Curriculum and Assessment Authority (VCAA) were chosen as they both required the use of the graphics calculator in their 2000 examinations (IBO, 1999; VBOS, 1998). Both authorities included a period, in which, the graphics calculator was allowed but not required in the examinations (IBO, 1992; VBOS, 1995).

EXAMINATION AUTHORITIES AND THEIR QUESTIONS

In this section sample questions from each of the examination authorities for the period 1993 or 4 to 2005 or 6 are described to exemplify the types of questions being used in graphics calculator required examinations.

Victorian Curriculum and Assessment Authority (VCAA)

During the period from 1994 to 2005 the VCAA question writers have regularly used questions requiring the sketching of trigonometric functions, which are shown in Table 1.

Table 1 A selection of graphing questions from the VCAA examinations over the years.

1994: The sea level height, y cm, is given by the equation $y = \frac{2}{5}\sin\left(\frac{\pi t}{6}\right) + 1$, where t represents the number of hours after midnight on 21 September 1994. Sketch the graph and state the period of the function and the amplitude of the function

2004: Sketch the graph of the function with rule $y = 3\sin\left(2\left(t - \frac{\pi}{4}\right)\right)$, where $-\pi \leq t \leq \pi$. State the number of solutions to the equation $3\sin\left(2\left(t - \frac{\pi}{4}\right)\right) = 1$, where $-\pi \leq t \leq \pi$

2005: On the axes provided, sketch the graph of the function $f : (-\pi, \pi) \to R,\ f(x) = \tan\left(\frac{x}{2}\right) + 1$.
Clearly label any axes intercepts, and any asymptotes with their equations.

It is apparent that the trigonometric function questions from the pre graphics calculator period (1994), required the students to sketch the function and indicate the period and amplitude. However, in 2004 and 2005, students were able to either input the function into the graphics calculator and then transfer the representation from the screen to paper or by knowing the shape of the function they could sketch it without recourse to the graphics calculator. The contrasting skills of sketching a function from an equation and the sketching (transferring) the graph from the screen of a graphics calculator is clearly different (Mason, 1995) and is representative of the differences found between a strictly pencil and paper medium and one that incorporates technology. A further discussion of the different skills required in a pencil and paper and technology environment can be found in Kieran & Drijvers (2006).

International Baccalaureate Organization (IBO)

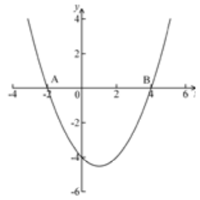
Between 1993 (pre graphics calculator) and 2006 the IBO question writers have regularly used questions requiring the sketching of graphs which are exemplified in Table 2.

Table 2 A selection of graphing questions from the IBO examinations.

1993: It is given that $f: x \mapsto \dfrac{x^2 + x - 12}{x - 2}$, $x \in R$ and $x \neq 2$. Show that $f(x)$ can be expressed as $\dfrac{(x + B)(x + C)}{x - 2}$, find B and C, where $B, C \in R$ … sketch the graph of $f: x \mapsto \dfrac{x^2 + x - 12}{x - 2}$ and show the intersections of the graph and the axes. Find the equations of the asymptotes and graph them

2000: Sketch the graph of $y = \pi \sin x - x, -3 \leq x \leq 3$, on millimetre square paper, using a scale of 2 cm per unit on each axis. Label and number both axes and indicate clearly the approximate positions of the x-intercepts and the local maximum and minimum points.

2005: The equation of a curve may be written in the form $y = a(x - p)(x - q)$. The curve intersects the x-axis at A(–2, 0) and B(4, 0). The curve of $y = f(x)$ is shown. Write down the values of p and q. Given that the point (6, 8) is on the curve, find the value of a. Write the equation of the curve in the form $y = ax^2 + bx + c$.



2006: Consider the functions f and g where $f(x) = 3x - 5$ and $g(x) = x - 2$. Let $h(x) = \dfrac{f(x)}{g(x)}$, $x \neq 2$ **Sketch** the graph of h for $-3 \leq x \leq 7$ and $-2 \leq y \leq 8$, including any asymptotes. Write down the **equations** of the asymptotes and shade the region whose area is represented by $\int_3^5 h(x)\, dx$

The 1993 question included many sub parts, which guided the students towards drawing the required function, a graph that could have been drawn on the graphics calculator without any of the intermediary steps. The 2000 question was interesting in that it was very specific on how the graph was to be drawn (graph paper is not provided) and with schools around the world taking the examination it was necessary to be very explicit on the requirements. The 2005 and 2006 expected the use of the graphics calculator. However, in each case there was a “twist” for 2005 the students were required to find a quadratic function using the given information whilst for 2006 they were required to sketch the graph of the quotient of two functions.

