

DIFFICULTIES WITH MATHEMATICAL WORD PROBLEMS

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***Abstract:** The extensive literature on students' 'difficulties' with mathematical word problems raises many issues for which the prevailing mostly quantitative paradigms and approaches to research appear inadequate. My own research, which draws on discursive psychology (Edwards, 1997) makes use of a word problem task to generate interaction. This discursive approach, which I set out and exemplify in this paper, offers a fresh perspective on some of these issues.*

Difficulties with word problems.

Research on students' proficiency in solving word problems is overwhelmingly quantitative in nature, generally entailing relatively large samples of students completing carefully designed batteries of problems. The prevailing paradigm within which hypotheses are made sees students as modelling the 'reality' described in word problems (Greer, 1997). Students' errors are caused by inappropriate or defective models, which may in turn relate to the way the problem is set out, or to the students' background. Thus, some of the key variables which have been explored include: the semantic structure, clarity and word order of the problem (De Corte and Verschaffel, 1991), the language of the test items (Yoshida, Verschaffel and De Corte, 1997) the mathematical proficiency and language background of the students (Bernado, 1999; Clarkson and Dawe, 1997) and the level of 'real-world knowledge' required (Verschaffel, De Corte and Lasure, 1994). A particular focus has been on students' apparent failure to interpret their solutions 'realistically'. For example:

1128 children are going on a trip in buses. Each bus can carry 36 children. How many buses are needed? (Greer, 1997: 294)

In responding to this and similar problems, students frequently leave their solution as a decimal; the 'correct' response requires them to round up to the nearest whole bus,

a finding that has been replicated around the world (Greer, 1997). Such consistent outcomes cause some perplexity amongst researchers. In concluding a study of their own on the application of realistic considerations, Reusser and Stebler (1997) ask:

Why is realistic mathematical modeling [*sic*] in the school context of word problem solving so difficult? What are the reasons that a significant proportion of students who participated in the different studies almost systematically:

- Gave mindless, even absurd answers to most of the problems?
- Showed scarcely any recognition of the indeterminacy or unsolvability of problems?
- Revealed a significant tendency to exclude realistic considerations from their interpretations of problems? (Reusser and Stebler, 1997: 323).

These questions largely concern *processes* of interpretation, thinking and meaning. The outcome-based quantitative approaches typical of research in this area are unsuitable for investigating such issues. Although my own research did not set out to investigate such questions, it has involved the development of a qualitative approach which can shed light on Reusser and Stebler's concerns. In this paper I outline this approach, illustrated with an example of the analysis of two students' interaction before making links back to issues arising in word problem research.

Researching multilingual classrooms.

The starting point for my research (e.g. Barwell, 2000a, 2000b, 2001) is an interest in how students for whom English is an Additional Language (i.e. not their first language) participate in the exchanges and discussions that take place as they work on their mathematics. Interpreting what students say in such multilingual, multicultural environments is, however, far from straightforward. What students or teachers mean by what they say depends on their experiences, their 'individual histories' (Bruner, 1996) which includes their linguistic experiences. In classrooms in which students come from several different cultural and linguistic backgrounds, the interpretation of meaning becomes problematic. A key part of this work, therefore, is the development of a way of researching the interaction of students which accounts

for the personal, subjective nature of meaning. Arguably these considerations are just as important in investigating interaction in classrooms which might not be seen as multicultural, since all students bring their own experiences and cultures. This is an issue which is often overlooked in classroom research, relying on an implicit assumption that meanings and interpretations are somehow similar enough to be considered unproblematic.

Despite the complexity of interaction in multilingual mathematics classrooms (indeed in any classroom), all interaction is necessarily patterned. In participating in communicative interaction, we reproduce these patterns, and yet adapt them to our current concerns as we do so. Each utterance is uniquely produced in its moment of interaction and yet each utterance reflects and can only make sense in the history of previous patterns of interaction. For Bruner (1990, 1996) the patterns of language and behaviour make possible the interpretation of situated meaning (1990: 19). Bruner goes so far as to suggest there are “agreed-upon, canonical relationships between the meaning of what we say and what we do in given circumstances” (p19), although this fails to capture the constant, creative process of re-production, which blurs and shifts the boundaries of the canonical. This blurred canon can nevertheless form the basis of resources that cultures provide for interpreting and making meaning in interaction, and which links the historical dimension of cultural patterns with the situated nature of their reproduction. Thus, although I do not have access to students’ individual meanings, I am able to look for the patterns which form part of students’ ‘tool kits’ as they are revealed through students’ interaction.

