

British Society for Research into
Learning Mathematics



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Introduction

The aims of this project were to investigate the metacognitive skills and concepts possessed by students of secondary school age in some typical mathematical learning environments, to explore the feasibility of raising the levels of their awareness by appropriate interventions, and to study the effects of such enhancement on the students' mathematical attainments.

The outcomes of the project include, as well as these results detailed below; the Teachers' Handbook containing the set of suggested enhancement activities, trialled and including examples of students' work; the Evaluative Instruments, partially developed but needing further improvement; and the set of Case Studies of the seven classes during the main experimental year. (*Documents available at present from the Shell Centre are the Summary Report (£1), the Teachers' Handbook (£6), Evaluation (£6)*)

Our interest in students' awareness of their learning arose from our work in a previous ESRC project *Diagnostic Teaching in Mathematics* (ESRC 8491/1) (Bell et al, 1985). In this, a teaching methodology based on identification of students' concepts and misconceptions and resolution of the latter by exposure, cognitive conflict and discussion, proved to be strikingly more effective than more usual methods, particularly for longer term retention. What became clear was that such teaching methods demanded a radical change in the students' conceptions of what was appropriate activity in a mathematics lesson. An orientation towards obtaining correct answers had to give place to a recognition that the aim was to acquire correct, well knit concepts and methods, and that this involved being willing to expose ones own ideas and approaches, even if wrong, and to look for personal satisfaction in the enlightenment provided by participation in a focused discussion. This in turn depended on an awareness of the nature of this type of learning and its distinction from memorisation and fluency practice. This led us to consider the possibility of achieving improved learning across the whole mathematics curriculum by increasing students' awareness of learning methods and their purposes.

A substantial amount of experimentation in the encouragement of metacognitive activity in school and teacher education settings has been built around the PEEL project, based in Melbourne, Victoria, Australia (Baird and Mitchell, 1986; Baird and Northfield, 1992). In this project, a substantial number of teachers at a particular school worked concertedly at developing methods by which the students (aged 15-16) took greater control of their own learning.

An experiment somewhat similar to our own, but with a single class of primary school (year 6) children, was conducted by Herrington (1992). His one-year programme sought to improve learning strategy awareness, mathematical achievement and confidence towards learning mathematics; it used some 70 short interventions involving concept mapping, a Think Board, self-questioning and writing. Significantly better gains than those of a control group were shown on learning strategy awareness, and non-significant improvements in confidence and mathematical attainment.

Our own project has focused on enhancing reflective activities and on providing lesson experiences through which students may acquire specific knowledge about learning tasks and processes; and this in real classroom settings.

Aspects of awareness: Aims and objectives for the teaching and testing

1. To increase awareness of the components of mathematical activity.

Distinguishing facts and skills, conceptual structures, general strategies and appreciation and attitudes towards mathematics.

2. To increase awareness of mathematical content.

Knowing one's way round mathematics as a discipline, and being able to identify one's own existing state of knowledge.

3. To increase awareness of mathematical strategies

Knowing that acquiring mathematical strategies is such as these for problem solving and investigating a legitimate goal for learning; being able to distinguish, identify and thus ultimately to deploy them.

4. To increase awareness of types and purposes of mathematical tasks

Distinguishing, for example, the relative purposes of investigative practice tasks.

5. To increase awareness of the purposes of different ways of working

Distinguishing between activity and learning; appreciating that learning involves a positive orientation towards recognising what is available for learning, and seeking to *understand* as to become *more fluent* in it.

6. To increase awareness of resources for learning and how to use them

Selecting appropriate resources, including the teacher, other pupils computer or printed resources.

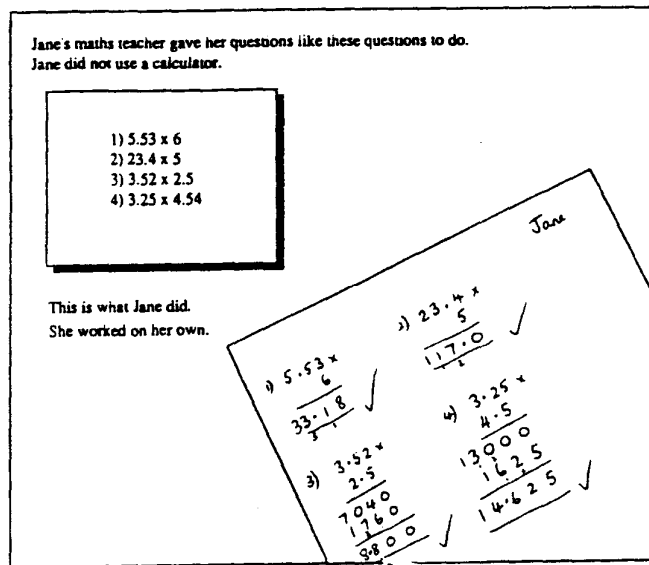
7. To increase awareness of general learning principles Appreciating, for example, the value of previews/reviews or of discussing causes of errors.

Evaluative Material

Three tests were produced covering different aspects of awareness .

. . The first 'evaluates awareness of the *purposes of different ways of working* and awareness of resources for learning, as well as exploring appreciation and attitudes towards mathematics. The second focuses on awareness of the *components of mathematical activity*, mathematical content and general strategies. The final booklet assesses awareness for *types and purposes of mathematical tasks*.

Some Pretest Results from the 'Purpose~' (c):'



What was the purpose?

2 - a main purpose, 1 - helped a bit, 0 - not a purpose, ? - not sure

This activity is to help Jane ...

- (a) to practice multiplying quickly and accurately
- (b) to know *when* multiplication is the right thing to do
- (c) to learn to work neatly
- (d) to think about what decimal numbers mean
- (e) to remember the roles for multiplying
- (f) to get better at discussion and explaining (g)
- to see how to use maths in her everyday life

Percents choosing this as a main purpose

Yr 7	Yr 9	Yr 10
52	58	79
17	22	32
9	12	15
25	37	46
26	41	62
6	8	9
43	21	15

The main perceptions of the purpose of the task become clearer from year 7 to year 10; these are the kinds of increase that one might expect. Similarly, the confusion with everyday life shows a decline, again corresponding with expectation. Perhaps the most significant observations relate to the purposes (b) and particularly (d) which we might interpret as showing that the students do not discriminate at all well between tasks aimed at the development of manipulative skill and those aimed at comprehension.

Ways of enhancing the meta cognitive aspects of learning

18 intervention strategies were developed, and appear in the Teachers' Handbook

Reflection and review activities

- a Students making up questions
- b Students reflecting on learning difficulties and misconceptions
- c Students reviewing and classifying
(Including students interviewing students, drawing concept maps
compiling review booklets, and planning an outline for a mathematics textbook)

Students as assessors

- d Students constructing tests
- e Students devising and using marking schemes
- f Students diagnosing errors
- g Students assessing themselves against statements of attainment
- h Students predicting their own performance

Students as teachers

- i Students teaching students

Students learning from text

- j Students writing meanings for headings and subheadings
- k Students using terminology and definitions
- l Students surveying the structure of text
- m Students sequencing pieces of text
- n Students composing text

Students reflecting on ways of working

- o Students conducting mini-debates
- p Students conducting small group discussions
- q Students observing students
- r Students describing what learning feels like

Many of these incorporate student role-reversal activities, for example, student as a teacher, an assessor, a textbook designer or an observer.

The effectiveness of the interventions

Reflection and review activities

These activities were readily seen by students as purposeful; student-produced posters and booklets designed to introduce newly arriving students to mathematics were one manifestation of this. *Diaries* on the

other hand, did not work well; they soon became a routine chore (as found also by Baird and Mitchell (1986).

Students as assessors

These interventions were used frequently and enabled students to appreciate more fully the purposes underpinning activities. Before students assessed work they were often called upon to devise a marking scheme; consequently they had to identify and order the underlying task objectives. This type of role reversal activity also provided a means through which errors or misconceptions could be constructively addressed.

When accompanied by a discussion of the relationships between topics and their relative importance *students constructing tests* provided a means of enhancing awareness of content. This technique was a useful diagnostic tool because it enabled teachers to determine the match between their intentions and the learning outcomes, and for students it provided a means through which they could realise more fully their own level of understanding, and on occasion, lead to a re-evaluation of their own state of knowledge. *Students constructing questions* was not confined to end of unit test construction; this ongoing intervention was also frequently used to enhance awareness of content. (See the illustration below)

Students as teachers

This was used by only two teachers, because it involves considerable time and relatively complex organisation and collaboration. For it to work successfully it seems essential that student-teachers are given time to prepare their methods and materials. On one occasion where adequate time was provided, the student-teachers were well motivated and organised.

Students reflecting on ways of working

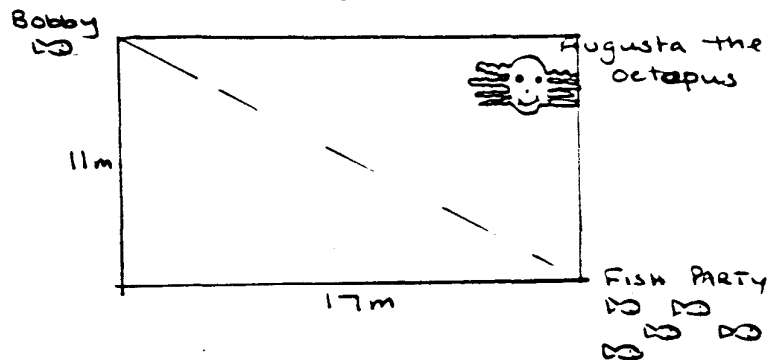
The review activity, *students interviewing students* was not trialled extensively but the available evidence suggests that it has value and is worthy of further investigation.

Students as observers was less frequently used, but the implementations that did take place provided positive outcomes.

Students conducting mini-debates was used by seven teachers, who all rated it highly. It involved students in constructing and defending arguments for and against different ways of working and resulted in vigorous and reflective discussion. It was adopted for in-service training. Some of the cards giving the students' briefs are shown below.

Alan Bell

BOBBY'S BIG PROBLEM



Bobby the fish is going to a party, but as he is swimming along he spots Augusta the octopus. Bobby has said he will babysit for Augusta's baby octopuses when she goes out, and she looks as if she is about to go out, but Bobby doesn't want to babysit now because he is going to a party.

He can swim 21 metres before Augusta can corner him. Will he reach the party?

Debating Cards for Group A

Cut out these cards and share them among your group.

Between you, prepare a 2 minute argument supporting each statement.

Share out this work so that it does not take too long.

1. You learn more from working on one hard problem, than from working on ten easy problems.

2. You learn more from getting things wrong than from getting things right.

4. You learn more by trying to explain something yourself, than by listening to someone else explaining.

5. You learn more by working on a lot of short problems than by working on a few longer investigations.

7. You learn more from listening to a good explanation than by working on problems .

8. It doesn't matter if you copy someone else's work, so long as you understand it.

Symbolising experience and experiencing symbolisation

Tony Brown

Manchester Metropolitan University

The aim of symbolisation .. is to store a realized experience in such a way that this can be preserved and communicated ~

In this quote from Barbara Dockar- Drysdale the topic of conversation is not mathematics. Rather she is talking about a process through which emotionally retarded children might be helped to organise their memories. This session was concerned with exploring the consequences of seeing symbolisation within mathematics in this way.

Dockar- Drysdale's work is directed towards heightening the awareness of her students in capturing their experience towards making sense of it. This seems to be to do with grouping aspects of experience in a way which enables the student to recollect and hold on to certain things. I have discussed this in relation to mathematics in Brown (1991). Categorising or what Gattegno calls *structuring and ignoring* is an essential part of getting to know the world. Symbolising has a dual function of holding onto things and of sharing things. But it may be that I use society's mode of symbolisation in holding on to things in a personal way.

As a student of mathematics it seems my task is necessarily both personal and social. I can experience mathematical phenomena in a personal way. Mathematics, insofar as it is the 'science of magnitude and number and all their relations' (Chambers Dictionary) can be unshackled from social conventions. I can decide a bag is heavy or light or I can estimate how many strawberries I can eat, without needing kilograms or a counting system. However, it seems I cannot go very far into mathematics, especially if I wish to share my experience, without entering into culturally derived notations (see Brown, in press). Given that the whole of mathematics is a social construct how can we speak of 'personal' mathematical activity?

In watching an animated geometry film by Nicolet these issues can emerge in a powerful way. I can watch and experience such a film whatever my age. However, I can only account for this experience in words, which immediately reveal my cultural and educational background. I can experience but I cannot share without using words. Gattegno (1988) distinguishes between Geometry and Algebra. The former might be seen as following the drift of the film - getting a sense of how it moves, feeling how shapes transform, predicting how the film will proceed etc. Algebra on the other hand is about categorising the experience of the film in words. In Gattegno's sense school geometry is largely algebraic with the insistence on partitioning phenomena into labelled things.

However, my everyday use of words is part of the experience I bring to the film and so I find it hard to think of the film without imagining 'circles', 'straight lines', 'tangents', etc. Cultural norms of classifying represent themselves in the

language we use, and this cultural dimension of our personal experience cannot be disregarded. To paraphrase Marx, 'Man does not speak language but rather language speaks man'. The organisation of the world implicit in language reveals the society which produced the language. Language use by an individual declares an affinity to the society which engendered it.

In this way the experience of mathematics cannot be disentangled from the society which produced it. Any naturalness in mathematical engagement by a child dissolves soon after leaving the womb, as this engagement becomes imbued with the linguistic framings the particular society favours. Lacan suggests the child moves towards having its world captured in language as it moves further away from identification with the mother. As the child increasingly speaks the language of the society he or she gets trapped within it (Brown, Hardy & Wilson, 1993). The personality of the child becomes immersed in the sociality of the society. We submit to the authority of a society and all the initiations into ideologies that entails. In so doing we suppress the powers of the personal to engage geometrically.

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A STORY OF A STORY: SOME METHODOLOGICAL ISSUES ARISING FROM

NATURALISTIC RESEARCH IN MATHEMATICS

- Case study is the way of the artist, who achieves greatness when through the portrayal of a single instance, he communicates enduring truths about the human condition. •

(Macdonald & walker 1975)

This is not meant to be an exhaustive inquiry but simply a brief account of some of the issues posed by case study and in particular the processes of validation. What it is that I am searching for when I seek to substantiate my interpretation of a case study ? In what sense are the insights that I yearn 'enduring truths ' ?