DISCUSSION

As a result of the analysis of the examination questions, in the period 2000 to 2006, the existence of four categories of questions where graphing was included was found, these categories were;

A. Graphics calculator active no graphing required (or graph already provided)

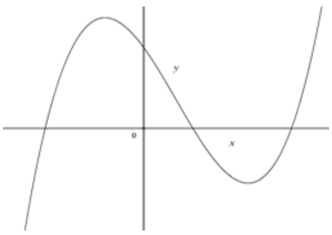
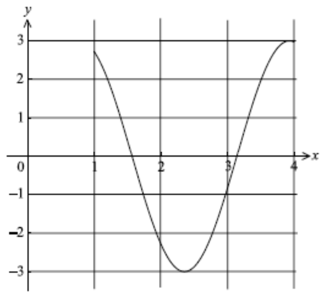
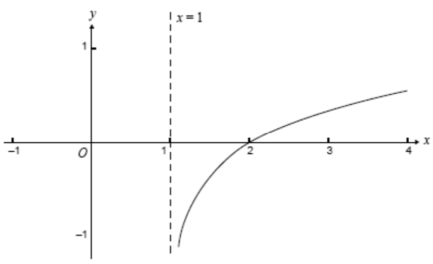
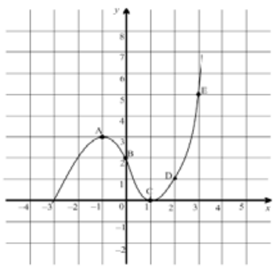
B. Graphics calculator active and graphing required

C. Graphing calculator inactive and no graphing required (or graph already provided)

D. Graphing calculator inactive but graphing required

These categories are exemplified in Table 3.

Table 3 Exemplification of the types of questions identified in this study.

A. *Graphics calculator active no graphing required (or graph already provided).* Part of the graph of the curve with equation $y = x^3 - 2x^2 - 5x + 6$ is shown below.  Write the equation in the form $y = (x + 2)(x^2 + bx + c)$ (VCAA, 2005)	**B.** *Graphics calculator active and graphing required.* Let $f(x) = 3\sin 2x$, for $1 \le x \le 4$ and $g(x) = -5x^2 + 27x - 35$, for $1 \le x \le 4$ The graph of *f* is shown On the same diagram, sketch the graph of *g*. One solution of $f(x) = g(x)$ is 1.89. Write down the other solution. (IBO, 2006) 
C. *Graphing calculator inactive and no graphing required (or graph already provided).* The graph of the function $f : (1, \infty) \rightarrow R,\ f(x) = 0.5\log_e(x - 1)$ is shown  State the domain and range of *f*. State why the inverse of *f* exists. Find the rule for the inverse function f^{-1}. (VCAA, 2004)	**D.** *Graphing calculator inactive but graphing required.* The sketch shows part of the graph $y = f(x)$ which passes through the points A(-1, 3), B(0, 2), C(1, 0), D(2, 1) and E(3, 5). A second function $g(x) = 2f(x - 1)$. Calculate $g(0)$, $g(1)$, $g(2)$ and $g(3)$. On the same set of axes, sketch the graph of the function $g(x)$.  (IBO, 2003)

Quadrant C also includes questions in which use specific wording to exclude the graphics calculator from the solution process (there are only a few such questions in the VCAA examinations). Quadrant D questions, which require students to construct a graph using key features, such as, intercepts are found in both examination authorities questions. Whilst quadrant B provides an interesting contrast to examinations questions from the pre graphics calculator period. Now instead of using calculus to draw the graph students are expected to transfer the representation on the graphics calculator screen to the paper, a different skill to the determination of points and then constructing a graph.

Ruthven (1996) proposed a set of questions that could be used in assessment with the advent of advanced calculators. The author's first question (Ruthven, 1996, p. 360) is indicative of the type of questions that would be found in quadrant C. However, authors such as (French, 1995) have expressed concern that questions testing generic concepts, such as those found in quadrants C and D, will increase the difficulty examinations. This is an area for further research. However, the results found by Brown (2005) would tend to indicate that examinations have not increased in difficulty, as the majority of graphing questions fall into categories A and B, which are considered to be routine in nature.

CONCLUSION

The questions in Table 3 are representative of the possible categories of questions that examiners could use in mathematics examinations in which the graphics calculator is required. The categories are described below

- **A.** Graphics calculator supported no graphing required (or already provided).
- **B.** Graphics calculator supported and graphing required.
- **C.** Graphing calculator inactive and no graphing required (or graph already provided).
- **D.** Graphing calculator inactive but graphing required.

These categories are also indicative of the types of classroom learning experiences that students will need in the classroom where the graphics calculator forms part of the instructional requirements. These learning experiences include visualizing a function using information about some key points; sketch a graph from the screen of a graphics calculator. Furthermore, students will still be required to sketch a graph without recourse to the graphics calculator especially with the increasing use of graphics calculator excluded examinations. Thereby broadening the range of graphing skills required.

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