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MISCONCEPTIONS OF MODELLING IN MECHANICS: A REVIEW OF THE RECENT A-LEVEL TEXTBOOKS IN MECHANICS

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Many A-level boards have included a modelling approach in the syllabus of their respective mechanics modules, and the recent plethora of textbooks, including those that represent the various exam boards (e.g. AEB, London, ME!), are structured according to this approach. This session will review the many recent A -level textbooks in mechanics that have included a modelling approach, and will attempt to show that these books are confused as to what modelling in mechanics actually is and the possible pedagogical consequences.

INTRODUCTION

Over the last decade, A-Level mechanics (formerly known as 'applied mathematics') has undergone profound changes. Before the changes, A-level applied mathematics was the application of the laws and principles of Newtonian mechanics to idealised examples. At that time, the vast majority of questions and examples involved point particles, inextensible strings, smooth pulleys and inclined planes without prior reference to any physical event (real or imagined). The average traditional and typical mechanics question mainly involved pure mathematics after the appropriate formula and 'rule-of-thumb' procedure was applied.

The most profound change in mechanics has been the inclusion of modelling. Modelling has changed the nature of the subject itself: from a highly 'theoretical' subject to one that describes the physical world by modelling it. Modelling in mechanics involves:

1. Representation of a physical object or person (real or imagined) as a *particle* or *body.*
2. Identifying the interactions with other bodies, and representing the interactions as *interaction variables.* The basic interaction variable in mechanics *isforce* (Hestenes, 1987).
3. Making assumptions so that the model can be simplified.
4. Applying the laws or principles of mechanics to the model.
5. Utilising the principles of pure mathematics to arrive at a result.
6. Comparing the result with reality.
7. Refining the model, if necessary, by including a previously omitted variable, and repeating the steps 4 to 7.

Within the modelling perspective, the typical traditional, and quantitative, mechanics question may be regarded as steps 4 and 5.

Along with the changes in the A-level mechanics syllabi are the parallel changes in the textbook literature (many of which are written on behalf of the various exam boards, reflecting their syllabus). The aim of this article is to review the new textbook literature on their treatment of modelling, and will show that much of the literature contains a confusion as to what modelling actually is and the possible pedagogical consequences.

MISCONCEPTIONS OF MODELLING IN MECHANICS AND THE POSSIBLE PEDAGOGICAL CONSEQUENCES

There is in the textbook literature a very deep and fundamental misconception as to what modelling in mechanics actually is. Consider, for example, the following extract *fromAEB Mechanics (1992):*

> *One of the simplest methods of finding a mathematical model for a given situation is to carry out an appropriate experiment to collect relevant data and from this data a formula relating the variables is found. This can be done using a graphic calculator, function graph plotter, spreadsheet or by simply drawing a graph .... This is a common method of mathematical modelling for scientific situations and most basic models in mechanics are found in this way. In this section you will have the opportunity to apply this method of approach to several physical situations* (page 2, my emphasis).

That 'this is a common method for scientific situations' simply isn't true. What the passage is actually referring to is empirical modelling, which is more akin to experimentation than it is with establishing models of phenomena involving the laws of motion. What is understood as 'modelling in mechanics' cannot be reduced to empirical modelling, and to claim that it is might have serious pedagogical consequences. For example:

> *The period of a simple pendulum: A simple pendulum consists of a small object attached to a fixed point by a string of length l. As the object swings back and forth the period of the pendulum is the time taken for one complete cycle. Investigate the relationship between the period and the length of the pendulum. Does the result depend on the mass of the object?* (page 3).

If the student manages to establish T *oc ll/2* then the student has empirically established a relationship between two variables, but consequently the student will be none the wiser as to the forces acting on the bob of the pendulum or have a qualitative understanding of the pendulum's motion. The student may realise that the result does not depend on the mass of the object, but the student still has to realise why! Empirical modelling may have a place in mechanics, but it does not constitute it's starting point and does not playa part in forming it's structure.

Understanding mechanics will not result from establishing connections from experimental data, and to argue that it will, might result in a catastrophic confusion as to the role of modelling.

The MEI A-level mechanics syllabus offers a choice between a modelling approach and an experimental approach for problem solving in their coursework component (see, for example, Edsall, 1992. A flowchart of the modelling and experimental approach is given on page 3 of Mechanics 1, MEI, 1993). However, to offer an experimental approach, as an alternative to a modelling approach, is to confuse the relation between modelling and experiment. For example, consider the following part of their 'experimental cycle' (Edsall, 1992):

> *Design an experiment ~ Conduct the experiment and derive practical results ~ Give a theoretical interpretation of results ~ Determine accuracy of solution of problem.*

To 'give a theoretical interpretation of results' presupposes the ability to model pnor to the 'experimental cycle'. According to Hestenes (1992), modelling precedes experiment such that experimentation is model deployment:

> *Experimental games can be classified as model deployment games. Deployment is the empirical component of modelling. Experimental deployment games differ from the theoretical deployment games just mentioned in that their objective is to test and validate models. Experimentalists might object that they are engaged in exploring the physical world for new phenomena, not merely evaluating models proposed by theorists. Experimentalists often underestimate the influence of theory even on their own activities, and positivism reinforces this tendency by claiming that theory is subservient to experiment* (author's emphasis).