Data Collection

On this occasion the primary data, an interesting vignette of mathematical activity was collected by hurriedly scribbled transcript. I am always very aware when working with case studies that I am telling a story about a story of an event. The primary source of data is not what actually happened but my portrayal of that event. Lakatos (1970) reminds us that we are not looking for facts to ground theories but for " interpretative theory to provide facts and explanatory theories to explain them." Is it possible to unravel what really did happen ? Having relinquished the notion of objective truth ,can I give a subjective account of what I experience entirely expunged of ideological bias ? I would assert not. Judgements and partiality are even inscribed in the language I use to describe the experience.

How accurate is my original story on this occasion and what did I miss in capturing it? The odd word perhaps but then there are the visual indicators missing of course, the pointing, the expressions. Then there are the syntax indicators, the pauses and intonations which break the undifferentiated sound string into meaningful ideas, I as scribe insert these myself. It appears that in order to reconstruct the meaning of even the simplest of statements I bring with me much more than just a vast internal dictionary I also bring a complex web of interpretational skills and responses fashioned by years of experience. The meaning of an utterance cannot be established solely by reflection upon the actual words, regard must also be paid to the intention and circumstances of the speaker. Fortunately as a participant in this event I developed an empathy for the speakers and along with the bare transcript I took with me much ephemeral evidence.

Covert Effects of Data Collection

As a participant observer in the classroom did my presence in any way influence the unfolding of the event ? Most probably very little on this occasion, I was the class teacher and the sight of me scribbling on the back of a worksheet was clearly so unremarkable that nobody even sought to question what I was doing or why. This however brings to my mind a very different episode which occurred a couple of years ago when I decided, on a impulse, to monitor the 'on task' activity of some students with an observation schedule. To ensure that I would collect some interesting data I choose a pair of girls known to be easily distracted. Typically the girls began to chat and I immediately became aware of my dilemma . Clearly it was an activity I had not

thought through because I was the person who would normally censure this type of behaviour and yet to do so now would prejudice my account of the very behaviour that I was monitoring. It was possibly at this point too that the girls became aware of my particular interest in them. The scenario must have seemed puzzling, I was glancing at them frequently, but not reproachfully, and made no attempt to rebuke them for their behaviour. The episode quickly evolved to a more bizarre level when my activity clearly became the focus of their discussion. In one easy step I had actually precipitated the behaviour I was attempting to monitor. The incident was amusing and very transparent but how often am I unaware that I am provoking certain responses by my very presence? Neither is it just the 'human effect', even a video camera which enjoys a reputation for furnishing an impartial and dependable chronicle of activity can affect the unfolding of an event. Aside from childish exhibitionism it is not easy to ascertain which behaviour is enacted specifically for the camera but one snippet which I picked up as I was scanning a tape was, "Quickly somebody say something or she will take the camera off us." How many vital pieces of evidence slotted into my jigsaw were just thrown in to stop me turning the camera away I mused? Neither is it just the social sciences that are prey to such soul searching nearly to be sure of the evidence they are collecting. Pawson reports that Collins (1975, 1981)

" Has produced some influential work which relates to how science operates in the absence of a secure observational base."

He cites the physics of gravity waves as an example of an area in which it is immensely difficult to collect data. The apparatus used to investigate Einstein's prediction that the gravitational forces on earth fluctuate as a result catastrophic events in outer space is a vast metal bar weighing several tons, it is suspended in a vacuum chamber to protect it from influence by other more localised forces. When minute variations in the gravitational forces are recorded, before attempting to explain them in the light of Einstein's theory, the debate rages as to how the scientists can be sure that the oscillation is caused by a super nova and not a man drilling up the pavement out side !

Observation is Theory-Laden

"Numbers gathered without some knowledge of the regularity to be expected almost never speak for themselves. Almost certainly they remain just numbers." (Kuhn 1961) . Most probably I already have a theory which prompted me to notice what I did. For example, I am continually noticing that observation is theory-laden. Very often after experiencing a new insight I find that in the next few days I am aware of instances of it everyWhere I go. I am often left wondering whether there has been a sudden epidemic, or if the cases were always there and I never noticed them before Language operates in a similar way, it does not determine thought most would assert, but it does regulate what we pay attention to. That is, I notice around me things that I recognise and can easily categorise or label. I do not of course always have one specific theory overtly in mind when I go out to collect data. I do however always have at my disposal a network of conjectures and suppositions, interpretative schema, within which I can situate new experience and observational data. I have built up this interpretative network over the years, distilling it from external data and internally generated reflective knowledge which I have sifted, classified, refined and generalised .

Validation Techniaues

" Any statement can be held true come what may, if we make drastic enough adjustments elsewhere in the system. " (Quine 1951) Possibly I may now wish to convince others, to whit, 'a friend and an enemy', that I have not made too major an adjustment in the system! It may be to add to my personal conviction, to increase the credibility of my research or simply to jump through an academic hoop ! I have various options open to me, one is peer validation which appears a seductively simple process but is I find problematic in certain respects. My primary concern is that the more explicit I am to my colleagues about what I am paying attention to and the theoretical framework I am using to construe it - the more likely it is that they will see what I see. And, conversely, the less explicit that I am, the less likely it is that they will be able to focus on anything very easily. As a consequence I feel it is very difficult to construct a truly critical community. What does it mean then if people confirm my way of looking ? It certainly means that it is less likely to be completely unreasonable. But it could just be that they have not questioned my hypothesis rigorously. If they all completely disagree with me it definitely means that I must rethink, but must keep my mind open to the possibility that I am surrounded by a lot of flat earth freaks! Whatever the response I can never be certain that my interpretation was valid, I can only be hopeful that it was not a completely unsatisfactory fit?

A second option open to me is that of involving the students who were the subject of the study. This again raises certain issues Tripp (1983), who has explored the avenue of co-authorship and the question of who 'owns' the data ,concludes that "researchers are not so much outright owners as majority shareholders. As such they may decide policy, but they also need to justify it.The participants as minority shareholders, are entitled to criticize publically." Where the study is of covert cognitive process of students however other issues are involved. Time has almost inevitably elapsed since the study and the students will have undoubtedly developed in their understanding and knowledge of the work which they were exploring. Subsequent interests and insights will in the interim, even if this is only a matter of days, have imposed new structures on their memory of the event. It is also clear that the researchers account of the event will have analytic purpose which will be incomprehensible to the student. Rather, their contribution would be essentially on the level of examining the researcher's interpretation of their statements. In the event of a disagreement occurring however what criteria could be applied to establish the favoured version? Bloor concludes that "Members' pronouncements on findings cannot be treated as a test of validity ... (but) can generate material which is highly pertinent to the researcher's analysis ... not as a test (but) as data."

Theoretical perspectives on validation

"That multiple research methods should be employed in a variety of settings in order to gain a total picture of some phenomenon. " seems a commonly held view but as Silverman (1985) continues "Putting the picture together is more problematic than such proponents of 'triangulation' would imply. What goes on in one setting is not a simple corrective to what happens elsewhere-each must be understood in its own terms." Viewing a situation from different perspectives may however give me more confidence that

I have questioned my interpretative framework rigorously enough to ensure that it is supportable. For, as Heidegger cautions, "Our first, last and constant task is never to allow our fore-having, fore-sight, and fore-conceptions to be presented to us by fancies and popular conceptions, but rather to make the scientific theme secure by working out these fore-structures in terms of the things themselves."

Gadamer developing his mentor's theme doubts the possibility of developing universally valid principles. His position is that practical situations yield moral knowledge which guides the choice and judgement of the inquirer, whose values and beliefs are continually being tested and modified through his interaction with the world. It follows, that the meaning of a situation is not an objective characteristic derived from setting aside values and beliefs but rather it is an attribute generated by applying them to the situation. Elliot (1987) observes that;

"one does not first understand things and then interpret their significance for one's values and beliefs and the practices they shape. Interpretation constitutes a moment within the process of understanding itself."

Habermas supports a different paradigm, however, which asserts "one cannot reduce all social understanding to an interpretative science" (Elliot 1987). Following in this tradition Kemmis and Carr have developed 'critical theory' the function of which is "to expose those false beliefs which sustain practitioners' misunderstandings of their practice, to identify those organizational arrangements which frustrate the pursuit of genuine educational aims and purposes, and to indicate to practitioners what needs to be done for those misunderstandings to be removed and the adverse effects of these organisational arrangements eliminated." (Carr 1983)

This seems to be a very powerful narrative supporting the notion that there is an ideal position from which to view the playing field. And, that it is inhabited by critical theorists who are capable of adopting an objective stance in order create a framework of awarenesses that we, the practitioners, may use to "eliminate false consciousness" (Gubba 1990). Critical theorems can be rejected by the practitioner only in circumstances of free and open discussion, presumably by practitioners who have passed successfully through the debugging framework ! Who one might ask is going to monitor the consciousness of the expert who is "concerned with the development of explanations for the ways political and social forces in the society ideologically distort teachers self-understandings and practices." (Elliot 1987) . Elliot himself believes that "The moral science paradigm of educational research incorporates its own critical perspective. It does not need to be supplemented by a critical paradigm based on absolutist and objectivist assumptions about the nature of human understanding. The advocacy of such a paradigm could itself do with a little ideological unmasking, for does it not once more allow the academic experts to play God with the teachers?"

Conclusion

So how does my case study rate on the validation front ? Well I find that if I assess my findings against commonly held criteria (eg. Lincoln and Gubba 1985 'trustworthiness' of naturalistic enquiry) there are many ways in which it falls short of expectations. Firstly it is not replicable or 'confirmable' ,if I waited and watched for a year I would most probably not see a

similar episode enacted before me. Neither would I expect the result to be 'dependable', it is particular case rooted in particular circumstances and there is no guarantee that, even if the circumstances were identical, which they couldn't be, that the event would unfold in the same way. I would hope however that the findings were 'transferable', not directly, but through their link to a theoretical framework. It is this aspect of the research which establishes its robustness. Kilpatrick observes; "Each empirical research study in mathematics education deals with a unique, limited, multi-dimensional situation, and any attempt to link the situation considered in the study with one's own 'practical' situation requires an act of extrapolation. Extrapolation requires, however, that one embed the two situations in a common theoretical framework so that one can judge their similarity in various respects."

Leaving what is to me a most difficult question, are the findings credible? Wherein lies the credibility? Is it in the eye of the beholder? Is it in the research? Or is it in the researcher? Would the findings be more credible if John Mason had stated them rather than me? Certainly were the findings to be highly controversial, the credibility of the researcher would become an important issue. The same issue is at stake when I use quotes to help me marshall an argument, I attempt to make the argument seem more substantial by demonstrating support from eminent people. So why do I feel the need to demonstrate the 'trustworthiness' of my research? The reasons may be many and may vary from personal gratification to academic necessity. Does the apparent rigour of the validation process have any affect upon the perceived 'worth' of the research findings? How is worth measured? Well certainly when looking at research aimed at creating knowledge for teachers 'relevance' and 'affect upon practice' might seem two possible criteria. But strangely there appears to be a negative correlation between the rigour of the validation process and the accessibility of the research to teachers. On a physical level, the most firmly substantiated research findings are often hidden away in Phd theses, possibly to be eventually filtered down in a simplified version to a professional journal. On a cognitive level the language is often exclusive in the extreme and psychologically there can be a barrier, a mystique, which both researchers and teachers are guilty of promulgating.

I do not expect my case studies to establish knowledge by predicting regularities or determining rules. Rather I would hope that they would do so by generating new insights about mathematical activity by illuminating fragments from a whole set of interlocking theoretical frameworks (eg. linguistic, educational, epistemological). There is, I would assert, no ideal position from which to mount a critique of a piece of research. The standards any critic employs are inevitably limited by their particular perspective. And from that perspective, to me, validity lies in credibility and the overall cohesion of the story which *you* tell with its network of interrelated theories.

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Researching Geometrical Intuition

BSRLM Conference, Manchester 20.11.93

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Introduction

The term 'intuition' occurs frequently in discourse about mathematics. Rarely, however, is the term analysed or investigated empirically. In this paper I will describe a recent exploratory study of the nature and role of geometrical intuition in the solving of geometrical problems.

What is intuition?

The most substantial work on intuition in mathematics learning is that carried out by Fischbein (1987). In my work I have used Fischbein's definition which is that intuition is a cognition characterised by the following properties (*ibid* p 43-56): *self-evidence and immediacy* (in that extrinsic justification is not needed), *intrinsic certainty* (note that self-evidence and certainty are not the same), *perseverance* (so that intuitions are stable), *coerciveness, theory status, extrapolativeness, globality* (in that intuitions offer a unitary global view), *implicitness* (so that although the result of selection, globalisation and inference, intuitions will appear to be implicit). The behavioural task of intuition then is to prepare and guide our mental or practical activity. This links with Fischbein's further suggestion that there are "problem solving intuitions", for which, however, he provides no empirical evidence. Thus, in attempting to study intuition, the context of problem solving appears to be an appropriate place to begin as the following quotation shows:

"If we seriously want to recognise the role of intuition in problem-solving and virtually all mathematicians recognise its contribution - then we need to fill out this stage of representation with concrete accounts rather than detailed and prespecific theoretical accounts. "

Noddings (1985 p 348)

Why study geometrical intuition?

The role of intuition in geometrical understanding has been emphasised for some time (for instance see Hilbert 1932 and Van Heile 1986). Indeed Fischbein's work stresses the role of visualisation in the generation of intuitions. Furthermore, for instance, the final chapter of Piaget's *The Child's Conception of Space* (Piaget and Inhelder 1956) is called 'The 'Intuition' of Space'. Since then, however, although there has been work on the intuition of number (see, for instance, Resnick 1986) there appears to be little which directly considers the intuition of geometry.

In addition, the advent of computer packages such as *Cabri-Geometre* provides new opportunities for studying the approaches used in solving geometrical problems. Indeed, such use of the computer may make intuition more accessible for study. In the context of Logo, Papert says:

"I see the computer as helping in two ways. First the computer allows, or

oblige, the child to externalise intuitive expectations. When the intuition is translated into a program it becomes more obtrusive and more accessible to reflection. Second, computational ideas can be taken up as materials for remodelling intuitive knowledge a turtle model can help bridge the gap between formal knowledge and intuitive understanding"

Papert (1980 p 145)

One aim of my current work is to discern whether *COOri* has a similar effect.

