AN ANALYSIS OF STUDENTS TALKING ABOUT oRE-LEARNING' ALGEBRA: FROM INDIVIDUAL COGNITION TO SOCIAL PRACTICE

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Abstract

In this paper we report on a study with the aim of investigating how afocus on langlloge and meaning can assist students in reconstructing algebraic knowledge. The project is set in the context (?f ongoing work with students in Higher Education who need to develop their understanding oj" algebra ~f they are to make sllbstantial progress within their linde/graduate sllldin. Jhe prr?ject is based upon a belief that students' dWiculties with algebra are lallgllage-reloted We 1101'(' collected extensive data by means (?l videotaped sessions involving the stlldents talking oh()/ft their OWl! understandings (?l algebra. The stlldents involved were drawn/i'om courses ill illl/wl t('ocher education and engineering. This paper presents a detailed allalysis (?l the re·V)()I/ses (if olle stlldent and discusses the ways in which this sh~lted our attention as reseorchers./i·om lookillg ot ollr data from the perspective (?l individllol cO}..,Tnition toward~' one iJ?lormed by social prucflce themy.

Introduction

The Re-Learning Algebra project grew out of the difficulties many students have \Aith algebra which have been observed in the course of working in the arena of Academic Maths Support at Sheffield Hallam University. These students have considerable prior experience with algebra and many have undergone years of drill and practice. They have encountered algebra as both an ahstract topic in its own right and also within various contexts. Therefore any additional help offered 10 sllch students clearly needed to take account of previous experience but also needed to have a different emphasis. An approach which was seen to be successful in practice involved encouraging interaction using group activities in which the students could share their understanding and experiellL:e The activities also addressed the use and development of algebraic language and have previously been reported on in Elliott and Johnson (1995).

Related Literature

The literature cited in this section of the paper helped to formulate our thinking and has informed our discussions during the course of this study. There has been much research on algebra at the school level e.g Lesley Booth (1984), including the studies concerned with school children's errors in algebra and an analysis of these errors leading to a categorisation of types of meaning associated with algebraic notation. Anna Sfard and Liora Linchevski (1994) developed their *tl1em\' oj re[fication* according to which there is an inherent process-object duality in the majority of mathematical concepts. They propose that the development of algebraic thinking is accomplished by means of a sequence of ever more advanced transitions from the operational to the structural. In particular they consider two especially crucial transitions: that from the purely operational algebra to the structural algebra 'of a fixed value' i.e. an unknown and then from there to the functional algebra of a variable. Carolyn Kieran (1989) emphasises the recognition and use of structu re as a major area of difficulty in algebra. She reports that for students, who tend to view the right hand side as the answer, 'the equation is simply not seen as a balance between right and left sides nor as a structure that is operated on symmetrically'. Carolyn Kieran and Nicholas Herscovics (1994) propose the existence of a *cognitive gap* between arithmetic and algebra which can be characterisee! as 'the student's inability to operate spontaneously with or on the unknown'.

We found a different emphasis in the work of Abraham Arcavi (1994) which seemed closer to our initial starting point for this study. He outlines what he terms as 'symbol sense' and which he compares to the notion of 'number sense' which has received far more attention His work is about describing and discussing behaviours and not about defining and describing research on students' cognition and ways oflearning. There is an emphasis on sense-making in mathematics and on recognising meaning.

Theoretical framework

Given the initial aim of this study, which was to investigate how a focus on langua.!-(e ane! meaning can assist students in reconstructing algebraic knowledge, we have sought to develop (I theoretical framework which takes account of this emphasis on language and meaning.

A key influence has been the work of Lev Vygotsky (1962), underpinning which is a Ct~ntral assumption that socio-cultural factors are essential in the development of mind. Intellectual development is seen in terms of meaning making, memory, attention, thinking, perception and consciousness which evolves from the interpersonal to the intrapersonal

In exploring the notions of sense and meaning further some useful ideas were drawn tl'om the field of activity theory. In particular Erik Schultz (1994) offers some interpretations of sense (111e! meaning, which we found to be helpful in interpreting some of our data He proposes that the purpose or intention ofa cultural product is the meaning and further that meaning is a kind of ' cultural intention' in a supra-individual fashion. Sense is the interpretation one makes of the meaning He argues further that language is a special kind of cultural product. We also found the work on activity theory of Kathryn Crawford (1996) to be relevant. She highlights how activity denotes personal (or group) involvement, intent and commitment that is not reflected in the usual meanings of t he word in English. In building upon Vygotsky's work, Leont'ev, Davydov and others made clear distinctions between conscious actions and relatively unconscious and automated operations. Operations are seen as habits and automated procedures that are carried without conscious intellectual dl'm!

Methodology

Data was collected by means of the video recording of a series of one-hour sessions With four groups of students during March 1995. The students involved were drawn fi'om courses ill Education Engineering. The groups had two or three sessions each.

