

## **DO DIFFERENT MATHEMATICAL OPERATIONS INVOLVE DIFFERENT COMPONENTS OF THE WORKING MEMORY MODEL?**

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*Studies looking at the connections between working memory and children's mathematical achievement are often limited by the general nature of the measures of either working memory or mathematics. In this study, thirty-five children in Year 5 (9 to 10 years of age) were given tasks designed to measure their phonological working memory, visual working memory, inhibitory skills and their mastery of addition and multiplication facts. The findings suggest that different components of the working memory model (Baddeley and Hitch, 1974; Baddeley, 1996) may be involved in these two different mathematical operations.*

### **WORKING MEMORY AND CHILDREN'S MATHEMATICS**

Working memory is a limited-capacity multi-component cognitive system responsible for the concurrent processing and storage of information in cognitive tasks. The current model comprises four components: a phonological loop responsible for the storage and rehearsal of speech based and phonological information, a visual-spatial sketchpad responsible for visual and spatial information and a central executive responsible for regulating the contents of working memory and integrating information from long-term memory. A fourth component, the episodic buffer, is responsible for combining information held in different forms into an integrated whole for a short time.

In studies looking at the connections between children's working memory and mathematical performance, various components of the Baddeley and Hitch (1974) working memory model have been implicated. These include the central executive in general (Lehto, 1995), the phonological loop and central executive (Logie et al, 1994), phonological loop and visuo-spatial sketchpad (McKenzie, Bull, & Gray, 2003), central executive and visuo-spatial sketchpad (D'Amico and Guarnera, 2005) and visuo-spatial working memory (Reuhkala, 2001). One reason for this apparent lack of consensus in the literature may be the use of general measures of mathematics that could obscure links between working memory and specific mathematical operations. Another possible confounding factor could be the use of single measures of working memory that do not discriminate sufficiently between the different components of the model and therefore obscure potential connections.

There have been a number of studies linking the phonological loop to the retention of the operands in mental calculation (e.g. Noël et al, 2001) and to the retention of partial results in multi-step calculations (e.g. Seitz and Schumann-Hengsteler, 2002). The existence of multiplication tables and the rote learning of multiplication facts suggest a role for the phonological loop. Articulatory suppression, which disrupts the

functioning of the phonological loop, has been shown to impair adults' performance with multiplication facts (Lee and Kang, 2002).

The role of the visual-spatial sketchpad in mathematical functioning is less well researched. Lee and Kang (2002) found that a concurrent visual-spatial task disrupted adults' performance on a subtraction task, but not on a multiplication task. Jordan et al (2003) found that children who showed poor mastery of addition facts performed significantly less well on a non-verbal intelligence task than children with better addition mastery. McKenzie et al (2003) found that visual noise had a particularly disruptive effect on the mathematical performance of young children, but that its effects were less marked among older children.

The mechanism by which visual-spatial working memory might affect mathematical performance is also uncertain. Klein and Bisanz (2000) gave non-verbal addition and subtraction problems to 4-year-old children and concluded that the limiting factor on their performance was 'representational set size': the maximum number of items that could be held in visual-spatial working memory. Hughes (1984) found that children were able to solve simple addition and subtraction problems almost as well when imagining using concrete objects as when using the physical objects themselves. Holmes and Adams (2006) have speculated that visual-spatial working memory might be the place where children move from concrete representations of number to more abstract ones. Could it be that visual-spatial working memory is important in young children's quantity manipulation operations like addition and subtraction and could therefore predict their performance on a task tapping abilities with simple addition facts?

There appears to be more consistent evidence that the central executive component of working memory is implicated in a range of different mathematical tasks in normally developing children (e.g. Gathercole et al, 2004), in children with mathematical difficulties (e.g. D'Amico, and Guarnera, 2005) and adults. The difficulty has been in identifying the precise role of the central executive. One important aspect of central executive functioning is the ability to inhibit redundant or potentially confusing information from working memory (Passolunghi and Siegel, 2004). Information about multiplication facts is thought to be stored in associative networks (Campbell, 1987). Activation of one node in the network automatically activates others and can lead to associative intrusion errors such as ' $4 \times 5 = 25$ ' where the incorrect answer given is a multiple of one of the multiplicands, but not the correct one (Barrouillet et al, 1997). It could be that the ability of the central executive to inhibit unwanted information predicts accuracy with multiplication.

## **THIS STUDY**

This study sought to avoid some of these potential problems by looking at very specific areas of mathematical performance and using tasks designed to tap very

specific areas of the working memory model. The study addressed three specific research questions:

- Does phonological loop functioning predict scores on a simple multiplication task?
- Does visual-spatial working memory predict performance on a task involving simple addition?
- Does the ability of the central executive component of working memory to inhibit unwanted information predict accuracy levels on a test of simple multiplication?

### **Participants**

Thirty-five children from two primary schools in the southwest England took part in the study. All the participating children were in Year 5 at the time of testing. All the participants and their parents gave written consent and were allowed to withdraw from the study at any time.