A Framework for Discerning Geometrical Intuition

One of the most productive researchers in the area of mathematical problem solving has been Schoenfeld (Schoenfeld 1985, for example). This particular work offers not only a theoretical framework within which to base the research questions in the present study, but also a rich source of guidance on issues of methodology and on the design of the empirical work. In his theoretical framework Schoenfeld makes only passing reference to the role of intuition.

In his work Schoenfeld has used a number of geometrical construction problems in which he found that "insight and intuition come from drawing" and that two factors dominate in generating and rank ordering hypotheses for solution, the first being what he refers to as the "intuitive apprehensibility" of a solution. In other words, Schoenfeld's study shows that geometric construction tasks may be ones in which it may be possible to discern the role of geometrical intuition. However, Schoenfeld makes little other reference to the nature and role of intuition in problem solving.

My approach was then as follows:

step 1: analyse Schoenfeld's transcripts of students working on the geometrical construction problems to help discern *critical decisions in the solution of these problems*. The conjecture being proposed here is that geometrical intuition plays a part in the critical decisions that problem-solvers make when tackling geometrical problems.

step 2: use the *categories of intuition* proposed by Fischbein to discern examples of the use of geometrical intuition by Schoenfeld's subjects at moments of critical decision.

step 3: by interpreting the definitions of the *mechanisms of intuition* given by Fischbein, the factors which participate in the generation of these geometrical intuitions will be isolated. Thus the second conjecture is that certain mechanisms of intuition are influential in certain episodes in the problem solving process.

In this way I devised a framework for discerning geometrical intuition.

Experimental work

The following methodology was developed:

- 1) The categories and mechanisms of intuition proposed by Fischbein are integrated into the framework for characterising mathematical problem solving developed by Schoenfeld.
- 2) Protocol analysis is adopted as the method for generating and analysing empirical evidence with the techniques of Schoenfeld being used to design the experimental work.
- 3) The focus for the experimental work is the solving of three geometric construction problems (as used by Schoenfeld) by pairs of subjects working with the computer-based

geometry package *Cabri-Geometre* and their solution process will be analysed by reference to the techniques developed in 1) and 2).

The experimental work produced three case studies of pairs of subjects engaged in solving geometric problems using the geometry package *Cabri-Geometre*.

Method of Analysis

The approach to the analysis of the empirical data generated by the experimental work carried out for this exploratory study can be summarised as follows:

- 1) analyse the problem-solving sessions for critical decisions.
- 2) analyse the problem-solving protocols for evidence of anticipatory and conclusive intuitions at moments of critical decision
- 3) analyse the protocols using the episodes proposed by Schoenfeld and to discern the participation of the mechanisms of intuition.

What did I find?

In this paper I only have space to describe problem 1 which is given below:

Problem 1 You are given two intersecting straight lines and a point P marked on one of them, as in Figure 1 below. Show how to construct, using straightedge and compass, a circle that is tangent to both lines and that has the point P as its point of tangency to one of the lines.



Figure 1

From Schoenfeld (1985)

Critical decisions in the solution of Problem 1 are as follows:

1. Constructing a perpendicular line through P
2. Constructing the angle bisector of the angle between the two intersecting lines

However, while each of the pairs made critical decision 1 almost straight away, only one member of pair 2 immediately saw that decision 2 was also critical. The other pairs constructed a second perpendicular, perpendicular to the lower of the intersecting lines, and proceeded to move this into approximately the correct position. For pairs 1 and 3 this prompted them into drawing the angle bisector of the intersecting lines and the problem was solved.

For pair 1 the suggestion to draw the angle bisector was made quite tentatively:

TC: Yes ... Ah! Now would the centre of the circle lie .. I'm just thinking something slightly different now, because I'm just trying to think, there must be a way of securing the centre accurately .. and I'm thinking .. does the centre of the circle .. sit on the bisector of the angle that's made by those two lines ..

In contrast, for pair 3 one of the students was more certain

KH: I tell you the other thing we could do and that's to bisect that angle to find out where they should cross.

My claim is that these are examples of the influence of geometrical intuition. This is how the students accounted for their actions. In the case of pair 1:

TC: .. [long pause] .. well, partly previous knowledge. I wasn't .. completely sure. I wasn't saying 'Oh, yes. This is what does happen'. I just had a sneaky feeling that we were missing something and I couldn't work out what it was, but I thought, well I'm sure the angle .. there must be some connection between the angle between the two lines and the centre [of the circle]. So, let's put the line in and see what happens.

It turned out to be right, but it was just a sort of stab .. well, it wasn't a stab in the dark completely ...

I can't think why, but I was sure we should be bisecting the angle.

In the case of pair 3:

KH: Ohhh! .. [laughs] .. That's quite interesting because, maybe, .. the fact that there's a cross there [where the two perpendicular lines intersect 'opposite' where the original two lines intersect] actually encouraged me to think well, we need to know where the cross is going to be. Perhaps if we hadn't have drawn the other perpendicular it would not have come so quickly.

Looking at that picture now I think .. it's .. er .. er .. I mean just having that sort of cross there on the screen opposite the angle there, I mean, that just spells it out. I think perhaps that's why it just came so quickly.

In both cases the students had some difficulty explaining their actions although both previous experience and the visual image played a part in determining the course of action they were suggesting. In this context, Fischbein says, "Experience is a fundamental factor in shaping intuitions" (Fischbein 1987 p 85). However, Fischbein then goes on to say that "There is little systematic evidence available supporting that view, ie evidence demonstrating that new intuitions can be shaped by practice" (*ibid* p 85).

In terms of the visual image, Fischbein claims that visualisation "is the main factor contributing to the production of the effect of immediacy" (*ibid* p 103). Fischbein then goes on to relate visualisation to the domain of mental models. The evidence available from this study seems to support Fischbein's views in the domain of solving geometrical problems. However, further work is needed in order to say much more than that.

Conclusions

I began with two conjectures. Firstly that geometrical intuition plays a part in the critical decisions that problem-solvers make when tackling geometrical problems and secondly that certain mechanisms of intuition are influential in certain episodes in the problem solving process. It seems that the first of these conjectures is well-founded: geometrical intuition does indeed play an important part in the critical decisions that problem-solvers make when

tackling geometrical problems.

The second conjecture, however, remains tentative. One reason for this is that the analysis was examining points of critical decision for the *successful* solution of the problem, instances of geometrical intuition may, inevitably, tend to form points of transition in the problem-solving process or occur during planning and implementing episodes.

Further work currently being considered could examine, for instance,:-

- the relationship between experience and intuition
- the nature and role of visualisation in the generation of intuitions
- developing the methodology in order to encourage the subjects to explain more
- the use of different geometrical problems
- the relationship between intuition and proof
- more on the effects of using *COON*, in particular, focusing on the potential of *COON* to provide a 'window' on geometrical intuition

On this last point, further work is necessary before any firm conclusions can be made. Correspondence on this point, or any other raised in this paper, would be welcomed (my e-mail address is DKJ@uk.ac.soton.mail)

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CAN WE TALK ABOUT CONSTRUCTIVISM?

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Constructivism is the dominant view of learning, at least within the mathematics education community. It is not difficult to understand why: it offers a theoretical rationale for the desire of most teachers to shift the locus of authority and control from the teacher to the pupils; it offers a justification for mixed-ability classes and individualised learning; the powerful metaphor of children constructing their own knowledge seems to describe the processes which are currently emphasised as thinking mathematically, particularly in problem-solving. I would argue that constructivism has been widely adopted because it appears to suit existing approaches to teaching, not the reverse, that it offers a particular view of the role of the teacher and the nature of the classroom. On the contrary one of the major weaknesses of constructivism is that it offers no connection between its theoretical foundations, that children construct their own knowledge, and what the teacher should do. Even in a banked lecture hall of 300 students and one lecturer 'transmitting' material for hours students are still constructing their own knowledge. All that is argued by constructivists is that some situations will encourage better or richer constructions than others. In order to make claims about the structure of the classroom, role of the teacher, appropriateness of materials, that encourage 'rich' constructions, constructivists need to draw on other values and beliefs, such as the 'good' ways to teach I mention above. In this report of the presentation given at the meeting I will focus both on the ideas presented and on those I did not quite complete, as well as attempting to incorporate at least some of the comments made by the other participants.

Constructivism's problem with 'social'

The limitations that I will argue are inherent in the radical constructivist position stem from the fact of other people, culture, and the social. The central principle of constructivism is that the individual is the source of meaning; indeed, constructivists will claim, how can meanings inhere in anything outside the cognizing human agent, and if they did how could they be absorbed by that individual?

„ ... we come to see knowledge and competence as products of the individual's conceptual organization of the individual's experience ... „ (E. von G. 1983 p. 66)

This principle was formed by Piaget, although carried many steps further by von Glasersfeld and others, in order to challenge both naive empiricism and platonism as theories of knowledge. We are neither born with the inherent notion of 'table' nor does unsullied observation provide us with the essence of 'table'. There is another view, which constructivists do not consider except when mistakenly conflated with these two absolutist theories, namely that 'table' is what cultures say a

table is, that is all, and individuals are enculturated into those meanings through human consciousness being actually about communication. I return to this below.

Constructivist researchers have recognised, in recent years, that social aspects of the classroom have been underplayed:

"Constructivism, at least as it has been applied to mathematics education, has focused almost exclusively on the processes by which individual students actively construct their own mathematical realities ... However, far less attention has been given to the interpersonal or social aspects of mathematics learning and teaching ... how ... does mathematics as cultural knowledge become "interwoven" with individual children's cognitive achievements? In other words, how is it that the teacher and the children manage to achieve at least temporary states of intersubjectivity when they talk about mathematics?" (Cobb, Wood & Yackel 1991 p. 162)

The answer that is offered is that one must take account of 'social interaction' and this answer is offered in various forms. I will describe the two main ones.

How do the radical constructivists (a la Piaget) deal with social Interaction?

The first of these forms of response is to re-examine what Piaget has to say about the function of social interaction.

(i) elaborating and justifying Piaget's position (Steffe, von Glasersfeld, Jaworski etc.)

Constructivism draws on biological metaphors to talk of equilibration of the individual's mental structures and the viability of the individual's theories or knowledge. By this argument, interactions with the world are necessary to the individual to test out her or his ideas and beliefs. This testing is driven both by the active and inquiring nature of all humans and by the very conceptualising itself. All interactions, though, are on the same level. There is no privileging of language and human communication:

"Sensory-motor material, graphic representations, and talk can provide occasions for the abstraction of mathematical operations, but they cannot convey them ready-made to the student." (von Glasersfeld 1992)

Thus I will challenge *my* belief that I can walk through walls by a sensori-motor experience and *my* belief in *my* own views about learning by testing them in argument with a constructivist. Perturbations will be set up by those interactions which will challenge the viability of those theories and the cognitive conflict will be resolved by me through striving to return to an equilibrium in *my* personal knowledge. I would argue that the wall will set up a much stronger perturbation; *my* reaction to someone disagreeing with *my* ideas is to think them stubborn, unreasonable and wrong. Of course they will think the same of me. The argument I am making here is that the reaction an individual makes to these perturbations is just as likely to be a splitting or 'monster-barring' as an accommodation. However, many constructivists will maintain that Piaget's formulation of the effect of social interaction is quite adequate:

"In this sense it is legitimate to interpret Piaget's work as a social-cultural approach in which he explained the mathematical development of children as self-regulating, autonomous organisms interacting in their environments. He seemed to take the social-

cultural milieu of the children as a given without attempting to alter their most general experiences. "

"Making sense", then can mean to construct ways and means of operating in a medium to neutralise perturbations induced through social interaction." (Steffe 1993)

I want to argue that whilst this position is coherent and consistent it is also very limited. It ignores so many aspects of social life and all of the power of communication, language and enculturation, and cannot accommodate any of the socio-cultural research of recent years. But I will briefly return to these below.

(ii) deveiooino a 'social constructivism' which incorporates both (Ernest, Cobb, Wood, Yackel, Voigt, Bauersfeld etc.)

The other move that is made by many of the constructivists is to try and incorporate an emphasis on communication into their constructivism in a complementary fashion. Below are three such arguments:

" ... we can observe that when we talk of students' constructive activities we are emphasising the cognitive aspect of mathematical learning. It then becomes apparent that we need to complement the discussion by noting that learning is also a process of acculturation. "(Cobb, Yackel and Wood 1992 p. 28)

" ... although the primacy of focus of each of conventionalism (*Ernest's term for inter subjectivity*) and radical constructivism is sacrificed in social constructivism, their conjunction in it serves to compensate for their individual weaknesses ... " (Ernest 1991 p. 86, italics my addition)

"The fundamental orientation of the work in our own classroom springs from the radical constructivist principle and an integrated and compatible elaboration of the role of the social dimension in these individual processes of constructing as well as the processes of social interaction in the classroom." (Bauersfeld 1992 p. 2)

I want to argue that social constructivism, sometimes taking one view of learning as observably occurring and sometimes the other, is incoherent and inconsistent. For one thing knowledge, concepts and even the filter for perceptions are individual and not shared, in the constructivist view. This filter is the conceptual state of the individual which makes her or him interpret or indeed ignore any interaction. But how is it that what constitute cognitive conflicts, disagreements and social interactions that are supposed to lead to dis-equilibrium and consequently adaptation do impinge on the individual and are shared, arising out of the 'social dimension of individual processes'? (Actually this is a criticism of radical constructivism as well as of social constructivism.) The strongest incoherence, however, is in taking communication and language as having no power to enculturate or to position individuals or to carry and regulate subjectivities at one time, but to accept those characteristics of discursive practices at another time.

ANOTHER ALTERNATIVE: REJECTING CONSTRUCTIVISM

There is another view of knowledge and of people that is not considered, one that transcends the separation of subject from everything else and without recourse to denying any validity to knowledge except that in individual minds. I would argue that it is offered by Lev Vygotsky's

cultural psychology; by the research on situated cognition of Jean Lave; by the studies of context and of affect and cognition of Jeff Evans; by the studies of classroom discourse of David Pimm and pupils' writing of Candia Morgan and Andrew Waywood; and the work influenced by poststructuralist discourse of Valerie Walkerdine and others. In my own recent and current writing (1992, 1993 and forthcoming) I have attempted to elaborate some aspects of this view. Here, I would summarise the view as follows:

Knowledge Isn't In the Individual's mind, nor 'out there' In objects or symbols.
 Knowledge Is as people use It, In Its context, as It carries Individuals along In It
 and as It constructs those Individuals. Knowledge Is fully cultural and social.
 and so too Is what constitutes human consciousness. Communication drives
 conceptualisation.

