A CULTURAL-HISTORICAL APPROACH TO TEACHING GEOMETRY PART 2: THE RESULTS OF A PILOT STUDY

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In the last proceedings (Rowlands and Carson, 2006) we discussed a curriculum initiative that aims to 'bring to life' the major primary events in the history of Greek geometry. In particular, the combination of the intellectual act of abstraction and the possibility of formalised, logical proof were discussed. In Part 2 the results of a pilot study to see whether year 9 'mixed ability' and year 10 'gifted and talented' students can be meaningfully engaged with these two primary events are discussed.

INTRODUCTION

In the last proceedings we outlined a curriculum initiative that aims to guide students through the processes of abstraction and proof as two 'primary events' ('key ideas') in geometry's developmental history. With the first primary event, students would be guided through 6 levels of abstraction and for each level the class would discuss and explain what has been carried forward and what has been left behind, culminating in discussing the Forms and the possibility of proving opposite angles are equal.

This article presents a summary of a pilot study to evaluate the likelihood of successfully implementing the two primary events with secondary school students. The extent to which the students engaged with the activities would be a measure of success. Because the author was the teacher the account given below is part subjective, but the extent of the engagement was verified by observers.

The pilot study consisted of one R. I. Maths Masterclass session comprising of 36 year 10 students and two Widening Participation sessions comprising of 'mixed ability' year 9 students. The next section explains the rationale behind the pilot study, followed by accounts of the Masterclass and Widening Participation sessions.

THE PILOT STUDY

This initiative was designed for 12 or 13 year old learners of 'average' development. An evaluation of such an initiative would need to take the form of an action research project involving the implementation of the initiative as a course taught within the classroom setting. The purpose of the pilot study was to see whether learners could be engaged in such an initiative and to what extent. The Masterclass and Widening Participation sessions were two opportunities to pilot the implementation of the two primary events but there were three major problems to consider:

The context, whether it is widening participation or masterclass, is artificial compared to a course in geometry within the normal classroom setting. However, the artificiality of the context makes it *more* difficult to introduce the two primary events. Within a normal classroom setting the introduction of the two primary events would be perceived by the students as part of the course, whereas in Widening Participation the expectation by the participants may be higher and, given this is not a course in geometry, the whole point of the exercise may be more difficult to grasp.

The learner's previous experience of geometry lessons may influence their engagement with the topic under evaluation. 'Gifted and talented' learners may have already been exposed to deductive proof, so introducing it might go against their expectations, with the whole exercise appearing superfluous.

The content and 'delivery' may not be appropriate to the learner's development (or 'ability'). 'Gifted and talented' 15 year olds may be cynical about going through the 6 levels of abstraction, since they are already very capable of thinking in the abstract.

To overcome all three problems it was decided to engage the class with the nature of proof before taking them back to the historical moment when the act of creating ideal objects made proof possible. This was achieved by beginning each session with the question 'the three angles of a triangle add up to what?' followed by 'how do you know?' In each session there were facial expressions of what may be described as 'cognitive conflict' with the second question. This was followed by discussion and an active engagement in proving formally the angle property of the triangle. This would set the context for what was to follow: an introduction to Thales and the importance of proof. By starting with proof it was hoped that the context and relevance of the two primary events would be provided, hence overcoming the first problem. As for the second problem, if students were already familiar with geometric proof then this would be revealed in the discourse, but hopefully in such a way that its importance could be discerned and thus not appear superfluous – thereby reintroducing proof rather than 'mistakenly' introducing it. If proof is reintroduced then the significance of the 6 levels of abstraction could be placed in the context of how proof was first made possible, hence relinquishing the third problem. As it turned out the learners were not familiar with proof, including the Masterclass.

THE MATHS MASTERCLASS SESSION

The duration of the session was one and a half hours. I began by stating the format of the session and the ground-rules for discussion, such as listening to each contribution, not to ridicule the answers given by others, etc. I then asked the question 'the 3 angles of a triangle add up to what?' and received the reply 180. I followed with 'how do you know?' For a few moments there were signs of cognitive conflict. Someone mentioned the tearing of the angles demonstration but a student responded that the demonstration only applies to the triangle torn, to which I added 'what of all triangles?' Another student suggested measuring the angles, but two students responded that the measurement would only apply to the triangle measured. I also added that I spent last Saturday night measuring the angles of a 100 triangles and only twice did I measure 180.0, with the mean average 180.7.

