MENTAL CALCULATION METHODS USED BY 11-YEAR-OLDS IN DIFFERENT ATTAINMENT BANDS: SUBTRACTION QUESTIONS IN THE 1987 APU SURVEY

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This is a continuation of a reanalysis of data on 11-year-old children's responses to mental arithmetic questions that were obtained as part of the APU's (Assessment of Performance Unit) mathematics survey in 1987, using categories developed by Beishuizen in the Netherlands. The sample of 247 children was divided into three bands of attainment as measured independently by their scores on a written test of concepts and skills. Differences in the responses within these bands are illustrated for three subtraction questions.

INTRODUCTION

The data described here, on 11-year-old pupils' responses to mental arithmetic questions, were obtained in the 1987 Assessment of Performance Unit's (APU) practical mathematics tests. Between 1978 and 1987, the APU, set up by the then Department of Education and Science (DES) in the UK, carried out a series of surveys of 11- and 15-year old pupils' mathematical performance (Years 6 and 11) in the schools of England, Wales and Northern Ireland. The Unit commissioned an independent research agency, the National Foundation for Educational Research (NFER), to conduct the surveys and report the results.

The APU practical tests were administered orally by experienced teacher-assessors in one-to-one sessions with pupils. The teachers acting as assessors were nominated by their LEAs and trained to administer the tests by the NFER research team. About 1 200 pupils were selected for the practical tests from the main probability sample of about 13 000 who took the more standard written tests of concepts and skills, and so were reasonably representative of their populations. About 200 schools participated in each practical test sample and assessors spent a day in each school, usually testing six age 11 pupils. Each pupil in the sample was administered questions from up to three of the dozen or so 'topics' developed for a survey, the mental skills test being one of the topics in 1987.

All the topic tests were untimed so that pupils were given as much encouragement as possible to demonstrate what they knew and could do. The one-to-one situation with its oral delivery of questions provided opportunities for controlled interaction between assessor and pupil that did not necessarily imply the use of concrete materials. This interaction, especially the 'probe' question, 'How did you get your

answer', provided rich information on pupils' methods. In the mental skills topic a booklet with the questions was placed before a pupil who was told by the assessor:

> I'm going to give you some questions that I want you to work out in your head. The questions are in this booklet. Here is the first question. Please read it to me and then work out the answer. After each one I'm going to ask you how you worked out your answer – that's not because you have got it wrong (or got it right), I'm just interested in how you worked things out.

Assessors were provided with a 'script' that included instructions about what to say to the pupil and how to present materials and questions. In response to a probe from the assessor, pupils could elaborate or explain their method further. The initial response and method were recorded as well as any changes in either resulting from a probe, a corrected answer counting as the outcome.

Pupils' standardised concepts and skills test scores (with a mean of 100 and standard deviation of 15) were taken as an independent measure of their general attainment in mathematics so that the mental calculation methods used by children in different attainment bands could be examined. A total of 256 pupils took the mental skills test, all but nine having taken a written test. The sample of 247 who had taken both tests was divided into three bands according to their written test scores as in Table 1.

Third	**Written test scaled score**	**n**
Top Third	107 - 128	81
Middle Third	95 - 106	80
Bottom Third	68 - 94	86
Total		**247**

Table 1: The written test scaled scores within the three attainment bands

SUCCESS RATES FOR THE TEST QUESTIONS

There were 12 calculation questions in the test, ten of which are being analysed in this project. Four of the ten questions were purely numerical, while the numbers in the other six were set in everyday contexts. There were two easy addition questions and a difficult multiplication question with a low success rate, but the remaining

seven of the ten questions had middling success rates between 48 per cent and 61 per cent.

The results for three of the ten questions are considered in this paper. These three questions, numbers 2, 5 and 8 in order of presentation in the test, can be described as subtractions, although this was not necessarily the operation used to carry out the required calculations (see Table 3). The questions are listed in Table 2 below, together with the percentage success rates for the sample as a whole and for each of the three attainment bands separately. As would be expected, the success rates fell from the Top to the Bottom Thirds of the sample.