As I argued at the start. constructivist principles and especially the metaphor of construction are powerful and dominant. However, radical constructivism is a severely limited view of learning, denying so much of life, and social constructivism is incoherent, an attempt to have one's cake and eat it. Starting from a fully socio-cultural view of the mind, the individual, learning and knowledge is a different position, one which seems to me to offer a much richer view of teaching and learning.

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Having Presence in a Mathematics Classroom

Pinder Singh

This presentation, which is a small part of a main research study, views non-standard trainee teachers' perceptions of 'having presence in a mathematics classroom'

.Badw:mmd:

The main research study is based on non-standard trainee teachers' constructions of what it is to be a mathematics teacher. The research questions are:

What shifts can be observed in trainee teachers' constructions of what it is to be a maths teacher?

What factors may be said to influence these constructions?

These trainee teachers from a two year B.Ed course in secondary mathematics are mostly from a different cultural and educational background to that of inner London secondary schools.

Olson (1988) recognised the role that teachers' cultures play in what they do in the classroom. He recommended that in order to understand their actions, one needs to interpret the rules they follow. Hoyles claimed that-

"teaching is a 'human' activity which involves the feelings and beliefs of the participants, each of whom have a personal and cultural history colouring their actions."

Hoyles, C.(1992)

If it seems that a teacher's culture and her personal history influence her actions and behaviour in a classroom and it would appear that a trainee teacher's construction of what it is to be a teacher could possibly differ from that of a tutor's construction. A tutor here is taken to include a teacher tutor as well as a university tutor.

The main research study considers several aspects of a trainee teacher's construction of what it is to be a teacher and the methodology consists of several other methods to collect relevant data.

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Data in this instance was collected by semi-structured interviews of three non-standard trainee teachers, three standard trainee teachers, two teachers (ex-students) and four tutors.

The interviewees were asked the following:

'You do not have presence in the classroom.

What does it mean?

Can you evaluate it?

Can you train others at it?

If so, how?'

Results.:

The trainee teachers who had not come from a British cultural and educational background had a rather confused picture of 'not having presence in the classroom'. A few comments in their own words are mentioned briefly:

(a) "No presence' implies 'no character' a feeling of rejection. Felt a failure of no value ... not worth anything. Did not bother to ask what is wrong."

(b) "The notion of having presence in the classroom is quite new to me. "You have to have presence in the classroom to be a good and effective teacher." was said to me, ... not unlike the advice "to enjoy life become a millionaire."

It conjured up in his mind "a picture of an authoritarian individual in Etonian robes with a ruler in his/her hand waving at cowering pupils." The trainee teacher understood that in current terms it meant having total control of the classroom, in other words being in charge.

The reality for the trainee teacher was somewhat vague. The problem for him was not who has presence in the classroom and who does not have it, but *why* some seem to have it and others do not. In his view, "the tutor directives have compounded the confusion: 'Be assertive. Avoid confrontation. Don't shout. '"

A trainee teacher who is of western culture but not acquainted with inner London schools expressed her view:

"When a teacher with her attributes (as a teacher) does not make her presence felt enough, she does not have a positive impact. It has to do with many factors but an experienced teacher manages to neutralise possible disadvantages and be almost the "centre" of a lesson, meaning a very strong reference point to the students".

Interviewees who came from an inner city environment recognised that having presence in a classroom was a multifaceted topic; voice, speech, communication, body language, classroom management, posture, appearance, acting ability and how to avoid confrontation were some of these facets. They also agreed it is far easier to know when someone has "got it" rather than why a person does not have it.

All the interviewees believed that one could be trained to some degree to have presence in the classroom, perhaps with the appropriate guidance;

"If a tutor is able to put herself in a trainee teacher's position then there is a better understanding of her words to the trainee teacher"

Trainee teacher from a non Western culture.

The trainee teachers were being offered sound advice and relevant debriefing. Yet a few of them did not seem to be making satisfactory progress. It seemed to take them much longer to make the necessary adjustments.

Conclusion

Trainee teachers from different cultures and educational backgrounds could have different perceptions and constructions about being a teacher. One needs to be aware of these differences and to be sensitive in dealing with them. Because of the complexity of this issue, it must follow that trainee teachers will achieve "success" in varying ways and in varying periods of time.

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Working in Mathematics Education with Namibian Primary Teachers

David Womack

Namibia (formerly South West Africa), is one of the most sparsely populated countries in the world, consisting mostly of thorny scrub, mountains and desert. After its Independence from South Africa in 1989, the European Community earmarked money to help free its education from the previous *apartheid* system. This had consisted of 11 separate education authorities; one for whites, one for coloureds and nine others for the different indigenous black ethnic groups - *in that order*. This particular project, called INSTANT (In-service Training and Assistance to Namibian Teachers), was implemented by the Vrije University of Amsterdam.

Part of the Project's task was to run in-service workshops in mathematics to as many of the country's primary school teachers as possible. Decisions were made *in situ* as to how best to do this and a number of 'training of trainers' workshops were held. This involved taking the 'best' rural primary teachers and preparing them to take workshops of their own with the teachers from their own immediate areas.

Visits were also made to the remoter areas of Namibia such as Kaokoland and Bushmanland in order to assess teachers' needs in mathematics. Most of the rural schools had nothing and so any practical suggestions made for teaching and learning aids had to be based on locally available waste materials such as empty soap packets, oil containers, bottle tops etc. A number of workshops were set up to in-service teachers who had not undergone any formal teacher-training. The workshops allowed teachers to demonstrate to fellow teachers, how they would teach particular topics using their own ideas and those of the tutor. Balances made of paper and paperclips, metre rules from newspapers and string timers were some of the measurement aids made and demonstrated. Geometrical figures such as squares, pentagons, hexagons; octagons etc. were constructed by simply folding paper, together with 3-dimensional figures made from old envelopes or strips of paper.

Whilst the Project was not specifically a research project, many insights into overfamiliar mathematical concepts arose in these workshops, providing much food for thought (and research). For example, are sorting, ordering and matching activities really necessary for learning about number? Are sets the basis of mathematics? In practice, many rural children learn mathematics successfully without these, (see:

Game, Set and Match; Times Educational Supplement, 8.10.93). Again, the *exact* meaning of *add*, *multiply*, *find the difference*, etc. is brought into sharp focus when trying to communicate the meaning of these terms in English to teachers whose first language is (e.g.) Herero and whose second language is Afrikaans. (See for example, *What's Simple About Arithmetic* T.E.S. 22.5.92)

Some time was also spent devising a 'rural schools' mathematics kit, which would provide teachers with the basic equipment for teaching clearly the early concepts of mathematics.

[Note: The conference session was not well attended, due no doubt to the failure of participants to see what relevance third world teaching had to their own particular research interests!]

CULTURAL INFLUENCES ON CHILDREN'S PROBABILISTIC THINKING

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Context

Probabilistic thinking is a subject that has been widely researched, mainly from a psychological point of view. So far research has been mostly 'culture free', despite indications of the important influence of culture on probabilistic thinking, on mathematics, and on cognition in general .

Objectives

This research **aims** to discover the relationship between children's understanding of probability prior to teaching, and relevant influences in their 'culture', Le. their beliefs, experience and language.

Method

The first stage of the research included interviews. These were conducted for two main purposes:

- a) Building communication: clarifying main concepts used; trying out and modifying items from previous research and new items; trying out interview techniques.
- b) Trying an holistic approach: looking for patterns linking the different fields of the child's probabilistic thinking, and between these and the child's culture (operationally defined as his language, his beliefs and his experience). These patterns could lead to hypotheses, with the intention of further validation in the second stage of research.

38 pupils were interviewed, all in their first year of high school (11-12 years old), from two inner-city schools in Manchester. 22 were of English origin, 11 of non-English origin and 5 of mixed origin. Each interview lasted for about 45 minutes.

The interviews were not uniform, due to the need to develop suitable questions and interview techniques, and due to the flexibility needed when an interesting aspect of the child's thinking emerged, needing clarification.

Although some quantitative analysis was done, the main emphasis remained qualitative, due to the small numbers and non representative sample. The perspective was basically a case-study perspective, trying to arrive at some cautious generalisations.

Results

Clarification of concepts: chance

The pupils' understanding of what is meant by 'chance', 'things happening by chance' or 'luck' was very varied.

Chance was sometimes understood as the **possibility, or opportunity** of something happening, that is, nothing to do with randomness, but in the meaning of 'having a chance'.
Examples:

Chess is analysed as a game of chance, because "you've got to try, and you've got a chance of winning and a chance of not winning." Trivia "has less chance, because you can't always get the questions right." Computer games - "that's chance, because you can always start again".
Football- "if you're in a difficult position, that's chance. My partner gets it in, I don't". (INTERVIEW NO 1)

Could you tell me what do we mean when we speak of "things happening by chance" in our lives?
Uke - chance to become something.

What do you mean by that?

When you leave school and go to college or university, to become something. (INTERVIEW NO 24)

Others defined chance as something to do with **uncertainty**, again, not necessarily involving randomness. Something that happens by chance might mean that you don't know the result.

Example:

I.: What would be something that does NOT happen by chance?

R.: When you know what's going to happen.

Later weather is discussed.

R.: I'd agree that was chance, cause you wouldn't know. (INTERVIEW NO 31)

Another interpretation of chance is - something that **just happens, without planning or Intention**. Examples:

I.: Could you tell me what do we mean when we speak of "things happening by chance" in our lives **1**

K.: It's like walking down the road and a stone fell out, kicked out on the road and you fell over. That would be by chance. Just happens by accident.

I.: What are things that happen not by chance?

K.: It's planned. It doesn't happen by chance. Like the U.N. people don't by chance arrive or send troop. You have to plan it.

K.: A road accident? By chance I suppose. Because ... unless the driver is trying to kill the person, or the person that gets run over is trying to kill himself. It's usually by accident if you stepped into the road in the wrong time; or two cars skid - it's usually by chance. (INTERVIEW NO 22)

Luck

Luck, being lucky, being unlucky, were very often associated with **something good** or bad that happened, not necessarily random. Examples:

The discussion is about a computer game called 'lemmings':
I've got a sister, she's younger than me. She's not very lucky. You've got to use a mouse and she can't use a mouse properly. So she's always killing these lemming. (INTERVIEW NO 21)

Example of luck: I got 39 out of 39 in a maths test. (INTERVIEW NO 6)

Sometimes luck involved an **unexplained**, perhaps **supernatural** factor:

About dice: My sister is very lucky. She gets 6 eight times in a row. I sometimes get a 6 straight away, sometimes I wait for ages ...
About tossing two dice: There is more chance to get different numbers. But my mum is good in getting the same number. (INTERVIEW NO 2)

Attributions

After clarifying what the child means by the terms 'chance' and 'luck', it was attempted to understand to what areas of life does he think these terms apply, i.e. where does he use them as attributions? This is important as an independent goal, of seeing how chance is mapped in the children's general view of the world. It is also important to see if random devices commonly used by researchers and teachers (dice, coins, counters, etc.) are seen so also by the children. This will be discussed in the next section.

When discussing the broader issue, several contexts were used, such as road accidents, the weather and success in sports, and their relationship to chance was discussed. The variation in views was quite broad. Road accidents were sometimes seen as known in advance or even planned by God; they are mostly seen as controllable by people's behaviour; some saw chance as playing a role. Weather was again sometimes seen as controlled by God, others saw it as causally determined, and the largest number saw it as related to chance. Sports, for instance winning in football, were mostly seen as dependent on skill, sometimes seen as dependent on chance, with some children combining the two in this way: if the groups involved are similar in ability - then chance is the crucial factor, if one group is better than the other - then it is a matter of skill.

It was also observed a few pupils used God as their major attribution, others tended to use causal attributions, and a few used chance a lot. As it will be shown later, these seemed to be types of reasoning modes, used along a lot of their responses. But many pupils, perhaps the majority, could not be described as revealing a certain type of reasoning, but rather used a mixture of types.

Model of dice, coins, etc.

Common devices used in probability teaching and research are dice, coins, counters, etc. It is often taken for granted that children see these devices as random. But, relating to their previously discussed system of attribution, do the children really see these devices as random?

Quite a number of children thought, in different degrees of certainty, that these results depend on how you throw, or handle, these different devices. This was especially so with coins. 8 children clearly thought the result depended on how you toss the coin. Some associated this with cheating; others - with experience; others could not explain why, but suggested some people are luckier than others. Specifically - quite a number of children prefer tails when tossing a coin (9 of the 12 asked, as compared to 1 that preferred heads and 2 that did not prefer either), some quoting the sentence 'tails tails never fails'. Most of these had no rational explanation for this, with exceptions such as explaining that 'heads' sticks out, and so it is heavier, causing the 'tails' result to be the more frequent.

Another assumption teachers and researchers sometimes take for granted with some of these common random devices is that they are seen as equiprobable. But some heuristics caused the children to see things differently. One such heuristic is 'availability' (estimating odds according to memories of similar past experiences see Kahnemann, Siovic & Tversky, 1982). Many children remember from their experience with board games waiting a long time for a 6 on the dice, often needed to begin a game. This makes them conclude that 6 is hard to get on the dice (17 pupils). For example:

When you want a number - it's harder to get it. It also depends if you cheat. If you do it slowly, the number might just come. If you're in a rush to get a number, if you're in a game, you throw and throw and don't get the number. If you're playing_ on your own, you just throw it, and it comes on the number you wanted. (INTERVIEW NO 5)

Sometimes this view does not emerge immediately, but only after some probing.

Another heuristic used, mainly in the context of coins, is 'representativeness' (expecting a sample to be representative of its parent population - see Kahnemann, Siovic & Tversky, 1982). This was used by 14 pupils:

After five heads on the coin, W. expects a tail. In fact, he expects: probably it will be heads, tails, heads, tails, heads, tails, and keep going like that. (INTERVIEW NO 9)

Some children used equiprobability - by did so automatically, reflecting the 'equiprobability bias' (expecting random events to be equiprobable by nature see Lecoutre, 1992) :

I.: Does a road accident happen by chance?

K.: You've got a 50-50 chance and not chance of it, because there's the chances that somebody will crash into you, and you've got the chance that you'll crash into them, half-half. (INTERVIEW NO 31, K.)