A series of tasks was devised which were designed to get the students talking toget her about their understanding of algebra. For example the first activity involved 'Algebraic Pairs' In this activity each group of two students is given a set of cards with a pair of algebraic expressions on each. The task is to decide of the two expressions are always, sometimes or never equal. Another activity was to ask them to explain what they understood by mathematical words such as expression, equation, function, variable etc. The sessions were carried out in a small TV studio.

The initial data analysis involved the three researchers simply viewing the video tapes and discussing reactions and questions arising. Following the tape transcription this process was repeated with the transcripts. Our discussions were further informed by our ongoing reading. We also held two internal university research seminars during this period.

In this paper we have chosen one particular section of the transcript which we found to be particularly rich but also very challenging to us to make sense of in terms of the sta 11ing point of our study i.e. how an focus on language and meaning can assist students in reconstructing their algebraic knowledge.

Data Analysis and Discussion

This particular section of the interaction took place at the end of the first session ""ith the:2 Year BEd students. They had been working on the Algebraic Pairs activities for the first palt of the session and then had spent the latter part in a discussion of mathematical terms such as expression, equation, function, variable etc. As the session was almost complete, the researcher provided the opportunity for any questions, reactions or general discussion. The result was an extensive and articulate series of responses from one student in particular - Anthony (AG) Anthony IS a mature student who had previously worked in industry as an engineer.

1. **BH** OK, I was going to think about further activity but seeing as we've only five minutes left, I
2.think we'll end. Unless, are there any particular things that struck you as we've been talking, that 3. you want to return to, words which conjure up ...
4. AG It's obvious as we start talking about maths, we start talking about functions, some people
5. have got a clearer view; that my image I realise now, when I'm teaching, I tend to opt for, I like
6. to see it as, that y = some function, it could be a=3b plus something. I keep returning to y =
7. some function ofx and ifI saw it in a textbook for example that 2(x+3) my automatic reaction
8.would be to write y=(2x+3) before I give it to the children to do. y=2(x+3) 9. **BH** What would you be thinking of asking them to do next')
10. AG I'd be asking them to multiply the brackets out to give me a y=2x+6 or asking them to 11. substitute a value ofx and tell me what y is because that on its own as a function - 2x+3. J 12. suppose to me it isjust floating about in mid-air with no relationship to anything II'S totally 13. intangible, what is it? what's it for? So ifI ask them to multiply that bracket out I got :2x+ 3
14. before, now I've got 2x+6, still doesn't lead to anything, doesn't mean anything doesn't tell me IS. what it's from or where it's from - so my automatic reaction is to put the y= in Otherwise you've 16. got that floating about and that is a function, then you've got function. To me. what is a function, 17. where does it come from, where does it come from?
18. **BH** Y ou'd be happy to relate it to y. What would that mean then for you')
19. AG There's a missing number y and a missing number x and if we put any valul' in It)r y, or any

20. value for x .. .If we can find a value for y then we can find a value for x and if vou get into a
21. quadratic there'd be two answers for y, so actually you're using something to solve a problem.
22.**BH** Just taking that, say it was y=2x squared plus 6 times something ... You said tvvo values. 23. AG Again, as soon as you get an x squared, I tend to think that it's probably going to be two 24. answers. Depends on ...

25. AG I'm coming from a realistic point of view in that I've got a specific problem of trying to find
26. out what this value ofy is and in doing that I've made an equation in order to solve my problem
27. and in trying to solve my problem I might find that there are two values of the '\.
28. **BH** Say we had that? What about ifI said y was minus 10')
29. AG Minus IO? Then there might not be a solution to it .... no real solution. No solution to my real
30. world. This idea of no real solutions - you've gone into a hypothetical world Y ou\ e gone out of a
31. real life situation. From my experience, in my situation, you've gone out of a real life situation,
32. you're going back to a hypothetical situation. You're going right full back in circles to functions,
33. that's something hypothetical - that's floating about, not related to anything or solVing anything. It's
34. not come from any real life situation, it's just a function, it's not related to anything else I think 35. that's why I have difficulty in seeing where it's coming from.