Experimental games presuppose modelling games, so despite offering an experimental approach as an alternative to a modelling approach, the only real option that MEI can offer is a modelling approach (that requires measurement - step 6 in the modelling process) or an experimental approach that presupposes the construction of a model.

Hestenes (1992) spells out the pedagogical consequences of confusing the relation between modelling and experimentation:

> *One cannot discover what one cannot conceive. Likewise, students must become familiar with the Newtonian World before they can recognise reflections of the Physical World within it and use it as a conceptual tool for understanding the Physical World.*

Like a game of chess, *beginners must know these rules before they can play Newtonian games with any assurance* (Hestenes, 1992). If a student is to conduct an experiment, then the student must understand the theory in order to make sense of the experiment (the design, construction, aims and objectives of the experiment) - and this is consistent with Toulmin' s (1967) argument that the scientist does not perform an experiment without a theoretical point in mind, and that it is the theory that structures the design of the experiment!. Experiments can only provide a means by which theory can be applied to make sense of the phenomenon under investigation. This is consistent with Chalmers' (1978) point that *precise, clearly formulated theories are a prerequisite for precise observation statements. In this sense theories precede observation* (page 27).

The MEI's experimental approach, as an alternative to the modelling approach, therefore becomes superfluous, and would probably lead to confusion as to the way mechanics actually models the real world.

The SMP (1991) textbook *Newton's Laws of Motion* expresses a similar confusion as to the relation between modelling and experimentation. On the one hand, the book presents the modelling process, in flow-diagram format, that is not dissimilar to steps 1-7 above; yet on the other hand, the tasksheets presented to illustrate the modelling process are all experiments *(The bricklayer's lament, Shoot* and *Pinball* simply involve the measurement of variables). This confusion between modelling and experimentation is all the more apparent in the Nuffield (1994) textbook on mechanics. On page 5 the reader is told that *[you] will be following in the footsteps of many of the key names in the development of understanding of mechanics: Aristotle, Galileo and Newton.* Later, on page 8, the book states:

> *Before the work of the Italian scientist Galileo* (1564-1642), *ideas about motion were dominated by Aristotle (384-322BC) who thought that a ball would fall with constant speed. Do you agree with this? Galileo certainly didn't, and, in the best tradition of applied mathematicians, he adopted a modelling approach to answer the question, 'How does afalling body move?' First, he observed the motion of such a body. His method was more than merely descriptive: he collected data which he could analyse. Collecting data about falling bodies presented Galileo with a major difficulty because there were no accurate timing* .... *(my* emphasis).

The book then invites a practical activity to validate Galileo's findings, with the assumption that the acceleration is constant and is 9.8m.s-^{2}. In fact, with the exception of presenting the modelling process as a flowchart, the treatment given to modelling throughout the book is that of data collection [Also, Galileo's method of data collection is conflated. According to Chalmers, 1982:

Contrary to the popular myth, Galileo seems to have performed few experiments in mechanics. Many of the 'experiments' he refers to while articulating his theory are thought experiments. This is a paradoxical fact for those empiricists who think that new theories are derived from the facts in

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[1] Driver (1994) pedagogically illustrates this point: *The slogan 'I do and I understand' is commonly used in support of practical work on science teaching. We have classrooms where activity plays a central part. Pupils can spend a major portion of their time pushing trolleys up runways, gathering, cutting and sticking tangling metres of ticker tape* ....*To what end? In many classrooms, I suspect,* '] *do and] am even more confused'.*

some way, but it is quite comprehensible when it is realised that precise experimentation can only be carried out if one has a precise theory capable of yielding predictions in the form of precise observation statements].