This 'equiprobability bias' was quite common in children's answers. One of the items that might be reflecting it was - a lottery in a maths class with 13 boys and 16 girls. 10 out of 23 pupils answered that to pick a boy in this class was just as likely as a girl. Part of the explanations seemed to reflect a 'natural' implication that chance is normally equiprobable, although mostly this was not explicit.

The 'outcome approach' (Le. the inclination to view probability as 'operative', as attempting to predict the outcome of an event - see Konold, 1989) was also common in the interviews. In fact - a majority of the children revealed this approach to some extent. For example - in Konold's weather problem (1989) they would tend to think that if after a prediction of 70% chance of rain it did not, in fact, rain - then the probabilistic prediction was wrong.

Links between culture and probability

The crucial question in this study is - does culture seem to play a significant role in the building of a child's probabilistic thinking? Although, due to the nature of the interviews this is not easy to generalise definitely, I claim that such a role exists. Examples of influences that appeared in the interviews are:

6 children with a high level of superstition, leading to views of dice and coins as not equiprobable, without any explanation, and to crude, unrefined probabilistic thinking.

A child with a strong influence of religion (see discussion below).

3 children with a strong tendency to causality and determinism, in areas normatively seen as involving randomness.

A child whose world of chance was dominated by tricks and suspicion.

A child with a view of chance that was extremely 'equiprobable', as in Lecoutre (1992).

In interview no 2 religion seems to have a strong impact on the boy's feelings toward chance, resulting in a mixture of rational and irrational elements in his analysis of probabilities. G.'s father is head of a Nazarene theological college, serving in the past as a missionary in Africa. G. sees some chance events as not really chance:

When I was in Africa a poisonous snake almost killed my sister. By chance I came outside so I could tell someone. But it was not luck, it was God. He saw that my sister didn't die. That wasn't chance ...Fruit machine, that's chance. I don't think God would like that.. I don't gamble .. I'm against it. You just get ripped off. Most places are fixed.

So some things may look like chance, but are actually acts of God. Others - have a touch of evil in them.

This could explain G.'s answers about probabilities, reflecting a duality of fairly correct theory with irrational ideas. Dice are seen, on one hand, as unpredictable:

You can't say what you will get. I sometimes get a 6 straight away, sometimes I wait for ages.

But on the other hand :

My sister is very lucky. She gets 6 eight times in a row ...

About two dice:

There is more chance to get different numbers. But my mum is good in getting the same number.

So certain people (the 'good' people?) have better chances than ordinary ones. In a question involving combinatorics G. reveals quite advanced combinatorics, but still gives an unexplained preference:

I.: What number would bet on when summing results of two dice, a 3 or a 6?

G.: I'd bet on a 6, because there are three possibilities to get it (3,3 ; 4,2 ; 5,1) and 3 has only one chance (2,1)

I.: And if you'd have to choose 3 or 11?

G.: It'll be difficult because both have one possibility. But I prefer 5,6.

So, to sum up, G. reflects a duality of rational with irrational ideas that seem to be influenced by religious belief. The hypothesis that probabilistic thinking is influenced by religious belief will be one of those investigated in the second stage of the research (questionnaires).

This example demonstrates, in my opinion, the influence culture, in its broad interpretation, has on children's probabilistic thinking. This is not to say that culture is the main influence. As shown in previous research (Green, 1982), ability is by far the major factor effecting probabilistic thinking. But culture does seem to be an additional factor, leading to several types of reasoning. The types that have been presented are probably not all existent types, and most children probably reflect not a 'pure' type of thinking, but some kind of individual combination. But this might be the link between culture and probabilistic thinking.

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Blind activity?

" ... the task in itself does not 'contain' mathematical concepts or structures. And 'blind' activity on a task does not ensure learning as intended." [Christiansen and Walther 1986: 250]

The research reported here arises out of concern that, very often, learning mathematics does not take place as teachers intend. Apart from the general recognition that mathematics is a difficult subject both to teach and to learn there is also substantive and convincing research evidence to the effect that teaching and learning mathematics is not the straight forward transaction that it might at first appear. We note the research reported by Denvir and Brown (1986) which attempted to develop a 'descriptive framework and diagnostic instrument' and then use this to inform the teaching programme of low attaining children; they write:

" ... acquired skills (...) were not the ones that had been taught" [1986: 153] and
"Children did not always learn precisely what they were taught" [1986: 163].

Further we note the teaching experiments reported by Bell and his associates (Bell 1983, Swan 1983, Underhill 1991) which demonstrated the marked difference in pupil's performance between those who had been taught using 'conflict' methods and those who had been taught using a 'positive only' approach.

In an attempt to explore further the nature of activity in mathematics classrooms I have been observing mathematics lessons in secondary schools and interviewing pupils as they are engaged in the tasks set by their teachers. My intention here is to share part of my analysis of extracts of the transcripts of these interviews and the theoretical model of classroom activity which I am using as an analytic tool.

The model.

Theories of cognition based upon social practice and constructivist arguments have explanatory power in the analysis of classroom activity. Although it is articulated elsewhere by Lerman that the joining of these two provides an incoherent account of learning I would argue that we will fail to provide an adequate explanation of learning activity unless we recognise the importance of both the social and the individual.

Lave (1988) describes cognition from the perspective of social practice in the following manner:

" ... cognition is constituted in dialectical relations among people acting, the contexts of their activity, and the activity itself." [1988: 148]

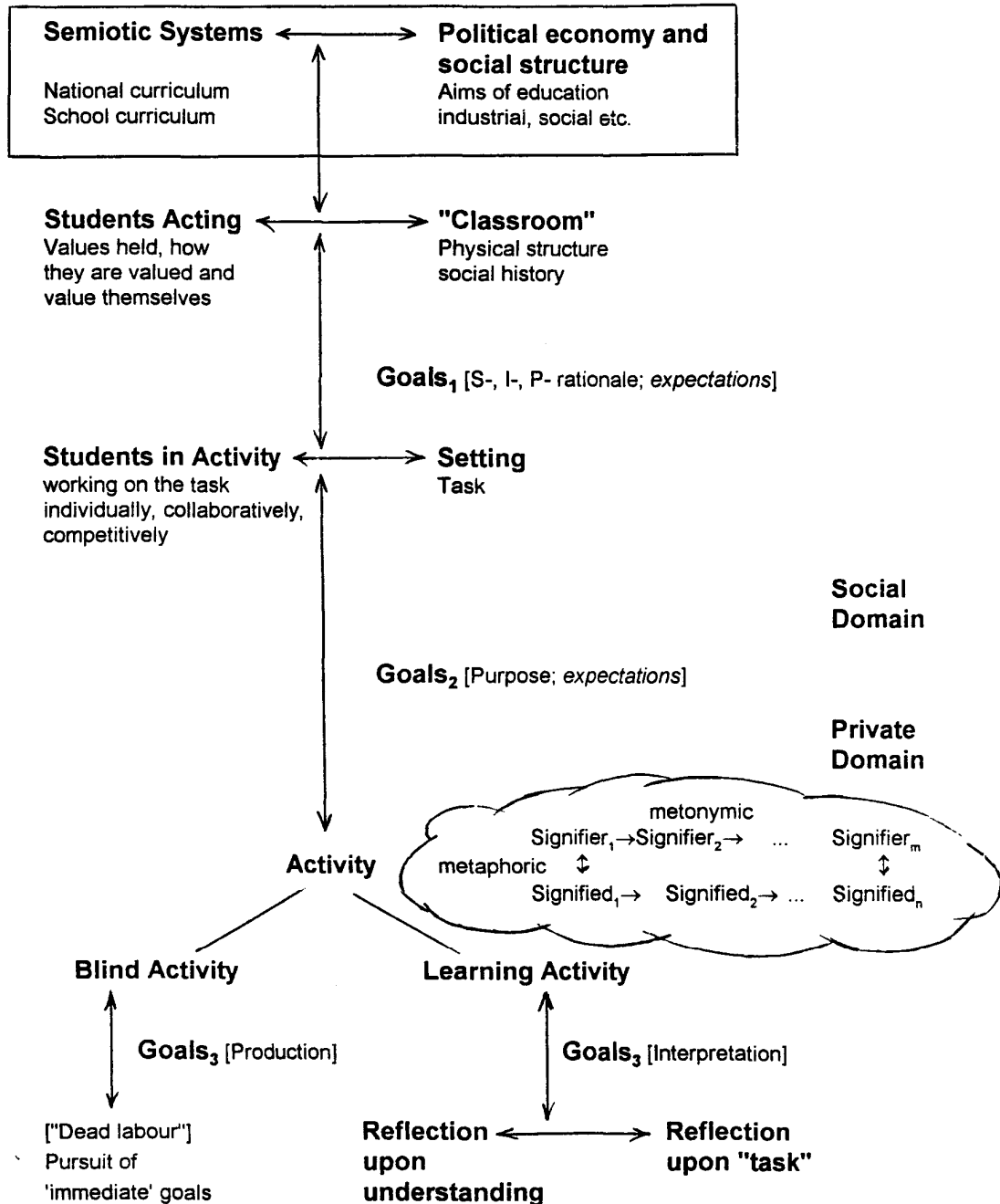
The model that I propose is based very closely on that illustrated by Lave [1988: 179]. Lave bases her argument on the observations of people in their everyday activity; dieters preparing meals, shoppers visiting a supermarket and workers loading pallets in a warehouse. However the activity of a classroom has significantly different objectives from those of people engaged in their everyday lives; as El'konin (1961) observes:

"The basic unit (cell) of educational activity is the educational task An educational task differs fundamentally from other types of problems in that its goal and its result consist of a change in the acting subject himself, not in a change in the objects on which the subject acts."

[Quoted by Davydov and Markova 1983: 60].

Whilst recognising 'the importance of the 'social' in the constitution of meaning the reflective activity of the cognizing individual is also seen to be central to an explanation of learning activity [Wheatley, 1992; Steffe and Wiegel 1992; Bauersfeld 1992]. Thus I present the following model upon which I base my analysis of classroom activity.

A model for the analysis of mathematical activity in school
 [based on Lave 1988: 179]



It must be stressed from the outset that the physical positions and juxtapositions of the objects and relationships of this model are due to the structure of the diagram and are not intended to presume any precedence or structure in the classroom. The arrows indicate dialectical relationships, thus students acting and the classroom are accepted as mutually constituted in dialectical relationship, similarly students in activity and the setting. To this point there is little to distinguish this model from that offered by Lave. I have labelled the vertical arrows 'goals' and indeed it is these goals towards which students work which are the focus of my present interest. Lave rejects the word 'goals' as she interprets this as a one-way relationship, she prefers to use 'expectations'. I have not followed Lave in this as I am specifically considering the rationale, purpose and interpretation/production of pupils and I am seeing these as one-way characteristics of pupils (although I accept that the consequence of the goals which pupils hold will be reflected in changes 'above').

The actual nature of mathematical activity I have indicated as metaphoric and metonymic processes, for this I must acknowledge, with gratitude the contribution of Paul Ernest. Again, it is emphasised that nothing should be read into the fact that this cloud of activity is shown to the right and apparently associated with learning activity, ideally I would superimpose it on the word activity.

Activity in the classroom could be 'blind' in that it does not result in any learning, the use of the word 'production' to describe the goals here will be explained below as I use the model to analyse transcripts. Interpretation is used to describe the goal of learning activity as this is seen as central to learning, [Goodchild 1992]. Learning activity is then seen as yet another 'level' of dialectic relationship, here between the learner's reflection on their own understanding and their reflection on the task. I find a useful sub-model for this is Neisser's perceptual cycle (in Christiansen and Walther 1986); in Neisser's model the schema directs the exploration which samples the object resulting in modifications to the schema and so the cycle continues.

Analysis of transcripts

1. In the first transcript examples of Goals2 are evident:

- T. So why are you doing it?
S1. Make us concentrate, I don't know.
T. Makes you concentrate?
S2. No, just make us do something.
T. Makes you do something? Do you agree with him?
S1. No because then if Miss wanted us to do something she'd get us cleaning or something like that wouldn't she?
T. Yes, yeah, so why do you think?
S3. Concentration.
S 1. Yeah I think concentration and I don't know because maybe there's something about this that we'll go on and do later on that can help us with other things.

(Year 8 students, average attainment, working on an investigation with polyominoes. The teacher~<; objective wa<; for the pupils to develop an understanding of and facility with congruence and rotations).

The pupils in this transcript offer three possible goals at 'level' two which indicate their perception of the purpose of the activity: concentration, make us do something and help us with other things. It is possible that the notion of concentration is seen as a universal aim of mathematical activity arising from the frequency with which the pupils are exhorted to 'concentrate' by their teachers; this opens up the whole question about the origin of pupils' goals and our responsibility as teachers in 'transmitting' appropriate signals through the language we regularly use in class: 'I want you to do .. .' rather than 'I want you to learn .. .'.

That a pupil could suggest the activity was simply 'to make us do something' is possibly an indication of the sad fact that many pupils do not see any purpose in their activity outside of the requirement to do what the teacher tells them. On the other hand although the purpose of the activity was not clear one pupil was able to express sufficient confidence in her teacher to suggest that there may be a point in the activity because it would 'help us with other things' not yet apparent. There is here a hint of goals at level, in that the pupils have a rationale for engaging in the activity because this is the accepted 'practice' of the mathematics classroom. I find it is useful to extend Mellin-Olsen's constructs of S- (socially significant) and I (instrumental) rationales [Mellin-Olsen 1987] to include a P- (practice) rationale.

2. This transcript provides an example of a 'level' three goal of blind activity.

- T. Do you think it's important while you're going through them to think about why the rule works?
 S. At the beginning so then you can understand it then.
 T. But once you've understood it you just do it?
 S. Yeah, why not?
 T. Why not?
 S. Because you know what you got, you haven't, it's going to take you ages to think each one while you're doing it, it's gonna take you four times as long. You'll only be able to do two sets of questions but otherwise you'll be able to do, say, four.

(Year 9 pupil, average attainment working at an exercise on approximations).

Although at the beginning of this extract the pupil acknowledges that understanding has a role it seems that he sees his goal as productivity and anything which gets in the way of this is to be avoided. It is interesting to note that my least successful interviews have been in lessons where the pupils have been engaged in routine, undemanding tasks and their clear desire is to do as much work as possible. Thus I label the pupil's goal when engaged in blind activity 'production' and this relates back to the words of Elkonin quoted earlier in which he draws attention to the difference between an educational task and other types of problems.