An initial analysis of this section suggested a number of links with the background literature and theoretical framework previously outlined. In order to help the reader make sense of tile transcript, it is worth emphasising at the outset that Anthony does not distinguish between the terms function and expression. In fact he refers to 2(x+3) as a function rather than as an expression at line 7. In relation to activity theory there are a number of references to a lack of purpose when dealing with functions. For example at line 13, Anthony asks 'what is it') what's it for')' and at line 14 says that it 'still doesn't lead to anything' and goes further to say that it 'doesn't mean anything' This statement fits with Erik Schultz's interpretation of meaning as the 'purpose or intentic)Il of the cultural product' which in this case is the word 'function'. Anthony's description ,dso suggests that he is working operationally for much of the time e.g. at lines 7/8. he says that 'my automatic reaction would be to write y=(2x+ 3)' and also at line 15 he says that 'my automatil' reaction would be to put the y= in'. His comments also suggest a lack of appreciation of the structural properties of equations e.g. at lines 10111 he would 'be asking them (the children) to ... substitute a value ofx and tell me what y is' This suggests a view, consistent with the work of Carolyn Kieran, o1' 'the right hand side as the answer'. His comments at line 19 'There's a missing number y and a missing number x' suggest that he has not made the transition, in Anna Sfard's terms, from the 'stnlctural' algebra of 'a fixed value' to the 'functional' value of a 'variable'. It seems from Anthony's C\ll11l1lents that he sees the purpose of an equation as being to find a missing number and not to express a relationship. In Carolyn Kieran's terms, the equality relationship is not fully recognised i.e. the equation as a balance between right and left hand sides and as a structure to be operated on symmetrically

To an extent these observations are typical of many students although they were surprising to the researchers, as Anthony was seen to be a mathematically capable, though not strong, student. However much of what Anthony had to say was left untouched by this analysis and we were left with a sense of the inadequacy of the various theoretical frames, through which we had viewed our data, to account adequately for what Anthony had to say. It seemed that there was evidence of resistance to 're-Iearn' algebra on Anthony's part and much that was being said about his sense of identity and also his view of the nature of mathematics. None of this seemed to have been addressed in our first readings of the data. As a result of wider discussions with colleagues we decided to look to social practice theory for a 'wide(r) angle lens' (Robert Dengate and Stephen Lerman, 1995) through which to view our data. In particular we turned to the work ofJean Lave and Etienne Wenger (1991) and that ofJean Lave (1996).

Jean Lave and Etienne Wenger stress the essentially social character oflearning and propose learning to be an aspect of a process of participation in socially situated communities of practice They discuss the notion of Legitimate Peripheral Participation (LPP) which describes the particular mode of engagement of a learner in a new community of practice, whose level of pal1icipation is at first legitimately peripheral in the practice of the expert. The move from peripheral part icipation to full participation is seen as a dynamic process, characterised by changing levels of participation. Writing in 1996, Jean Lave describes the direction of movement as a *Ie/os* and gives the example of 'becoming a respected, practising participant among other tailors or lawyers, becoming so imbued with the practice that masters become part of the everyday life of the Alley or the l1losque for other participants and others in their turn become part of their practice' She proposes that this might form the basis of 'a reasonable definition of what it means to construct identities in pracl ice'

Returning to the analysis of the transcript, it seems that there is considerable resist,lI1ce on Anthony's part to re-construct his view of algebra. His view ofa function is that 'it is totally Intangihle' (I 13) and 'with no relation to anything' (113). It is 'floating about in mid-air' (I 12), withoul meaning e.g. 'what is it?' or purpose 'what's it for?' (113). It seems that Anthony's view of mathematics is only meaningful if 'you're using something to solve a problem' (121). Having a problem to solve is real e.g. 'I'm coming from a realistic point of view' (125) and equations are simply tools to solve 'my problem' (127) e.g. 'I've made an equation to solve my problem' (125). In formulating his views on the nature of mathematics, Anthony also seems to be saying signiticant things about hi~ own sense of identity. His background is that of an engineer working in industry over many years and his path into Higher Education and teacher training would have been via vocational routes Ant hony seems to be calling on his previous experience (as expert) in this particular community of practice and also on his developing expel1ise in the practice of 'school teacher' to emphasise his identity as a part of the 'real world' e.g. 'my experience, my situation' (I 31). This contrasts with his view of the community of practice of mathematicians, as exemplified by the researcher, who inhabits' a hypot hetical world' (130) and who has departed from the real world e.g. 'you've gone out of a real lite situation' (I 31). He stresses his view that the researcher/mathematician is going nowhere e.g. 'Y ou're going right full back in circles to functions, that's something hypothetical - that's tloating about, not related to anything or solving anything. It's not come from any real life situation, it's just a function, it's not related to anything else.' (131-35). However he does seem to express some sympathy and desire for a greater level of participation in the practice of being a mathematician when he says' I think that's why I have difficulty in seeing where it's coming from.' (1 35) This also seems to reflect his peripheral participation in this particular community of practice.

It seems that our interest at the outset of this study, in language and meaning, has given us a picture of some of the ways in which our students are working on re-Iearning algebra. However it has also revealed much more - a complex set of phenomena and questions with which to revisit both our data analysis and also the ongoing development of our own practice.

Acknowledgement

We would like to thank colleagues who attended seminars of the Mathematics Education Research Group and the Learning and Teaching Institute at Sheffield Hallam University during 1996 and also participants at the session at the BSRLM Conference in November 1996 for their contributions to the development of our thinking about this study. In particular we would like to thank Protessor Peter Ashworth and Mick Nott for their insights.

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