Patterns alone, however, are not enough to deal with the complexity of interaction and its analysis. Discursive psychology (Edwards and Potter, 1992; Edwards, 1997) offers both a detailed theoretical perspective of interaction and a methodological approach designed to avoid the problem of the subjective nature of meaning. My interpretation of this work forms an important staging post in the development of my own methodology. Discursive psychology conceptualises language as primarily “a

medium of social action rather than a code for representing thoughts and ideas” (Edwards, 1997: 84): social action is foregrounded as the primary function of language, which is seen as having evolved through social interaction, and therefore as being structured both by and for the on-going moments of social interaction. The patterns of language through which these different actions take their form derive from each individual’s experience of social interaction, their cultural and linguistic history. This focus on the actions performed by talk therefore leads to an analysis of classroom discourse which asks “not what do children think but how do children think” (Edwards, 1993: 216).

Discursive psychology also offers an approach to discourse analysis which emerges from the theorisation of interaction presented above. The process of analysis is based on the principle that language-in-use makes explicit that which participants are concerned with, and as a result, also makes their interpretations available for analysts (Sacks, Schegloff and Jefferson, 1974: 728-729). Building on these ideas and work in conversation analysis and ethnomethodology, Edwards and Potter (1992) outline five distinctive aspects of the discourse analysis of discursive psychology: (1) analysis is of naturally occurring talk and prepared texts; (2) analysis is concerned with the content of talk and its social organisation, particularly its sequentiality; (3) analysis is concerned with action, construction and variability. Different ways of talking are used in different circumstances and for different rhetorical purposes; (4) the rhetorical organisation of talk and thought is designed to counter potential alternative versions which may arise; (5) it is the consideration of ‘cognitive’ issues such as intention or meaning in terms of how they are dealt with in discourse that leads to this approach being characterised as ‘psychological’, including looking at how participants construct and rhetorically deploy psychological concepts such as meanings or intentions in interaction (see also Edwards, 1997; Barwell, 2000a). The power of this approach in examining intercultural interaction derives from its focus on what is done through talk, rather than what is meant, leading to an examination of how students

think together through their interaction. These ideas are exemplified in the analysis below, following a brief outline of the research context.

Research background.

‘St Thomas Primary School’ has approximately 150 students from a variety of cultural and linguistic backgrounds. In Year 5 of 1999-2000 there were six students recognised as EAL. I hoped to collect recordings of these students as they worked in order to obtain naturalistic records of their interaction. As classroom-based recording proved impractical, however, small groups of students were withdrawn from the classroom and recorded while they worked on a task together. Although not identical to classroom situations, the teacher frequently asks students to work in pairs or small groups in this way. Furthermore, the combinations of students were based on groupings which I had observed during mathematics lessons. Thus although the interaction was not completely natural, neither was it particularly artificial.

The (pilot) research design involved selecting a topic from the teacher’s schedule for the term. In this case the topic concerned calculators, including some work on using calculators in the context of money. Six pairs or threes of students were recorded both before and after the calculator topic working on a task drawn from the week’s work. The lessons were also video-recorded and both audio and video recordings were transcribed. The task used in the taped pair work was to write word problems ‘about money’ and ‘to do with calculators’. I had previously observed the teacher ask students to write and solve word problems. I had also observed that students were sometimes asked to work together on such tasks. A calculator was provided. The students were also asked to solve their problems. It is noticeable that research on word problems rarely involves students writing their own problems. Where this does occur, it is generally the nature of students’ problems which are analysed (e.g. English, 1998), rather than how students go about writing or solving them. In

principle, however, such an approach has the potential to reveal insights on both students' mathematical thinking and their mathematical experiences (Silver, 1994).

The discussion and analysis offered in this paper involves two students: 'Safia' comes from a Somali speaking background and arrived in the UK about 18 months previously; 'Rahim' is of Punjabi/Urdu speaking background. Both were assessed in Year 5 as having English language proficiency of level 2 ('starting to communicate in English') bordering on level 3, on a 4-point scale. Although this research concerns learners with English as an Additional Language, I will not comment particularly on this in the following analysis. It is enough for the purposes of this paper to observe that the analysis provides evidence of what such learners can do, in contrast to deficit models which seek to identify 'barriers' or difficulties. The transcript excerpt below was recorded after the calculator topic and shows the solution of Safia and Rahim's second word problem (below), which they solve immediately after writing it together. Their problem, as written is:

Chris want to the shop and brot 2 crips wich cost £4 each and brort 5 drinks wich cost £10 each. What is the total

Excerpt: solving the word problem [1].