3. The extracts from a transcript below reveal the different responses to the repeated question 'why' and indicate goals at different 'levels'.

- T. *(read~)* Round off each of these to three
 S. Significant numbers, figures
 T. Figures, right. And so partj (99993) what did you round that to? 10
 S. Hundred thousand.
 T. Hundred thousand? [So Yeah.] Why did you do that?
 S. Because the next number, that's the three significant numbers and the next number's higher than five so you round it up to the next whole number. 15
 T. So you rounded ninety nine thousand nine hundred and ninety three [So Yeah.] Up to a hundred thousand?
 S. Oh ten thousand! *(reading her own answer incorrectly).*
 T. That's all right, yes. [So Yes.] Why?
 S. I don't know, because I have to, it's had to round it ~to three significant figures .. 20
 T. O.K. you've got ... that answer's correct [So Yeah], um and so you've obviously done it in the right way [So Yeah] but why does that way work?
 S. Don't know.

(and much later in the interview)

T Why do you think you're doing this? 15
 . Because Miss G. told us to. 2
 S.

(Year 9 pupil, average attainment working at an exercise on approximations, line numbers relate to complete transcript)

Why? asked in line 12 elicits a response of goal at 'level' three "the next number's higher than five so you round it up", the student's reveals a syntactic process. I argue that this does not necessarily compare with metonymic transformation practised by mathematicians as illustrated in the model since it is possible that the pupil acts without any consciousness of an underlying metaphor. When a mathematician works metonymically then she/he is in possession of a metaconcept which explains and justifies the actions even though there may be no concurrent metaphor in her/his mind. This student is following a rule without any notion of its meaning. When "why does that way work?" is asked (line 23) the pupil responds "Don't know", although later in the interview (not reproduced) the pupil reveals an understanding of all the necessary underlying concepts such as place value she is clearly not bringing these to the interpretation of the activity. It seems then that her 'level' three goal is one of production rather than interpretation and on the basis of this and other parts of the interview (not reproduced) I would argue that her activity may be described as 'blind' rather than learning.

Why? asked again in line 19 elicits a goal at 'level' two. The purpose of her activity is to follow the instructions in the text, "because I have to, it's had to round it off to three significant figures." This seems to be characteristic of so much of pupil's activity in mathematics, the task is not really about anything in particular, it is about the manipulation of numbers empty of any meaning and there appears to be little purpose in engaging in the task other than "because I have to."

In line 152 why is asked yet again and a level one goal is provided. The pupil's rationale for working is because she has been told to do this by her teacher and the accepted social practice in this classroom is that pupils will engage with the tasks set by their teacher, no other reason is necessary or sought.

Discussion

The validity of the interpretations of the transcripts is not at all evident from the brief extracts discussed above but this is not the prime issue here, they are offered rather by way of illustrating the model provided and help to define further the notion of 'blind' activity. A number of points are in need of clarification.

A syntactic interpretation is not in itself evidence of 'blind' activity. Skemp has described relational and instrumental understanding and indeed it is reasonable to see the possibility of relational learning and instrumental learning. Learning a syntax based rule may rest on instrumental understanding but it is still learning. 'Blind' activity is used to describe a pupil's activity which is motivated by and for the production of answers without any interpretation of the activity or reflection upon the answers, it is work for the sake of work alone and is no more a learning activity than "cleaning or something like that" (I accept that in some circumstances even cleaning may result in learning, so perhaps the example is not so good!).

An approach which incorporates the practice of routine skills may be justified as a proper task for the mathematics classroom in that it raises pupils' level of proficiency and may help retention, this too some may argue is not 'blind' activity. I would want to explore the pupils' goals before coming to any judgement on this issue. It is very easy to dismiss rule bound activity, and syntactic interpretation as 'blind' but there is a value for these in mathematics in that pupils may learn what to do or how to represent and meaning comes later, as Pimm [1986]

has suggested in relation to a young child's learning of number words. Pupils want to know 'how to do' I have been asking pupils what makes a good maths lesson for them and a frequent response is "one where I know what I'm doing" which lays emphasis on the production goal of their activity rather than interpretation.

A question arises as to whether it is possible to prompt particular types of answers which may be interpreted as indicating different goals, and whether I have attempted to lead pupils through using 'effective' questions. As far as the transcripts discussed above are concerned the answer is clearly 'no' since the interviews preceded the development of the model and the attention to pupils' goals by some months. In my future work the need for extreme caution is recognised so as not to fall into the trap of leading pupils to particular responses because they will fit and confirm the model I hope to use to analyse their activity.

In the third transcript examined above it appears that there may be a coherence between the goals:

Goal₁ (P-rationale) → Goal₂ (purpose → Goal₃ (instrumental interpretation
'it's what I have to do,
it's the rule) or production)

Again, there is insufficient evidence here for such a conclusion. I hope to be able to present a more convincing account of this on the basis of my continuing programme of interviewing pupils in the course of their activity in mathematics classrooms.

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Simon Goodchild

Low Attainment and the Pursuit of Algebra

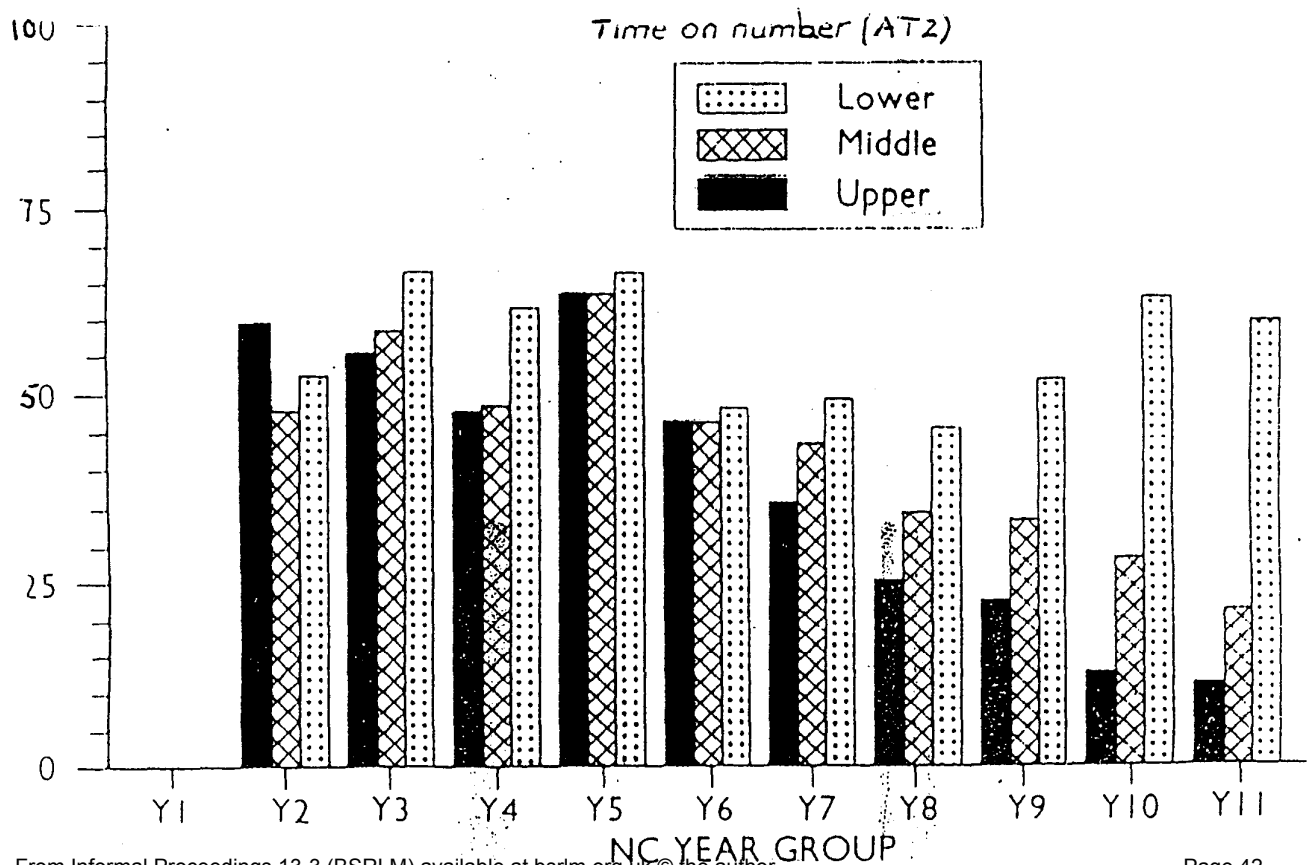
Introduction

*I never understood Algebra at school. It never made any sense to me, but I can't say as I've ever felt handicapped since by not being able to do **it**.*

When it comes to algebra and we have to operate with x and y there is a natural desire to know what x and y are. That at least was my feeling: I always thought the teacher knew what they were and wouldn't tell me.

(Quoted in Mason 1985)

Both these quotes from Mason's book *Routes to Algebra* - the first by a practising teacher and the second by Bertram Russell - illustrate the feelings of many pupils towards the study of algebra. In particular for pupils who are termed low attainers in Mathematics the study of algebra is in many ways considered to be an area of the curriculum that is better not studied. This results in the delivery of a rather narrow Mathematics curriculum being offered to a substantial body of pupils. This is illustrated in the graph below which shows that for low attainers the Mathematics curriculum is often reduced very largely to arithmetic.



Area of Research

The main aim of the study is to investigate the use and understanding of algebraic concepts of a group of low attaining pupils.

The low attainers in this study are pupils in the bottom Mathematics set out of eight in year 8 of a mixed comprehensive school. Some of the characteristics of the pupils are that they:

show a low level of achievement in their school Mathematics.
have low esteem - particularly in relation to their ability to do Mathematics.

lack confidence in their capacity to tackle mathematical problems.

have difficulty in performing basic number tasks.

A number of sub questions for research are:

1. Are these characteristics sufficient for identifying low attainers or are there other characteristics which affect their ability to work mathematically?

What aspects of algebraic thinking are accessible to these pupils and is number competence a pre-requisite?

What are the methods that these pupils use for performing numerical operations and what effect does it have on their ability to generalise?

2. How does the way in which these pupils are introduced to algebraic concepts influence their understanding.

How does the learning environment affect pupils potential for making progress in the learning of algebra?

Does the computer help in the search for an environment in which the ideas of Algebra can be naturally explored?

3. Does using an appropriate environment such as Logo, enhance the algebraic opportunities available to "low attainers"

In particular does the environment provided by Logo create this possibility, and help the student to work in a creative and reflective way?

4. How does working in Logo affect the way in which the pupil views work in other areas of Mathematics?

In what ways do do pupils make links between the symbolic language used in communicating with the computer and working algebraically in other systems

In order to investigate these areas of concern an exploratory pilot study was conducted in the period March to June 1993. It was conducted with a group of 12 low attainers in the bottom

Mathematics set in year 8 at an 11-18 comprehensive school in Bristol. The sessions were all conducted in the computer room where there was a suite of networked RM Nimbus machines. Although the pilot was conducted with the whole group, for the purpose of this pilot a detailed study was made on the learning of one of the pupils.

The aim of the study was three-fold:

To explore an effective way of working with the pupils.

To refine the research questions and the area of study

To test the validity of some tasks designed to promote the use and understanding of variables.

Plan and Methodology

The methodology for the study is represented by the diagram below:



This was not intended as a pre-test and post-test situation but rather as an opportunity for the pupils to talk about what they had been doing.

In the prepared activities it was intended that the pupils would have the opportunity to use variables in the following ways:

1. variable representing a known physical quantity
 - a. use of a single parameter in a procedure
 - b. use of >1 parameter in a procedure
2. variable used as an operator
eg use of scale factors.
3. variable operated on
eg creating rectangles in which there was a specified relationship between the length and the width.
4. variable used to represent a functional relationship.
- 3.

Some Aspects of the Interviews

In the initial interview it appeared that the pupils had little experience of using letters. They tentatively tried to make an interpretation of the expressions discussed and tried to maintain some kind of consistency. There seemed to be a vague idea that the letter n represented "any number" but did not really understand what this might mean. They lacked confidence in their ability in that "don't know" was frequently an initial response to a question, and they were easily influenced by the responses of others in the group. In number work they appeared to think in

terms of units, tens, hundreds ... millions. There was also a limited view of such things as numbers between 4100 and 4200 only one number was specified 4150.

Two sets of activities were used. The first set of activities were designed to enable the pupils to gain an understanding of some of the basic processes in Logo, create some simple procedures, explore the possibilities for making a shape bigger and smaller, and take a design made up of a series of parts and explore the changes in the shape as different values were given to the variables in different parts of the design. The second set of activities that the pupils were involved in were to do with working with function machines. The activities involved the pupils in working out what particular Logo procedures did, writing their own Logo procedures for particular input/output tables - some of which were provided by the teacher and some of which were generated by themselves. Finally they worked on comparing procedures that produced the same outputs.

In the interviews which were conducted following each set of activities a number of points of interest can be noted.

The pupils seemed to develop a clear understanding of functions as operations and their inverses. This was shown for both single and multiple operations both within and outside the logo environment. They demonstrated an understanding of the use of a variable or parameter as a scale factor and the effect that different values of the parameter had on the shape which had been constructed. It also seemed that they were able to use a variable or parameter as a place-holder for numbers. In addition they showed some evidence of being able to use notation to represent relationships described in words.

In each of case it would seem that the use of Logo has been influential in the subsequent understanding that was demonstrated outside the Logo environment. All the ideas were developed in Logo which meant that the ideas were explored visually on the screen and so there was immediate feedback which immediately confirmed or questioned conjectures that were implicit in the inputs made. In this way the pupils appeared to develop an understanding of concepts such as operations and their inverses, and scale factors and this understanding was used in non-Logo contexts.

Reflections and Issues

In this study the initial objectives were to involve the pupils in problem-solving situations that would allow them to use variables in a number of ways.

To a large extent these objectives were satisfied as has been illustrated in the discussion above. The pupils did use variables in a variety of ways. They were well-motivated. They were on task for whole sessions of more than an hour. They did appear to gain in confidence. They started to consider errors that were made as being interesting things to explore and find out why the errors occurred. But I am still not sure that I am clear about their

perception of variables. Further in the work that they did I feel that it was very much teacher directed and did not allow the opportunity for much self-initiated work which could have given a better picture of their understanding.

