SUM AND DIFFERENCE PROBLEMS AT KEY STAGE 2

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A comparison is made of the responses of 11 year olds to two questions involving the sum and difference of two numbers. The first question was asked on a Key Stage two National Curriculum test and was presented in algebraic form. The second question was presented as a word problem and was designed to be more accessible to children. Fewer children were successful in solving the problem expressed in words, though more attempted it than the version presented algebraically. We explore the methods used to tackle both questions and the incorrect answers given as well as offering a tentative comparison between the two.

INTRODUCTION

This paper concerns children's responses to two questions about the sum and difference of pairs of numbers. The questions are drawn from different phases of a project being carried out jointly by the Centre for Mathematics Education at the Open University and the Mathematics Test Development team at the Qualifications and Curriculum Authority. The first phase of the project concerned analysis of children's responses to the 2001 Key Stage 2 National Curriculum tests. The question discussed here is shown below and we will refer to it as 'P and Q'. Children were allowed to use calculators in answering questions on this test.

p and q each stand for whole numbers.

$p + q = 1000$

p is 150 greater than q.

Calculate the numbers p and q.

Phase two of the project involved a specially designed paper consisting of questions related in some way to those analysed in phase one. The new question concerning sum and difference was about two girls who had 60 stickers between them, with one having 8 more than the other. Children were asked for the number of stickers one of the girls had. We will refer to this question as 'Stickers'.

BACKGROUND

Problems in which the sum and difference of two numbers are stated have been around for centuries and are often attributed to Diophantus in the middle of the third century AD. Methods of solving such problems have developed over the centuries and a study by Harper (1987) set out to consider whether the development of methods over the secondary age group showed a similar pattern. A sum and difference problem was given to 144 grammar school pupils with 24 pupils drawn from each of six year groups in the school. The solution method of the youngest students (11 to 12 year olds) is particularly relevant to the current study. Only 4 of the 24 students in this age group produced solutions using the methods being considered by the

researcher. All these solutions were classified as 'rhetorical', meaning no use was made of algebraic symbols, but a procedure was given for calculating each number. Some students also used methods classified as 'trial and error' and other methods were classified as 'incomplete' and 'erroneous'.

Other studies have asked children to solve sum and difference problems presented in context. Some have also included interviews with children about their methods of solution. One interview appears in a study carried out with secondary school students (Hart 1981). A student was interviewed about a question concerning 10 sweets shared between two boys so that one has 4 more than the other. The student said that one child would have 9 and the other 1. The interviewer concentrated on the phrase '4 more than' in an attempt to probe the solution, but the student stood by their answer, even replying "yes" when the interviewer said "The one that's got 9 has got 4 more than the boy with1?"

In the interview described above, adult intervention did not seem to influence the student's answer. In another study (Laborde, Puig and Nunes 1996) adult intervention had a stronger influence, though another factor may be that the two numbers concerned had an odd total, meaning that the incorrect method probably used in the study above would not give a sensible answer. In this case students were asked to solve a problem in which 75 books were shared between two classes so that one class had 13 more books than the other. The students being interviewed started by finding half of 75, checking and laughing at their answer of 37 ½. At this point the interviewer asked 'Did the classes have the same number of books?' As a result of this and further prompts, the students moved to a correct solution, starting by subtracting 13 from 75.

METHOD

Phase one of our study was based on analysis of children's answers to National Curriculum test questions. The scripts used came from the sample used for the official analysis (QCA 2002) and consisted of 458 papers selected to represent children drawn from a range of schools and to include approximately a third of children at each of the National Curriculum levels 3, 4 and 5. Responses were initially classified as correct answer, incorrect answer or no response. Responses giving correct answers were then classified according to the method shown; those giving incorrect answers were classified according to the answer.

Following phase one, questions were considered with a view to re-wording them to see if this altered children's ability to answer. In the case of the P and Q question there were three main changes. The first was to remove the formal algebra, the second was to set the question in a context which it was hoped would make sense to the children, and the third was to make the numbers smaller. It was expected that all these changes would make the question more accessible.

For phase two the questions were trialled informally, and then assembled to form a paper which was completed by 364 children drawn from 7 schools. Children were not

allowed to use calculators in answering these questions. In selecting the sample, the intention was to match the national profile, particularly as far as levels of attainment were concerned. Teacher assessment levels were supplied for all pupils, with 20% assessed at level 3, 53% at level 4 and 27% at level 5. Analysis of individual questions was carried out in the same way as for phase one.

FINDINGS

P and Q Question

Initial analysis of children's answers is shown in the table below.

