

## **Crossing Boundaries in the Study of Mathematics Teaching: An Uneven Odyssey of Learning and Unlearning**

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This presentation examines our work in the borderlands between mathematics and education. We trace across our six-year collaboration the development of one strand of our work: the reasoning of justification. We identify four phases of the work that have been significant in the development of our research, and discuss the challenges and benefits of crossing boundaries between our fields for the study of mathematics teaching and learning.

One of us (DB) is an educational researcher and former practicing primary school teacher, the other (HB) a research mathematician recently engaged in efforts to improve mathematics instruction. The plenary presentation both was and was about a collaboration between us. We recounted both the process and some of the substance of our interdisciplinary research over the past eight years, in the borderlands between disciplinary mathematics and mathematics teaching and learning. The records of our boundary-crossing work together revealed a series of phases, which we labeled in this presentation as “chapters”:  
Chapter I (1996): Finding and pulling the mathematics OUT of teaching;  
Chapter II (1997-1998): Interlinearity — Finding the mathematics IN teaching;  
Chapter III (1999-2001): Intertwining—Exploring the mathematical problems of teaching.

The central questions we address in our work are:

- What mathematics knowledge and resources -- skills, understanding, sensibilities -- are entailed by teaching?
- How are such mathematical knowledge and resources used in the course of teaching?

- How might teachers be helped to develop usable mathematical knowledge and resources?

In the plenary presentation, we focused primarily on the first two questions.

Our primary method is first to investigate -- not disciplinary mathematics, not curricula -- but practice itself, through primary records (video, student work, etc.). To immerse the audience in the nature of this approach, we started with a video segment of a third grade mathematics lesson, with which we invited the audience to explore the first two questions above.

This launched the story of Chapter I of our work. Using portions of a large multidimensional data base of an entire year of mathematics teaching (from which the above segment was chosen), HB, with guidance from DB, tried simply to document all of the significant mathematical events (tasks, student productions, teacher moves, class discourses, etc.) that bore on the questions above. This was hardly straightforward, as it involved interpretations of non-standard utterances and interactions, and often unconscious evaluative stances. This stage of the work not only produced an abundance of observational material, but it also served as a setting in which to negotiate the norms for our collaborative work in this area, reconciling those from our quite different professional formations.

Chapter II involved our interpretations and analyses of these observational data. We operated with Word files. Starting with an HB annotation of the primary data, DB might comment on, or, more often, challenge some of the claims made. Then HB would respond, and such a back-and-forth would ensue, always in the same file. This dialogue was interlinear, and we could track the succession by using a different color for each installment. These inter-linear, multicolored documents became, eventually, a rather large secondary set of data for our further research.

In the third phase of our work, which we referred to as Chapter III we began to develop some theoretical frameworks responding to aspects of the first two of our central questions above. Our presentation sampled a part of this work devoted to the “reasoning of justification,” in particular what is involved in teaching and learning such reasoning at the primary level. We identified two foundations for this, which we call the “base of publicly shared knowledge” and “mathematical language.”

The presentation concluded with some of the things we have learned through this work:

- The intertwining of interdisciplinary perspectives is a rich resource for the study of teaching and learning.
- The central importance of records of practice and of the analytical work.
- The reconciliation of norms, methods, discourse across disciplinary boundaries must be an ongoing process of negotiation, learning, invention.