Keiran (1988) suggests that the development of an understanding of algebra requires the student to develop the concept of a function as a rule or an operator, and also to use the symbolism to build a support structure. The pupils certainly did this in the Logo environment. Most of their work was using functions as operators and the symbolism of Logo was of necessity their support structure. Even so it does not seem to me that these are sufficient conditions for developing an understanding of algebra. To this needs to be added what I believe is the most powerful aspect of algebra - the facility to express generalities, to be able to move from the specific to the general. It is this aspect of algebra that needs to underpin all the initial algebra work with the low attaining pupils as it is this aspect which will empower them to progress further and help them to develop a structure within which they can work.

Interestingly Bell (1993) in an unpublished paper has suggested that the raw material of Mathematics is Number and Space, and that symbol systems arise as a means of denoting action in these areas. This seems to be a logical idea as from a historical perspective the development of algebra was indeed a means of representing a problem and then acting within that system to solve the problem. It seems to me that this is the way that Logo needs to be viewed for these pupils. It is a succinct, functional language which allows the pupil to represent a problem using symbolic expressions and further it allows the pupil to generalise the problem or situation. Within the system it seems that there are four steps that the pupil needs to go through in order to develop a conceptual understanding of the algebra of Logo or for that matter for any other algebra.

- be able and willing to operate with a system of symbolic expressions.
- be able to learn the linguistic aspects of the algebra
- be able to learn to manipulate the symbols of the system
- be able to develop a global view about how to generalise within the system.

So it is important now to specify the aspects of generalisation that are evident within the Logo system. This should then give the base line for the development of appropriate activities. It seems to me that there are a number of aspects as listed below:

Operation with variable/parameter unseen
(FD ... RT ... etc)

Procedure with named variable/parameter
(SQUARE :side)

Using a variable/parameter to create the general from the specific.

(eg scale factor)

Using more than one variable/parameter to create the general from the specific.

(eg squares to regular polygons)

Using a relationship to create a generality

Using multiple procedures to create the general from the specific.

It does seem that in this list the move from the first to the second aspect is a substantial step and as such needs to be considered very carefully.

Future Work

In the initial interviews more work needs to be done on:

pupil ideas on ordering numbers
pupil ideas on numbers between numbers
pupil ideas on negative numbers
pupil methods of doing arithmetic operations in different contexts
pupil ideas on decimals

Activities based on the ideas of functionality and generality as specified above need to be developed. These activities should also match the algebraic principles discussed above. Some of the follow-up work on functions could also be related to graphs using a graph-plotter.

The work developed need to lend itself to work both at and away from the computer. Linked to this will be work which enables communication possibilities to be developed, since this is an area where these particular pupils have little confidence. The work of Brousseau and Vergnaud and the theory of situations would seem to be a possible vehicle for developing this possibility. It would also be a means of allowing the pupils to be creative and taking some control over their learning.

Finally both Sfard and Tall have articulated learning structures which emphasize the fact that concepts are learned at two levels - a process level and a structure level. In the work that is done with the low attainers it will be necessary to try to identify those activities which encourage learning at one or other or both of these levels.

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Investigating Children's Mathematics Anxiety: The Effect of Teaching Approaches

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The concept of Mathematics Anxiety has received much attention recently, largely owing to the suggestion that it affects many people and threatens both performance and participation (Suinn, Taylor & Edwards, 1988). There are many different definitions. In this study the definition in mind was Richardson and Suinn's of 1972:

••.. feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations."

There are two assumptions inherent in this definition. Firstly, research has concluded that while Mathematics Anxiety is related to general anxiety (Hendel, 1980), test anxiety (Dew, Galassi & Galassi, 1984), and other academic anxieties (Marsh, 1988), it is also specific (for example, Richardson & Suinn, 1972; Adams & Holcomb, 1986; Hembree, 1990), *i.e.* it exists in people who are not otherwise anxious (Morris, 1981). Mathematics may be particularly susceptible to the adverse effects of anxiety because of features like precision, logic and the emphasis on problem-solving ability (Richardson & Woolfolk, 1980).

Secondly, while it is agreed that anxiety can have a motivational role and therefore a positive effect on performance (Wigfield & Meece, 1988), it is also agreed that the higher mental processes such as problem-solving and divergent thinking which are required for mathematics will be negatively influenced by Mathematics Anxiety (Jones, 1986; Skemp, 1986; Fairbanks, 1992).

Much of the research on Mathematics Anxiety has concerned itself with adolescents and adults, while considering that childhood is a period of rapid change (Jones, 1984) and that the foundations of attitudes are formed early (Suinn *et al.*, 1988), it may be that anxieties have their roots in primary school. This study will consider the presence and effects of Mathematics Anxiety in primary school children.

Background: The South African Project

The study was carried out in South Africa, because of the current project in teaching mathematics in many primary schools, which has a problem-centred approach. The approach may be considered to be more socio-constructivist and less behaviourist than the more traditional transmission approach, with the emphasis more on conceptual understanding and less on computational skill. The type of learning anticipated is more relational than instrumental (Skemp, 1976), more discovery than reception (Ausubel, 1968), and more meaningful than rote (Ausubel, 1968). The description of such

¹ This research is supervised by Dr Julia Anghileri, Homerton College.

a classroom follows from Human (1990), Human, Olivier and Murray (1991a; 1991b) and Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti and Perlwitz (1991).

In such a classroom, children are free to design and choose their own methods and strategies, and ideas are then shared in small teacher-monitored groups. Mistakes are treated by discussion and reaching consensus and the pupils decide what is right or wrong. The teacher has a facilitative role, initiating negotiation and communication, choosing activities to facilitate development and introducing the taken-as-shared terminology and symbolism of wider society. Word sums (problem-solving) are used as the primary vehicle for learning. Social norms, such as explaining methods, listening while others explain, reaching consensus and accepting responsibility for learning, are continually renegotiated.

This is then compared, in terms of the children's Mathematics Anxiety, to a classroom in which the teacher demonstrates and explains standard "vertical" methods of computation for each operation and asks questions, and this is followed by individual pencil-and-paper seatwork (Romberg & Carpenter, 1986). The teacher decides what is right or wrong and intervenes in the case of mistakes. Later word sums may be used as application of methods. Social norms are more static and involve more discipline, rewards and teacher authority.

The curriculum for both approaches includes geometry, measurement and number concept. The problem-centred approach, however, emphasises the development of a versatile problem-solving ability as a primary objective, introduces variable and relationship concepts earlier, allows the use of calculators for checking answers, has no ceilings on number ranges and varies problem types rather than focusing on one at a time.

The rationale behind the project is twofold. Firstly, the view of how mathematical knowledge is acquired has changed from a behaviourist view, advocating drill and practice, to a more conceptual approach and an emphasis on meaningful learning and understanding of the underlying structures of mathematics (Resnick & Ford, 1981). Learning is seen as an active and constructive activity (for example Piaget, 1970; Lachman, Lachman & Butterfield, 1979; Kilpatrick, 1987), rather than the passive acceptance of ready-made knowledge. Children are seen as having a substantial amount of informal knowledge (for example, Ginsburg, 1977; Skemp, 1986) which should be built on, rather than ignored, as their strategies are often more conceptually based than standard methods (Schoenfeld, 1986; Carraher, Carraher & Schliemann, 1987). One way of achieving this is the early introduction of word sums (Carpenter, Hiebert & Moser, 1981). Also, social interaction is seen as facilitating learning opportunities in the mathematics classroom (Webb, 1991), through, for example, stimulating new ideas (Ausubel, 1968), cognitive rehearsal and clarification of one's own point of view (for example, Dees, 1991), conflict and controversy (Perret-Clermont, 1980; Doise & Mugny, 1984), extending schemas to incorporate others' interpretations (Skemp, 1986), exposure to higher quality thinking within the Zone of Proximal Development (Vygotsky, 1978) and metacognition (Romberg & Carpenter, 1986). Small group work can also make children more comfortable (Webb,

1991), less anxious and more confident (Ausubel, 1968) than when interacting in a whole-class situation.

Secondly, the availability of calculators means that it is less relevant for the children to focus on executing calculations in a quick and automatic manner than it is to emphasise planning and evaluating solutions of problems (Human *et al.*, 1991a). Changing needs of society suggest that standard pencil-and-paper methods are less important in preparing children for further mathematical participation than basic number concepts and an attitude that mathematics can and should be understood (Human *et al.*, 1991b) rather than the attitude that it is the form of a mathematical answer that counts (Schoenfeld, 1986).

The Hypothesis

It has been suggested (for example, Skemp, 1986) that Mathematics Anxiety may be created in the classroom. Because aspects like working in small group, using existing knowledge and emphasis on process rather than product have been named as possible ways to prevent and/or lessen anxiety in the classroom (Morris, 1981), less anxiety was initially expected to exist in children exposed to the problem-centred approach. However, because aspects like having to explain and justify solutions may actually *cause* anxiety, and pupils may be tempted under pressure to use quicker and more elegant methods which they don't understand, different *types* (profiles) of anxiety may be expected to exist in the two approaches, the mathematics anxieties being inferred from responses to a questionnaire previously constructed and validated for this purpose.

Based on research, such as Suinn *et al.* (1988) and Hembree (1990), in each approach negative correlations were expected between total responses to the Mathematics Anxiety questionnaire and performance in mathematics, as measured by internal tests in the schools concerned (only one school from each approach was used). The two approaches were not being compared in terms of performance, as aims and objectives differed for each approach.

Method

A 20-item Likert-type scale constructed to measure Mathematics Anxiety was translated and revalidated for use in South African schools. Items described a variety of mathematics-related situations inside and outside the classroom, for example calculating change in a shop and explaining a mathematics problem to the teacher.

143 children, mainly nine and ten years old took part in the study, of which 58 were boys and 74 girls, 116 were in the project school and 27 in the non-project school. The children were all white and

Afrikaans-speaking and in schools in the Orange Free State¹. The choice of schools was limited by the avoidance of troubled areas of the country. Also, the project has proved successful with regards to understanding and progress (Human, 1990) and participation and attitudes (Bouwer, 1991), and has spread rapidly from a small number of schools in the Cape Education Department to all schools in the CED as well as some schools in other provinces, such as the Orange Free State, sometimes without the necessary training and support. In other words, project ideas may be present in nonproject schools and it is becoming increasingly difficult to find a purely "non-project" school with which one can compare project schools. The non-project school was therefore in a rural area, 100km away from the city which contained the project school.

The children all completed the Mathematics Anxiety questionnaire. The average total scores, average item responses and correlations between total scores and average performance over the year were calculated for the total and subgroups. Factor analysis, using principle component analysis, was carried out on the questionnaire for the total sample and the project school sample (the non-project school sample being too small), to see if patterns obtained in a pilot study would be replicated by either or both groups. In the pilot study carried out in Cambridge, two factors had emerged, which were called Number/Sum Anxiety (items related to doing sums and working with numbers) and Social Anxiety (items related to doing maths in a social situation).

Results

The translated instrument proved to be reliable ($\alpha=0.84$). Validity could be inferred (Suinn *et al.*, 1988) from the relationship with performance as criterion, the single primary factor and the children's own experience of the questionnaire as reported in interviews.

The average total score for the Mathematics Anxiety questionnaire was 32.03 for the total sample (where the minimum score was $20 \times 1 = 20$, and the maximum was $20 \times 3 = 60$). The average for the non-project school (35.63) was significantly higher than the average for the project school (31.19) ($p < 0.001$). There was no significant sex difference ($p > 0.05$).

The items with the highest average responses related to a maths test for which one has not been warned and to teacher questions to find out how much one knows about mathematics. The item with the lowest average response related to games involving numbers. The non-project school reacted to the following items with significantly more anxiety: the maths test, teacher questions, explaining a problem to the teacher, working out change, using symbols like + and x, adding $97+45$ and playing games with numbers.

¹ Acknowledgment with thanks to Dr Peet Venter and the OFS Education Department, Ms Hanlie Murray and Prof Piet Human of the Research Unit for Mathematics Education, University of Stellenbosch, and the staff and children of the schools concerned.

The correlation between Mathematics Anxiety and performance was negative and significant ($p < 0.01$) for both the total sample and the project school. It could not be calculated for the nonproject school as the sample was too small.

Factor analysis showed one primary factor for both the total sample, and the project school, which is assumed to be Mathematics Anxiety. The findings of the pilot study were therefore not replicated. Scoring the children on two scales, Number/Sum Anxiety and Social Anxiety, with intuitive allocation of items to each scale, showed a significant relationship between the two scales ($p < 0.001$). In other words, children were mainly either anxious or not anxious across the board (for both scales), with the exception of 16 cases who showed low Sum Anxiety but high Social Anxiety, and 13 who showed high Sum Anxiety but low Social Anxiety.

Discussion

There was, as hypothesized, more anxiety in the non-project school children, if this can be inferred from responses to the questionnaire. There was no evidence of different profiles or patterns (factors) of mathematics anxiety, as had been found in the Cambridge sample of children. It should be noted that a recent study of South African students' Statistics Anxiety also reported only one factor (Pretorius & Norman, 1992), so there may be cultural differences which contribute to this finding.

The total scores were inversely related to the children's performance in mathematics, but the direction of this causality is necessarily ambiguous (Dreger & Aiken, 1957): Anxiety may cause poor performance (Hembree, 1990) or poor performance may cause anxiety (Tobias, 1987). The relationship may also be more indirect than direct (Adams & Holcomb, 1986; Meece, Wigfield & Eccles, 1990)

The limited sample, especially with regard to the non-project school, means that conclusions should be drawn with care. Any differences obtained could be ascribed to a variety of factors, including approach to teaching, general atmosphere (e.g. urban and rural) and teaching styles and personalities. The latter was evident from observations of the non-project class and the three classes in the project school. More schools would be needed in each approach to eliminate these other variables, and this is the next step in this research.

Conceptually, Mathematics Anxiety is difficult to separate from general anxiety and even more difficult to measure. Ideally, several measures should be used (Jones, 1984). Interviews were conducted with some of these children and should lead to some understanding of the specificity, dimensionality, causes and effects of Mathematics Anxiety. Analysis of these interviews is in progress.