S um/ if you add/ four pounds to ten pound/ which will make fourteen/ I *'that' =*
 130 don't need to use that *calculator*

R no you have to use that/ two pound because there are two fourteen/
 four pounds/ and/[(...)

S [no no if you just bought two crisps/ and it costs four
 pound [if and/ each yeah/ oh

135 R [each

S not one

S oh yeah/ forgot

R you've got your two [four pounds

S [four add four

140 R no let me do it for you

- S add [ten
 R [cause there's not two tens
 S equals/ [eighteen
 R [eighteen pound
 145 S pound
 R add/ add/ ten/
 S look
 R ten
 S four/ add/ four/ add/ ten/ equals eighteen
 150 R where's ten?/[each
 S [ten pound/ oh [ten pound/
 R [you've got five drinks
 S yeah five drinks
 R you put you put it too expensive
 155 S I don't care/ (...) done it harder than that (there)/ add/ four/ add/ ten/
 add/ ten/ add ten/ add/ ten/ equals/ forty eight/
 R let me just double check/ ten pounds/ add ten pounds/ add ten pounds *calculator*
 S you have to add the four two
 R I know/ ten twenty thirty forty/[fifty
 160 S [ten twenty thirty forty fifty sixty
 R fifty/ add four/ add four/ equals fifty eight pounds
 S no you done it/ wrong/ but/ look// you've got four *takes calc*
 [add/ four/ add/ ten/ add/ oh/ I took ten/
 R [add/ four/ add/ ten/
 165 S I done it wrong/ four/ add/ four/ add/ ten/ add/ ten/ add/ ten/ add/ ten/
 add/ ten/ equals/ fifty eight/ you're right/ fifty eight
 R fifty eight/ pounds/ get the little sign

Analysis.

At the start of this excerpt, Safia offers a solution to the problem, adding “four pound to ten pound” (130) and then signals that she has finished by saying that she doesn't need to use the calculator (131). Rahim responds “no you do have to use that [the calculator]”, thus tactfully indicating to Safia that he does not regard her solution as

satisfactory whilst maintaining continuity with what she has said. He then begins a mathematical justification for his position, but is interrupted by Safia: “no no if you just bought two crisps/ and it costs four pound...” (133-134). At this point Rahim interjects “each” (135). Although he overlaps with what Safia is saying, he has proposed a possible completion for Safia’s sentence. His attention is on the word ‘each’, thereby giving it significance. In this way he guides Safia’s attention to ‘each’, indicated by her repetition of the word, at which point she stops (134). Rahim then reinforces the attention on ‘each’ by glossing it “not one” (136). Rahim has thus switched attention from mathematical justification to the wording of the problem. It is tempting here to infer that Rahim has ‘understood’ the source of Safia’s ‘error’. This is not an inference that can be made within the discursive psychology frame, however. In the rhetorical process of persuading Safia, Rahim recruits both mathematics and the words in the problem. Safia does now agree with Rahim, adding a face-saving “forgot” (137). Again, it is not possible to decide whether Safia really forgot or if she is just ‘covering up’ her mistake. We can say, however, that by invoking a plausible ‘forgetting’, she accounts for her concession to Rahim’s argument, an accounting which reaches back to her first attempt at a solution.

There follows a short sequence which comes to a provisional solution of eighteen pounds (145). Once again, Rahim gently reopens consideration of the solution-so-far, adding “add/ add/ ten/”, prompting Safia to run through it again: “four/ add/ four/ add/ ten/ equals eighteen” (149). Rahim first of all directs attention to the ‘ten’, “where’s ten?” (150), which Safia takes as a request to explain something about ‘ten’, but as she begins (151) Rahim adds “each” (150). This ‘each’ breaks Safia’s flow: “oh” (151). Rahim now offers further grounds for his gentle challenging of Safia: “you’ve got five drinks” (152). He has now focused their joint attention successively from ‘ten’ to ‘each’ to ‘five drinks’. His careful understated rhetoric leads to Safia trying another solution (155-156). Meanwhile Rahim offers a narratively oriented (Barwell, 2000a) comment on their problem “you put it too expensive” (154).

Interestingly Safia relates ‘expensive’ with ‘difficult’: “done it harder than that” (155).

