with, to arrive at a mathematical result which is applied back to the context. This can be shown in diagrammatic form as follows:



This reflects the structure of ‘mathematical modelling’ that is taught at higher levels of mathematics, and it is commonly supposed that this sort of modelling is what is needed for the everyday application of mathematics as well. However, Gravemeijer (1997) reports that two conceptions of modelling can be found in the literature. The one described above is modelling as a form of translation between the real and mathematical worlds, and originated in the work of Polya. A contrasting view is that of modelling as a form of organising of the real world using mathematics, which started with the ideas of Freudenthal.

It can be argued that word problems fit the first kind of modelling, but most actual mathematical functionality is better described in the second way. In ordinary settings people take mathematics to the context, rather than abstracting mathematics from it, and the mathematics usually remains attached to the context, as observed by the researchers cited in the first section. This is not to say that modelling of the first kind cannot be used in real settings: it can and it is. However, for most people in most contexts it is the second account of modelling that is more accurate, and assessment of functional mathematics is better served by an approach which reflects that, rather than operating on an assumption of the first kind of modelling, as word problems do.

## 6. BEYOND WORD PROBLEMS

In order to have demands that are more likely to bring an engagement with the context, which is necessary to assess mathematical functionality, Verschaffel et al (1997) say that what is needed is “more complex and more authentic problem situations” - but what does that mean? What has to be changed?

Van den Heuven-Panhuizen (2005) boils it down to two criticisms of word problems:

“the context is not very essential – it can often be exchanged for another without substantially altering the problem”; and “the reality that is presented is often not in tune with the real situation of the actors in the problem”

Reusser and Stebler (1997) add a third:

“Students not only know from their school mathematical experience that all problems are undoubtedly solvable, but also that everything numerical included in a problem is relevant to its solution, and everything that is relevant is included in the problem text”

## **7. HOW EACH OF THESE MIGHT BE OVERCOME**

### **(i) The ‘essential’ context**

A word problem starts in the mathematical world, with a piece of mathematics to be ‘applied’, and the problem setter ‘translates’ this into a context, contrived for the purpose. Another context would indeed have done as well, and students understand that what they have to do is concerned with mathematics, not the situation as such. The alternative is to start with a context, not the mathematics. The mathematically related questions that are then asked must be more genuinely about that situation.

### **(ii) The reality of the ‘actors’**

An obviously ‘mathematics lesson’ question is likely to get similar responses to a word problem, even if the situation being referred to is more complex and authentic as a context. The alternative is to choose questions that might actually be asked.

### **(iii) The closed context**

In word problems all the required information about a specific event is given, and all that is given is required. Giving the information ‘on a plate’ reduces the need to engage with the situation. The alternative is to describe a broader context that includes data relevant to a range of events, only some of which will be asked about.

## **8. THE CONSTRAINTS OF A TEST**

An assessment that is based on authentic contexts, described in ways that are broader than the questions asked about them, and where the questions are realistic in the sense of being genuine questions that might actually be asked, can be addressed in different ways, and perhaps more easily in contexts other than in a test. In a test there are constraints that impact on the means by which these ambitions can be realised.

### **(i) What it is reasonable to ask in a time limited test.**

Because of reading issues and time to process, there is only so much complexity that can be included in a test – and would it be enough to assess functionality?

Introducing more complexity can be helped by the inclusion of ‘pre-release’ material that candidates can spend time with before the test, but this too is bound to be limited.

### **(ii) Mark schemes**

In a test, assumptions have to be made about the understanding about a context that reading the description of it will provoke. A test cannot afford to have students drawing in realistic considerations from their own experience, because it would be impossible to allow for that in the mark-scheme. The answer needs to use the given information, but not any information that a more ‘active’ reading of the description of the context would bring. As Cooper and Harries (2003) say, a test of functional

mathematics must look for “a *particular* realistic consideration, but not realistic considerations *in general*”.

## 9. CONCLUSION

The challenge of getting students to act functionally in a simulated reality in order to assess whether they have the capacity to cope out of school is not easily met, although some straightforward suggestions can be made to improve the prospect of it. However, a test seems to make it that bit harder, both through the constraints on describing a realistic situation, and through the limited information a test gives about the thinking that informed a response.

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## **NCETM/BSRLM WORKING GROUP: DEVELOPING A FRAMEWORK FOR RESEARCHING CPD IN MATHEMATICS**

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The overall aim of this workshop is to build UK research capacity related to Continuing Professional Development for teachers of mathematics. Whereas there is an extensive research base in this area, much of the research derives from CPD initiatives in the USA. We take as a starting point that CPD for teachers of mathematics is about being stimulated to re-think, to experiment, to make fresh distinctions and to probe those distinctions to see if they are informative in enabling choices related to teaching and learning that influence learners' mathematical experiences and activity. We also take as a starting point that CPD could take many forms which include: individual inquiry; participation in professional organisations; participation in school-based or local authority groups and networks; participation in research or curriculum development projects and participation in courses. Within this broad landscape we propose to develop a framework for characterising professional development, which involves asking the following questions:

*What kinds of professional capacity are being developed?* For example, is it envisaged that the CPD will strengthen specific aspects of participants' core subject knowledge and their capacity to bring that knowledge to bear in their teaching? To what degree may this also require the CPD to address wider aspects of participants' ideas about teaching and learning mathematics, and their approaches to it?

*What forms of interaction are afforded?* For example is the CPD situated within a social network and who are the members of the network? How do the members of the network communicate? What are the characteristics of the interaction and what sorts of questions are being asked?

*What sorts of stimuli are provided?* This relates to the types of tools, artefacts and language that are worked on by the participants. It could include mathematics problems, questions, research evidence, theoretical ideas, ICT. This will involve explicitly considering how the NCETM portal is being used as a stimulus

*What forms of experimenting/reflecting are evoked?* This relates to the ways in which the CPD team experiment with their practice and the risks that this involves.

*What constitutes evidence for effective CPD?* This involves identifying the probes that members of the CPD team use to provide evidence for the effectiveness of CPD.

*What forms of reporting occur?* How is the team expected to document their engagement in professional development, which could include a video-story, a presentation, a piece of written work. Identifying the audience for the reporting will also be important.

*Are wider contributions to the field being made?* Does the CPD produce qualitatively new professional knowledge from which others in the field could benefit? How could such knowledge best be expressed in transposable forms?



## **British Society for Research into Learning Mathematics**

BSRLM is an organisation which acts as a major forum for research in mathematics education in the United Kingdom. It is both an environment for supporting new researchers and a forum for established ones. It is open to and welcomes membership from anyone involved or interested in mathematics education.

BSRLM is associated with an e-mail list in operation to facilitate effective communication between members and others in mathematics education worldwide. To join this list, send the single world message <subscribe> to [maths-education-request@nottingham.ac.uk](mailto:maths-education-request@nottingham.ac.uk)

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