I asked how we can show the angle property for all triangles and someone suggested constructing a line parallel to the base of the triangle drawn on the board so that the angles of the triangle can be related to the angles that make up a straight line. I demonstrated the proof as he said it, adding that we can assume that alternate angles are equal when a line cuts two parallel lines. I asked whether he had seen the proof before but he said he could not remember. We then discussed the nature of this proof with respect to all triangles and I mentioned and explained the term 'logical necessity'. I also explained that proof forms the heart of mathematics.

This was 20 minutes into the session. At this point I formally introduced the session and outlined what we were going to do. I then introduced the class to the Egyptians, their practical geometry and Thales' contemplation in the desert of ideas existing above and beyond the observed configurations that once served to represent those ideas. I then took them outside to sketch the four stakes and two intersecting ropes and to take them through the 6 levels of abstraction.

One has to bear in mind that this is not in school time and hence compulsory. Nevertheless, out of the 36 students, 35 sheets were handed in. 11 were highly detailed, with notes and comments written in earnest next to the drawings. You can sense pride in these notes, with 6 stating that they want the notes/drawings back. Although it was made clear that these sheets will be collected (and returned if desired), the notes are in a sense personal and not constructed for evaluation by the teacher. 15 of the remaining sheets contained the necessary drawings accompanied with relevant notes but the commentaries concerning level 6 were scant. The remaining 9 contained diagrams and notes but were in some ways incomplete. Interestingly, one sheet from this sample was unique in constructing a parallel line to prove that opposite angles are equal using alternate and corresponding angles.

The following is a sample taken from the sheets. This is a small sample because much of what has been stated has been expressed (albeit in different ways) by others. Of course, much of what was written was prompted by discussion.

Level 1: ***Literal representation***

'Leaving behind angles, measurement.'

'Shadows, grass, tone, daises, flies, detail, colour and 3 dimensionality have been left behind while the 'shape', the outline, the essence of the image remain.'

'Left behind: 3D, colour, reality, texture, actuality, physicality, daises. Taken: idea, basis, images, impression, representation.'

'Drawn forth: rope, tree, stakes, mud, grass, daises [the detail in the accompanied drawing shows each, including the texture of the rope]. Left out: building, traffic, road, people.'

'Taking a basic idea of the image and abstraction, but the actual configuration is easier to see and you are leaving behind the 3 D image.'

'Can only be seen from your point of view. Can't see it from a different angle.'

'We are taking an image representing how our eyes perceive what is before us'.

Level 2: ***More abstract representation***

'Left behind the physical nature of the object. Taking the shape with you.'

'Now the image is left behind and only the position and presence of the stakes and rope remain.'

'Taken: shape, idea. Left: reality, size, perspective and above [referring to level 2].'

'Drawn: the distance between the points. The points of attachment to the ground. Left: everything else'

'This is a more basic symbolic view and the actual thing is more graphic.'

Level 3: ***Model***

'Allows freedom of sight.'

'Now the size is left behind and some of the appearance, but scale and everything else of the original stakes remain. It can be seen from different perspectives, so it is superior to the 1st level of abstraction.'

'Able to see anyway we like.'

'Drawn forth: the straightness of ropes and stakes.'

'More perspective. Less actual representation. More accuracy.'

'Rotation can be done.'

'We take something similar to reality, We can perfect reality, proportion'.

Level 4: ***Private Concept***

'No sticks, no dots. General idea of 2 intersecting lines – no specifications, no measurements.'

'All you take is the general idea of two intersecting lines. It is general because there is no specificity about the length of lines, where they cross or the angles at which the lines cross. All we have is that they do, at some point, cross.'

'Imagination – 2 lines intersecting – no sticks – general idea - no specifics (where lines cross, length, angles) – no scale – just 2 lines.'

'The intersecting lines remain; everything else is lost. Angles in mind are immeasurable, as are lengths – they can be changed at will.'

'Different to everyone elses. Cannot be measured. Can be changed.'

'No one can see it. It's in my mind'.

Level 5: ***Authorised concept***

'The concept that has been established by a community, in this case mathematicians.'

'Not necessarily true (e.g. world being flat).'

'Agreed concept found in textbook'.