Having apparently agreed on what their question requires, Safia performs a calculation out loud. Rahim again gently challenges Safia, inviting himself to “double check” (157). He also works out loud, which allows the calculation to be accountable. His total is different from Safia’s, which she accounts for by asserting “you done it wrong” (162). We cannot say if Safia really believes Rahim to be ‘wrong’ or not: we can only observe the accounting action her assertion performs. She begins to rework the calculation again, interrupting herself “oh/ I took ten”. It is not clear what she means here, but we are not concerned with meaning in this way. Instead we examine the action performed – in this case she is accounting in advance for a forthcoming backing-down, “I done it wrong” (165). It is not possible to say whether or not she did take ten or not; she is again plausibly accounting for ‘doing it wrong’. Furthermore, her form of words serves to counteract her previous account that Rahim was wrong. She can then repeat the calculation, again out loud, and agree with Rahim (166).

Finally, looking at the excerpt as a whole, there is an interesting interchanging between numbers as quantifiers (ten pounds) and numbers as numbers (ten) – compare lines 149 and 151, for example. There appears to be a loose but general pattern here. The more situationally precise numbers as quantifiers tend to be used at moments of dispute or tension. When Safia states “four/ add/ four/ add/ ten/ equals eighteen” (149), for example, she is routinely restating her calculation. When Rahim invites her say more, she reverts to using the quantifier: “ten pound” (151). The quantifiers seem to have the effect of clarifying and focusing attention on the narrative context of the problem. This context was of course established during their preparation of the problem immediately before this excerpt.

Summary.

In solving their problem, the discussion between Safia and Rahim hinges around the word 'each'. Other forms of analysis may try to decide whether or not Safia understands what 'each' means in the problem, or find some other way to explain her apparent difficulty with this word. Such questions are problematic. I am not familiar with Safia's first language. I am familiar with a language (Urdu) which does not use a specific word 'each' in problems like Safia and Rahim's. My unfamiliarity with Safia's language and cultural background make such considerations difficult, if not impossible. What I can say about their discussion, is that Rahim twice directed Safia's attention to the presence of 'each' in the problem, and that she then changed her calculation. The social nature of the students' discussion should also be noted. Rahim's challenges are carefully managed to direct attention to the mathematics that underlies the language of the problem, without threatening Safia's face. (This is in contrast to Safia's work with a more dominating student discussed in Barwell, 2000b). Finally, the use of quantifiers seems to link the students' discussion with the narrative context of their problem. This then is a brief example of a discursive analysis of two students working on a word problem. How can this approach be of use in examining questions from word problem research?.

Discursive psychology and word problems.

The above analysis suggests that discursive analyses of interaction may be able to shed some light on Reusser and Stebler's (1997) questions. Safia's initial solution to the problem she prepared with Rahim, would seem to be 'mindless': she adds together the two costs stated in their problem. It is not possible to say, however, whether this would be her 'final' solution if working alone, since it is not possible to separate out particular thoughts or beliefs from each individual, or detach thoughts from the overall situation. Their interaction unfolds contingently, turn by turn. In this excerpt, this unfolding ends with the two students coming to agree on a correct

solution. This is accomplished by paying attention to the language of the problem, particularly the word ‘each’. There are also indications that they are aware of the ‘realistic considerations’ of the problem, including the use of quantifiers at moments of intellectual tension and Rahim’s remark that the values in their problem are ‘too expensive’. The social aspects of their interaction are an additional and perhaps unexpected factor in the way their discussion unfolds. Rahim’s role seems to be in some sense dominant, though again we cannot say that he, for example, consciously supports Safia’s work, since his actions are contingent on hers.

This analysis shows how a detailed examination of students’ interaction as they work together on writing and solving word problems can reveal insights that are not available from quantitative studies. These insights relate to the process of thinking together, to the factors which students account for in their discussions, as well as a more general reflection of students’ previous mathematical experience, in particular their experience of word problems. An important feature of the approach exemplified here, is that it does not rely on prior assumptions about the nature of students’ thinking. In particular, it is not necessary to consider students as ‘modelling’ word problem scenarios, in some ways placing the research on a stronger footing.

NOTE.

1. Bold indicates emphasis. / is a pause < 2 secs. // is a pause > 2 secs. (...) indicates untranscribable. ? is for question intonation. () for where transcription is uncertain. [for concurrent speech. & for utterances which continue on a later line. Italic capitals for letter sounds – *T* for ‘tee’.

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