### **Procedures**

The children were tested in two separate sessions about a week apart. They were given two tests of simple addition and simple multiplication, one an answer production task, the other a multiple-choice task. Both tasks were timed and the number and nature of any errors noted. The children's predominant calculation strategy for each task was noted.

Each child was given three different working memory tests. Phonological working memory was assessed using the non-word repetition task from the Working Memory Test Battery for Children (WMTB-C). Visual-spatial working memory was assessed using a variation of the visual patterns task (Dela Sala, 1999) adapted for presentation on a PC. The ability to inhibit confounding information was assessed using a novel task in which two shapes (one red and one green) were presented for two seconds. In some cases the larger shape was red, in others it was green. The children had to name the green shape. The number of intrusion errors (instances where the red shape was named) was measured.

### **Findings**

In all the mathematical tasks, the children were instructed to answer as quickly as possible, but to be accurate. No errors were made on the addition tasks. Correlations revealed that there was a significant and positive correlation between times on the multiplication tasks and the number of errors indicating that there was no trade-off between speed and accuracy as the faster children also made the fewest errors.

T-tests revealed that there were no significant differences in any of the mathematical tasks or working memory tasks between male and female participants

## **Phonological Working Memory**

Pearson correlation tests revealed significant relationships between phonological working memory and multiplication fact mastery times and between phonological working memory and multiplication multiple-choice times (see Witt and Pickering (submitted) for details). There was no significant correlation between the phonological working memory tasks and the addition tasks.

## **Visual-Spatial Working Memory**

The findings regarding this component of the working memory model were harder to interpret. There was a significant correlation between visual-spatial working memory and the addition multiple-choice times but not with the addition answer production task. The correlations between the visual patterns task and the measures of multiplication mastery were non-significant.

## **Central Executive Working Memory**

There was a significant correlation between the number of errors made on the inhibition task and the total number of multiplication errors made, the correlation between the number of errors made in the inhibition task and the number of associative intrusion errors made in the multiplication tasks narrowly failed to reach significance. However, if operation errors (e.g.  $4 \times 5 = 9$ ) are included as 'intrusion' errors, the correlation between inhibition performance and multiplication errors was significant

Surprisingly, there was a very significant correlation between performance on the inhibition task and times on both the addition answer production task and the addition multiple-choice task. The correlations for the multiplication tasks were not significant (see Witt and Pickering (submitted) for details of the statistics).

## **DISCUSSION**

The data presented here suggest that different mathematical operations may be drawing on different working memory capacities. The significant correlation between multiplication fact mastery and the phonological loop component of working memory suggests that multiplication facts are encoded in terms of their phonological features. It is thought that strong associations between the multiplicands and the correct product are formed by repeated exposure to them together. Children with good phonological working memory are more likely to retain the two multiplicands while they are calculating the product and are therefore more likely to associate the product with the multiplicands. Children with poor phonological working memory are more likely to forget the multiplicands, making it much more difficult to form a strong association between them and the correct product. Rapid access to and direct recall of multiplication facts from long-term memory is a faster and more accurate strategy than counting based multiplication strategies.

The data concerning visual-spatial working memory are more difficult to interpret. The significant correlation between the visual patterns task and the times for the

addition multiple-choice suggest that visual-spatial working memory skills may be important in the establishment of addition facts in children. The ability to manipulate sets mentally would help children move from slower, concrete representations of quantity manipulation problems (addition and subtraction) to faster mental representations. Work by Hughes (1984) has already shown that young children can be almost as accurate using these mental representations as they can when using concrete ones. The ability to visualise and mentally manipulate sets may help young children retain addends and calculate simple sums more accurately leading to a stronger link in long-term memory between the two addends and the correct sum.

The correlation between the inhibition task and the number of multiplication errors indicates that the ability to prevent unwanted associations from contaminating working memory may be important in avoiding associative intrusion errors in multiplication. This may be important in avoiding two types of error: those in which an incorrect multiple of one of the multiplicands is chosen as the answer (e.g.  $6 \times 7 = 36$ ) and operation errors in which the two multiplicands are added instead of multiplied (e.g.  $7 \times 6 = 13$ ). This latter type of error may be particularly prevalent in younger children who are beginning to multiply, as the majority of their mathematical experience will have been with addition.

The significant correlation between the inhibition task and performance on both the addition tasks can be explained in different ways. The highly visual nature of the inhibition task might lend further support to the suggestion that visual-spatial skills are important in early addition. It may be that the inhibition task is tapping more general central executive functioning, which has been shown (e.g. Gathercole et al, 2004) to be important in several mathematical operations. Specifically, children who made few errors on the inhibition task may have ready and fluent access to information in long-term memory, which may facilitate their accessing stored addition facts. More research is needed to understand the (possibly many) ways in which the central executive component of the working memory model is involved in children's mathematical performance.

The data presented here go some way to explaining the lack of consensus in the research into the role of working memory and children's mathematics. It lends support to the idea that different mathematical operations are dependent on different components of the model and this should be taken into consideration in future studies when choosing tasks measuring both mathematical and working memory performance.

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