'- agreed upon by mathematicians – in text books etc. – established by a community. (Is something true because its in a textbook? Is something in a textbook because it is true?) Just because people think something does that make it true? Belief does not make something true. For something to be really true we need proof. What is the nature of truth? Thales (first person to prove anything) asked how do we prove opposite angles are equal? Opposite angles are equal because of logical necessity, it cannot be any other way. It's not possible for them not to be true.'

Level 6: *Platonic Form*

'The idea is always true even though it may not ever of happened. The idea which is totally independent of human minds.'

'What is truth? Truth is something abstract that has no place; it is everywhere at all time. Something does not need to be proven to be true – it is true already, but we have to prove it to know as humans that it is true. Because potentially the triangle could have existed, it is true, although it has not necessarily happened.'

'These forms are more real than physical reality! Did they exist before humans. Humans discovered it they did not invent it, so it must have existed before – Plato's argument. The concept can exist without anything else.'

'Truth doesn't depend on human agency, the truth is what is real. . . . Whether people are still around, things will still be true even if there is nobody to say it is true'.

'Not opinion, not consensus'.

'Real truth lies beyond what we can perceive as truth.'

'Where does this immortal truth reside? Its just there. Everytime 2 lines intersect. It's independent of whether there are 2 lines that intersect. ('Platonic Form' – named after Plato circa BC 450). Potentially true. There was the potential for triangles before humans, but no way of knowing if there were. Concept of triangle independent of humans. Humans make things an actuality – potential for anything. Thales changed geometry from practical to concept – objects in the mind. Only then can properties be explored. Ideas that are abstract and concept, actually more real than physical objects.'

After level 5 and for the remaining 30 minutes the class went back into the classroom for a discussion on the Forms leading to proof. At this stage there was very little note taking as all the students seemed fully engaged with the discussion. The discussion developed into whether mathematics is invented or discovered. At some stage I led the class through a formal proof concerning opposite angles by asking questions. I then asked them who was it that actually performed the proof, to which a student stated 'we did', followed by smiling faces. This was a cue to introduce the slave-boy in Plato's Meno, who under Socrates' questioning was able to prove Pythagoras' theorem without having any previous mathematical experience, raising the question as to how the slave-boy knew the answers to the questions. I concluded the session by placing the Forms in their cultural-historical context.

YEAR 9 WIDENING PARTICIPATION SESSIONS

These consisted of two groups of about 30 students each. The session for each group was half an hour less that the Masterclass session. The session format would essentially be the same, but the ‘mixed ability’ of the class and the shorter session meant a focus on the 6 levels of abstraction and the opposite angle proof.

After preparing the groundwork the sessions began with the two questions. On both occasions there were suggestions of measurement and the tearing demonstration. Both groups followed the discussion on the limitation of these suggestions and were able to respond to questions leading to the formal proof after I suggested the construction of a line parallel to the base. I praised both groups for their ability to prove, told them the importance of proof and briefly introduced them to Thales.

Both groups went outside to undergo the 6 levels of abstraction and all took the exercise seriously. When everyone closed their eyes there was no silly behaviour.

There was, however, very little or no commentaries on the sheets. This is perhaps to be expected as this was a younger group of ‘mixed ability’. Nevertheless, many of the drawings were of considerable detail. The students were able to give examples of what was carried forward and left behind for levels 1 to 4.

The discussion of level 6 was restricted to the existence of geometrical objects, leading to the opposite angle proof. By their engagement with level 6, the majority of students appeared to follow the idea that a geometrical object is not something we can draw and were able to answer questions concerning the dimensions of geometrical objects, whether or not we can see them and whether they existed prior to humans. They were able to answer questions leading to the opposite angle proof.

CONCLUSION

The aim of the pilot study was to see the viability of engaging secondary school students with the two primary events leading to the nature of proof. Was the study successful? A ‘yes’ to the question can be asserted on the grounds that the level of engagement by each group was consistently high, verified by observers. The sessions could have been disasters, with students regarding the exercise pointless with low level disruptive behaviour as a result. The active participation of all the students was visible and was not a result of politeness to the occasion. This prompts the next stage in the research, comprising a more objective evaluation of the initiative. This would involve the observation of a series of lessons within the classroom setting coupled with an evaluation of learning outcomes.

Of course, any evaluation of the project will involve the extent to which the teacher can engage the class as it would the implementation of the two primary events. The ability to pose the appropriate question at the right time so as to lead the class to the desired outcome, the ability to handle diverse responses and the ability to captivate using historical narrative are examples of skills that can be developed and formalised as an outcome of evaluating the initiative.