Summary of answers (458 scripts)		
	Number	Percentage of total number of scripts (to 1%)
Correct answer	104	23%
No response	88	19%
Incorrect answer	266	58%

Table 1: P and Q, Summary of answers

Just over a quarter of those answering correctly showed no evidence of their working or method, despite the fact that a box was provided for this purpose with the label 'Show your method. You may get a mark.' From those scripts which did show a method, we identified two methods based on the structure of the problem. Children using the first method found half of 1,000, then half of 150, then added or subtracted the latter from the former. The second method involved subtracting 150 from 1,000, finding half of what was left to give the first number, then adding 150 to find the second number. Other children used trial and improvement, or spot and check. There were a small number of solutions we were not able to classify. There were no correct solutions using algebraic methods such as substitution. (See table 2)

Analysis of correct answers (104 scripts)		
	Number of scripts	% of correct answers to 1%
Correct p=575, q=425 no working	27	26%
Find half of 1,000, half of 150 etc.	16	15%
Subtract 150 from 1,000, find half etc.	22	21%
Trial and improvement using pairs totalling 1,000	3	3%
Trial and improvement using pairs with difference of 150	9	8%
Other trial and improvement	4	4%
Spot and check	17	16%
Other working	6	6%

Table 2: P and Q, analysis of correct answers

Of those children not giving correct answers, 35% either made no response, or did not complete their response. A small number of children either gave the correct answers in the wrong order, or gave one correct answer. Looking at the other incorrect answers showed that many children gave pairs of answers which matched one but not both of the constraints given. 159 children gave pairs with a total of 1,000 and 30 gave pairs with a difference of 150 while 32 gave other pairs. (See table 3)

Analysis of incorrect answers and no response (354 scripts)		
Answer	Number of scripts	% of incorrect answers to 1%
Incomplete or no response	123	35%
P+425, q=575 (wrong order)	8	2%
P=575 or q=425 (one correct answer)	2	1%
350, 650 (either order)	40	11%
Other pairs totalling 1,000	119	34%
Other pairs with a difference of 150	30	9%
Other pairs	32	9%

Table 3: P and Q, analysis of incorrect answers

Stickers question

Initial analysis of the stickers question showed the vast majority of children giving an answer, with only 4% making no response. However only 15% of children gave the correct answer. (See table 4)

Summary of answers (364 scripts)		
	Number	Percentage of total number of scripts (to 1%)
Correct answer	55	15%
No response	16	4%
Incorrect answer	293	81%

Table 4: Stickers, summary of answers

For this question a large box was provided under the answer box with a label saying ‘How did you work out your answer?’ All the children giving the correct answer wrote something in this box and in most cases it was possible to classify what was written according to method. The most commonly used methods were the two ‘direct methods’ involving halving, as described for P&Q. Some children used spot and check or trial and improve and there were a small number of other methods.

Analysis of correct answers (55 scripts)		
Method	Number	% of correct answers to 1%
Find half of 60, half of 8 etc.	20	36%
Subtract 8 from 60, find half etc.	13	24%
Spot and check	6	11%
Some attempt at trial and improvement	9	16%
Other methods	7	13%

Table 5: Stickers, analysis of correct answers

Most children not giving the correct answer did give an answer and most, though not all, of these also showed how they had worked it out. The most common incorrect answer was 38, arrived at by finding half of 60 then adding 8. Far more children gave this answer than the correct answer (192 compared to 55). The next most common incorrect answer was 68, given by 42 children and arrived at by adding 8 to 60. It is possible that these children did not understand the structure of the question, though they may have used the word ‘more’ as a clue that addition was required and simply added the numbers given in the question.

Analysis of incorrect answers and no response (309 scripts)		
Answer	Number	% of incorrect answers to 1%
38	192	62%
22	16	5%
68	42	14%
52	16	5%
32	7	2%
Other	20	6%
No answer	16	5%

Table 6: Stickers, analysis of incorrect answers

COMPARISON AND CONCLUSIONS

Perhaps surprisingly, more children were successful on P and Q, though more attempted stickers. Not all children giving correct answers to P and Q showed their method, perhaps because the instructions were phrased differently, so some caution is needed when comparing methods used for arriving at correct answers to the two questions. Despite this it is worth noting that there were some similarities. In both

cases more children got correct answer by 'direct method' based on the structure of the problem than by trial and improvement or similar methods. Caution is also needed in comparing incorrect answers as one question asked for pairs of answers and one for only one. However in both cases the most common incorrect answer was obtained by halving the total then adding the difference. Some differences in the question, the sample and the conditions of the test make it difficult to draw firm conclusions about comparison of the two questions. However it does seem reasonable to conclude that the changes made in designing the second question did not lead to many more children being successful as may have been anticipated. It may be that the problems with the original question were not due to the formal algebraic notation as originally suggested, or that these problems were simply swapped for another set of difficulties associated with word problems. Another possible explanation is that the real difficulties were with the structure of the problem and that misconceptions associated with this structure persisted however the question was presented.

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