No.	Question	Total Sample n=247		Top Third n=81	Middle Third n=80	Bottom Third n=86
2	64 – 27	61		77	70	38
5	I buy fish and chips for £1.46. How much change should I get from £5 ?	60		83	56	42
8	I catch a bus at 9:43 am and arrive at my stop at 10:12 am. How long does my journey take?	48		72	51	22

Table 2: **Percentage success rates in Total Sample and in each Third for Questions 2, 5 and 8**

THE CLASSIFICATION OF STRATEGIES USED

The mental calculation strategies used by pupils have been classified using categories developed by Beishuizen in the Netherlands and England (see Beishuizen and Anghileri, 1998). These have been placed under two broad categories, Complete and Split number strategies. Children using a Complete number strategy would operate on one of the numbers in the calculation without splitting it into tens and units, or, in the case of a measure, without splitting it into

its separate units. Split number strategies are the reverse: both numbers are split into tens and units or both measures into their separate units. Some examples are given in Table 3. For a fuller list of examples see Foxman (2001).

Question	Complete	Split
64 – 27	64 – 20 – 7 27 + 30 + 7	60 – 20; 7 – 4 (or 4 – 7) → 40 – 3 (or + 3)
Change from £5 for £1.46	£5 – £1 – 46p £1.46 + 54p + £3	£5 – £1; £1 – 46p → £4 + 54p
Bus 9:43 to 10:12	9:43 + (17 + 12) min 9:43 + (2 +15 +12) min	(10 – 9) hrs ; (43 – 12) mins → 1hr + 31mins

Table 3: Some examples of the Complete and Split mental calculation strategies

In addition to the mental calculation strategies, a number of children described a standard written algorithm as their method, working from right to left, and others used unclassified methods or were unable to respond. Table 4 lists the percentages of the sample using Complete, Split or Algorithm strategies (C, S, and A) and the success rate within each of these categories. Unclassified and non responses are not included in the table.

The following points are noted about these results:

- *Complete number strategies were the most successful for all three questions*, marginally so in the purely numerical calculation, 64 – 27, but overwhelmingly in the bus journey time question. This is true within all the attainment bands in all three questions, with the sole exception of the bottom band for 64 – 27, where those choosing the standard algorithm did slightly better than Complete number strategy users.
- Of the two broad categories of mental calculation strategies, *Complete number methods were far more successful than Split number methods*, even more so in the two lower attainment bands than in the top band.
- For all three questions, *Complete number strategy use declined from the Top to the Bottom attainment band, while Split number strategy use increased from Top to Bottom.*
- *Complete number strategies were used far more frequently than either Split methods or the Algorithm for working out the in context questions.* Only in the case of the purely numerical question 2 was the standard algorithm a more popular method, particularly for the middle attainers.

No.	Question	Attainment Band	% success				% sample using strategy		
			C	S	A		C	S	A
2	64 – 27	Top (n=81)	85	56	81		49	20	26
		Middle (n=80)	85	27	78		34	19	45
		Bottom (n=86)	58	26	63		22	41	19
5	Change £5;£1.46	T	87	58	83		77	15	7
		M	73	40	33		53	25	15
		B	61	22	57		48	31	8
8	Bus trip 9.43;10.12	T	81	50	33		79	12	4
		M	77	5	0		59	26	3
		B	51	0	0		42	42	0

Table 4: Subtraction questions. Percentage of sample using Complete, Split or Algorithm strategies, and their success rates within Top, Middle and Bottom Attainment Bands

CONCLUSIONS

There are two principal conclusions that arise from the data on these three subtraction questions:

1. It is unlikely that mental calculation strategies were taught in primary schools in 1987. Despite this it is clear that a large proportion of 11-year olds of that era had invented mental calculation strategies, especially for the calculations in context illustrated here.

2. Of the two categories of mental calculation strategy, Complete and Split numbers, the former were considerably more effective than the latter. It is therefore not surprising that the top band of attainers used Complete number strategies more than the middle band, who, in turn tended to use them more than children in the bottom band. Conversely, pupils in the bottom band used inefficient Split strategies more than those in the middle and top bands of attainment.

The main significance of these findings is that the two mental computation strategies represent different attitudes towards numbers. The Split strategies suggests that numbers up to 100 are viewed as consisting of tens and units and children using them attempt to deal with these values separately. Such strategies can frequently lead to the sort of errors that occur when using the written standard

algorithm as the examples given in Table 3 illustrate. By contrast, Complete number strategies treat numbers as wholes. Furthermore, the calculation steps are sequential so that subtotals are operated on as they occur and do not have to be stored separately in memory.

This reanalysis of APU data from 1987 strongly suggests that Complete and Split number strategies are differentially used by children at different levels of attainment and are differentially effective at all levels. Unfortunately, the National Numeracy Strategy appears to treat them under the same heading of 'Partitioning' strategies (Thompson, 2000).

REFERENCES

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