According to Chalmers (1978), one of the widely held common-sense views of science is that scientific theories are derived in some rigorous way from the facts of experience acquired by observation and experiment. According to Koulaidis and Ogborn (1995), a common description of a scientific law is that it is a generalised statement or relationships among natural phenomena; and according to McComas (1996), one of the myths of science is that experiments are the principle route to scientific knowledge. With respect to the teaching of science to schoolchildren, Driver (1994) has spelt out the pedagogical consequences of this common-sense view of science:

> *Through the eyes of those initiated in the currently accepted theories of science, common school demonstrations, such as trolleys and ticker tapes, experiments with batteries and bulbs, or work with ray boxes, mirrors and prisms, appear to offer self-sufficient support for the underlying principles they are designed to demonstrate, whether it is Newton's laws of motion or the laws of reflection of light. If children fail to abstract and understand these principles from their experiments, it may be seen as the children's error for either not observing accurately or not thinking logically about the pattern in the results. The constructivist view of science, on the other hand, indicates the fallacy here. If we wish children to develop an understanding of the conventional concepts and principles of science, more is required than simply providing practical experiences.*

These currently accepted theories of science reflect an inductivist/empiricist/positivist VIew of science, and positivism (which asserts that scientific law is a generalised statement of sense perception) is the 'dominant ideology' in science (Bernal, 1969) and in education (Hwang, 1996). According to Heather (1976):

> *The positivist attitude has become something we have inherited from earlier generations and, like other aspects of inherited culture, it forms and constrains the way we think. It has become ....part of the 'natural order of things' which we tend to accept with unquestioning compliance.*

There is much evidence of this attitude in many of the textbooks over the years. For example, according to the A-Level book *Physics: Concepts and Models* (Wenham et aI, 1972), where the authors assume *that students using the text are familiar to some extent with the ideas and content of such courses as the Nuffield O-Level Physics course* (page v):

> *At its simplest, a law is a summary of observed, measurable behaviour. Philosophers have discussed at length the impertinence of scientists who assume that what is true for a necessarily limited number of observations made in the past is true for all similar observations both now and in the future* (page 5).

However, as Driver (1994) points out:

> *The most simplest view of the scientific enterprise is, perhaps, the empiricist's view, which holds that all knowledge is based on observation. Scientific laws are reached by a process of induction from the 'facts' of sense data .... .This inductivist position was criticised when it was first suggested by Bacon nearly 400 years ago, yet it has reasserted itself early in this century in the heuristic movement and later in some of the more naive interpretations of the discovery method adopted by the Nuffield science schemes* (page 43).

Modelling in mechanics is not the measurement of connected variables, and to proceed in that way will not develop an understanding of force and motion necessary to identify the forces involved in an unfamiliar situation. Mechanics is not the study of the connection between variables, rather, it is a paradigm that has meaning, and the meaning can be facilitated by textbooks provided the authors and the respective exam boards understand the nature of mechanics and the way it models the world.

REFERENCES

AEB Mechanics, (1992). *Mechanics Core.* Heinemann, London.
Bernal, J. (1969). *Science in History.* Vol. 3. Penguin Books Ltd. Harmondsworth, England. Chalmers, A. (1978). *What is This Thing Called Science?* The Open University Press, Milton Keynes.
Chalmers, A. (1982). *What is This Thing Called Science?* Second edition. The Open University Press, Milton Keynes.
Driver, R. (1994). 'The Fallacy of Induction in Science Teaching', *Teaching Science,* ed. R. Levinson. Open University PGCE Series. Routledge in association with the Open University, London.
Edsall, D. (1992). 'The Role of Coursework in Sixth Form Mechanics'. *Teaching Mathematics and its Applications.* Vol. 11, no. 4, pp 150-154.
Heather, N. (1976). *Radical Perspectives in Psychology.* Essential Psychology Series (No. F8), ed. P. Herriot. Methuen, London.
Hestenes, D. (1987). *New Foundations of Classical Mechanics,* Reidel, Boston.
Hestenes, D. (1992). 'Modeling Games in the Newtonian World'. *American Journal of Physics.* Vol. 60, no. 8, pp732-748. Aug.
Hwang, Ahn-Sook, (1996). 'Positivist and Constructivist Persuasions in Instructional Development', *Instructional Science.* No. 24, pp343-356.
Koulaidis, V., and Ogborn, J. (1995). 'Science Teachers Philosophical Assumptions: How Well do We Understand Them?' *International Journal of Science Education.* Vol. 17, no. 3, pp273-283. McComas, W. (1996). 'Ten myths of Science: Re-examining What we Think We Know About the Nature of Science'. *School Science and Mathematics.* Vol. 96, no. 1, Jan.
MEI, (1993). *Mechanics* 1. Hodder and Stoughton, London.
Nuffield, (1994). *Mechanics* 1. Longman, Essex.
SMP, (1991). *Newton's Laws of Motion.* Cambridge University Press, Cambridge.
Toulmin, S. (1967). *The Philosophy of Science,* (first published in 1953). Hutchinson and Co., London.
Wenham, E., Dorling, G., Snell, J., Taylor, B. (1972). *Physics: Concepts and Models.* World Student Series Edition. Addison-Wesley Publishers Ltd. London.