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Students' Understanding of Differentiation and Integration

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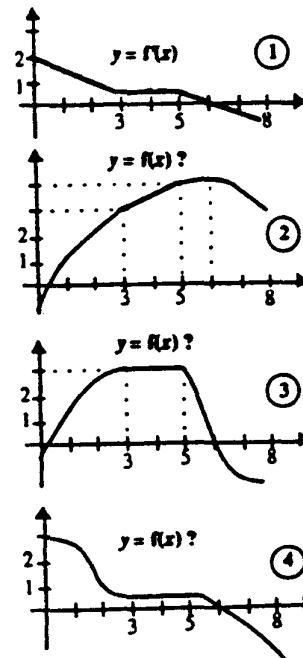
This study is a small part of a wider undertaking which examines students' cognitive difficulties in understanding differentiation and integration in both computer and non-computer environments. Here the results of a pilot study which took place at sixth form college are described and discussed.

The focus here is to analyse A-level students' errors and difficulties on a question. This question (taken from Cornu 1983) shows a graph which is the derivative of one of three others. The students have to choose one, and give reason(s) for their opinion. This question (see Figure) were administered to 18 students who have been using graphic calculators through the course.

Graph 1 is the derivative $y = f'(x)$ of a function $y = f(x)$ defined for $0 \leq x \leq 8$.

Which of the graphs 2, 3, 4 could be the original graph $y = f(x)$?

Give reason(s) for your choice.



Two of those students could not attempt the question. Among those students (9 students) who gave correct answer only one of them made a full explanation, the others gave partial explanation:

" Between 1 & 6 $f'(x)$ is positive so $f(x)$ is increasing, and after $x = 6$, $f'(x)$ is negative, so $f(x)$ is falling. Also on (2) the line becomes straight as $f'(x)$ crosses the x-axis."

" after $x = 6$, there is a negative gradient."

Seven of the students chose the wrong answer in this item. Three of those students chose graph 3 although four of them chose graph 4. A typical explanation given by a student who chose graph 3 is given below:

"It shows the stationary points in the correct areas."

Even more interesting are the explanations of those who chose graph 4:

"The gradient graph does not increase at any point."

" Crosses x- axis at 6. Level between stages $x = 3$ and $x = 5$."

Briefly, the results show that 50% of the students gave the correct response with a correct reason. Empirical research done by Tall (1986a) also revealed that 67% of the experimental students who used Graphic Calculus (Tall, 1986b), chose the right answer with a correct explanation, while only 8% of the control students did. Thus it is likely that visualization in the graphical context can help students to understand the relations between differentiation and integration.

In the wider study the following will be addressed.

- What are the misconceptions beyond these difficulties?
- Is it possible to diminish or eliminate these difficulties with the use of computer?If so, How?

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CONVERSATION AS A METAPHOR FOR MATHEMATICS AND LEARNING

Paul Ernest

Much can be learned about theories in mathematics education and elsewhere by examining their underlying metaphors. Elsewhere I have compared the underlying metaphors of mind of various learning theories (Ernest, 1993). For example, I have argued that the metaphor of mind for information processing constructivism is that of a computer, an unfeeling thinking machine; that of radical constructivism is an evolving and adapting but isolated organism, a cognitive alien in an unknown or hostile environment; and that of social constructivism is that of persons-in-conversation. It is this last metaphor that I wish to pursue (and extend) here.

For some years I have been developing a social constructivist theory of mathematics and learning (Ernest 1991a), and persons-in-conversation plays the part of the central metaphor in this theory in a number of ways. In this paper I wish to indicate how this metaphor can be developed to describe mathematics itself, mind and the teaching and learning of mathematics.

The term 'social constructivism' is used in a variety of ways by different people (and indeed, by myself, over time). My view of the problematique of social constructivism is twofold. It comprises, first, an attempt to answer the question: How to account for the nature of mathematical knowledge as socially constructed? Second, How to give a social account of the individual's learning and construction of mathematics? This account must accommodate both the personal reconstruction of knowledge, and personal contributions to accepted mathematical knowledge. Below I sketch the contribution the metaphor of conversation makes to both of these problem areas. However I should indicate that in the second area my views have developed and shifted significantly. In Ernest (1991a) I attempted to bring together views of socialisation through language acquisition (drawing on Wittgenstein and Vygotsky) and a neo-Piagetian or radical constructivist account of the individual's construction of meaning. Although I offered criticisms of the radical constructivist position (e.g. Ernest 1991b) it is only gradually that I have come to realise how incompatible the neo-Piagetian/radical constructivist position is with a view of mind thoroughly based on the metaphor of conversation. So here by social constructivism I mean *social* constructivism, and not social *constructivism*.

CONVERSATION: AN EMERGENT METAPHOR

Conversation in the form of written dialogue has been used in philosophy from the time of the Ancient Greeks. It has also been explicitly adopted as a central epistemological concept by many philosophers and theorists. But I need to distinguish between three different meanings of the term. Conversation originates at the interpersonal level, where persons in one or more shared 'forms of life' engage in direct conversations, based in common 'language games' (Wittgenstein). This living, actual conversation with others in real-time is based on shared experiences, understandings, values, and mutual respect. At this level conversation is one of the basic modes of interpersonal human interaction, perhaps even the most basic one, if understood inclusively enough.

Mediated forms of conversation involving written texts (understood broadly to include all forms of notation, inscription and sign systems) represent an important extension of the notion (Gadamer). However, the transition from spoken to written textual forms of conversation is a crucial one. It creates a different relationship between the author and what is uttered, and allows the text to be objectified and preserved beyond the moment of utterance. It allows mathematical texts, proofs in particular, to be construed as monological, with all answers anticipated and incorporated in the text. However, the reading of any text remains dialogical, with readers interrogating the text and creating answers from it. Thus the second form of conversation is at the cultural level, the 'conversation of humankind' (Oakeshott). This is the direct sum of interpersonal conversations in oral cultures. However the rich, complex, symbolic culture of the history of mathematics as we know it is only possible through

extended conversation, based on the production and use of texts in permanent form (but not limited to just that).

Third, there is internalized private conversation. Many theorists including Plato, Gergen, Harre, Mead, Shotter, and Vygotsky argue that thought itself is internalized conversation, and that socially situated conversation between persons plays a crucial role in the formation of mind. Consequently it is also a central underlying feature of the subsequent use of mind. Even the private and individual functions of mind are socially constructed, although once formed they can take on a life of their own and operate a long way removed from any collective or public conversation. Mathematicians for example, can operate in isolation for extended periods of time, having internalised some of the conversational roles and procedures they learnt through conversation of the first and second kinds. These include, most notably, the role of proponent, in which a line of thinking or thought experiment is followed through sympathetically, for understanding, and the role of critic, in which it is examined for weaknesses and flaws.

All three forms of conversation are social, be they interpersonal, cultural or intrapersonal, for they involve an alternation of human voices: present or removed, real or imagined. Conversation is also a construction, as are all linguistic and other cultural entities and phenomena. My proposal, therefore, is to adopt conversation (understood broadly to accommodate its three aspects) as a basic epistemological notion for a social constructivist philosophy and theory of mathematics.

The argument for accepting conversation as epistemologically basic is that language and discourse play an essential role in the genesis, acquisition, communication, formulation and justification of all knowledge, mathematical knowledge in particular. Conversation is the dialogical deployment of language, with its social exchange dimension, and its dialectics, with ebb and flow, assertion and counter assertion, is essential for communication and feedback, in the form of acceptance, elaboration, reaction, criticism and correction. This underpins the justification of objective mathematical knowledge, and the ratification of personal knowledge, as I describe below.

In general epistemological terms, Collingwood, for example, similarly proposes a dialectical 'logic of question and answer' in place of the (mono) logic of propositions. Again, Rorty adopts conversation explicitly as his philosophical basis for epistemology and mathematical knowledge.

If, however, we think of "rational certainty" as a matter of victory in argument rather than of relation to an object known, we shall look toward our interlocutors rather than to our faculties for the explanation of the phenomenon. If we think of our certainty about the Pythagorean Theorem as our confidence, based on experience with arguments on such matters, that nobody will find an objection to the premises from which we infer it, then we shall not seek to explain it by the relation of reason to triangularity. Our certainty will be a matter of conversation between persons, rather than an interaction with nonhuman reality.

Rorty (1979: 156-157)

CONVERSATION IN MATHEMATICS

Currently, there is a move in some quarters to reconceptualise mathematics and the philosophy of mathematics in fallibilist, human-centred and even social terms (Davis and Hersh, Kitcher, Lakatos, Tymoczko, Ernest 1991a).

This reconceptualisation represents a break from the traditional absolutist views of mathematical knowledge which see it as monological in character. Monologicality is a central assumption of Cartesian rationalism and the modernist outlook based on it. Mathematical knowledge is presented as if it is God-given, not uttered by human voice, let alone by a one of several voices (albeit a dominant one) in a dialogue or conversation. Monologic thus produces perfectly ordered and structured texts with no trace of author/listener, with an implied single meaning, which is the utterance of Authority. It can be viewed as either a perfect polished achievement of dialogue; or a degenerate form employed by Authority to impose power (in mathematics, society or the classroom).

Instead, my argument is that mathematics is dialogical, and that conversation permeates mathematics in deep and multiple ways. The underpinning metaphor of conversation stresses dialogic, comprising alternating voices in a shared quest for understanding, based on the logic of question and answer, and on uncertainty. This resonates with the fallibilist turn in the philosophy of mathematics. However, the claim that mathematics is conversational, dialogical or dialectical can be understood in multiple ways (I do not distinguish these three notions here, but use them as loosely equivalent.). These include: its linguistic/textual basis; its concepts and content; the foundations of proof; and the underlying epistemology and methodology of mathematics.

Mathematical Language

Mathematical activity is primarily a symbolic activity, which uses written inscription and language to create, record and justify its knowledge (Rotman, 1993). Viewed semiotically as comprising texts, mathematics is inescapably conversational and dialogical, for by its very nature it addresses a reader (Volosinov, Bakhtin). Beyond this general feature, an analysis of mathematical texts, proofs and algorithms, reveals the verb forms employed to be both in indicative and imperative moods. The declarative case of the indicative mood is used by the writer to make statements, claims and assertions, which are claims about the outcomes of certain processes. The imperative mood is used for both inclusive and direct imperatives, which are shared injunctions, or orders and instructions issued by the writer to the reader. Thus mathematical texts comprise specific assertions and imperatives directed by the writer to the reader, i.e. they are one-sided segments of dialogue (Rotman, 1993).

Mathematical proof is a special form of text, which since the time of the ancient Greeks, has been presented in monological form. This reflects the absolutist ideal that total precision, rigour and perfection are attainable in mathematics. Thus the monologicality of the concealed voice uttering a proof itself belies and denies the presence of the silent listener. But as it is an argument intended to convince, a listener is presupposed. The monologicality of proof tries to forestall the listener by anticipating all of her possible objections. So the dialectical response is condensed into the ideal perfection of a monologic argument, in which no sign of speaker or listener remain.

Mathematical Concepts and Content

A substantial class of modern mathematical concepts and content have an underlying conversational or dialectical basis. These include, for example, aspects of analysis (e.g. limit definitions: "You give me ϵ , and I'll give you δ "), statistics (hypothesis testing: H_0 versus H_A), probability (analysis of wagers, betting games), game theory (alternation of moves by opponents), constructivist logic (the interpretation of quantifiers $\forall x \exists y$: "You choose x , and I show how to construct y "), number theory (John Conway's game theoretic foundations of number), set theory (game theoretic version of Axiom of Choice, Cantor's dialogical diagonal arguments), recursion theory (interpretation of quantifiers in arithmetical hierarchy). Thus it can be said that conversational and dialectical interpretations can be given to a significant range of concepts from some of the main branches of mathematics, and form an a necessary characteristic of some others. Thus the dialogicality of mathematical content is widespread and deep.

Origins and Basis of Proof

Dialectics and conversation provides the origins of mathematical proof and logic, and a foundation for certain modern conceptions of logic and proof. Mathematical proof, certainly in its axiomatic form, developed in Classical Greece, probably due to the widespread practices of disputation and dialectical reasoning, which were central to the public democratic institutions and cultural practices of the day. (The word 'dialectic' is derived from the verb meaning 'to discuss'). Szabo and others locate the source of deductive mathematics and logic in dialectical argument, disputation and conversation. Thus it seems that Rorty's conversational reading of persuasion in proof reflects its very origins, and is not just a postmodern reading of it.

In proof theory, some of the main developments also treat mathematical proofs as if they are offered in a dialogue. In them a proponent attempts to convince an opponent of her claims, whilst the opponent challenges what is asserted, but accepts a number of agreed basic rules of reasoning and facts). Thus these developments are evidently dialogical. They can be found in Heyting's intuitionistic proof theory, Natural Deduction, the method of Semantic Tableaux, and in Lorenzen's constructive logic (used by Habermas as a basis for his conversational Theory of Communicative Action). Hintikka also proposes a system of Game Theoretic Semantics for tableaux.

Thus both the beginnings of logic and mathematical proof and many of their modern developments suggest that mathematical proof is at root dialectical, based in human dialogue and on conversational exchange.

Epistemology and Methodology

The epistemology and methodology of mathematics, including the nature and mechanisms of mathematical knowledge genesis and warranting can be accounted for in an explicitly and constitutively dialectical way. The social constructivist account of the conversational basis of mathematics is based on primarily on the work of Wittgenstein and Lakatos (Ernest 1991 a, forthcoming). Wittgenstein offers the basis of a social theory of meaning, knowledge and mathematics resting on dialogical 'language games' embedded in 'forms of life'. This basis makes it clear that conversation rests on shared experiences, habits, understandings, assumptions and participation in communal activities. Wittgenstein thus shows the situated, contextual basis of all knowledge. He also provides an account of mathematical proof and necessity, based on socially accepted notions of 'following a rule' as opposed to the objectivity of mathematical knowledge understood in an absolutist or transcendent way.

Lakatos offers a multifaceted if incompletely formulated theory which crucially reintroduces history into the philosophy of mathematics. At the heart of this is his heuristic or Logic of Mathematical Discovery (LMD), which is a dialectical theory of the history, methodology and philosophy of mathematics. Lakatos' LMD can be explicated as a cyclic process in which a conjecture and an informal proof are put forward (in the context of a problem and an assumed informal theory). In reply, an informal refutation of the conjecture or proof are given. Given work, this leads to an improved conjecture or proof, with a possible change of the assumed problem and informal theory. This pattern is evidently conversational and dialectical.

The proof procedure seems to me to be a remarkable example of the dialectic triad of thesis, antithesis and synthesis. The progress of mathematical thought - in this case starts with the primitive conjecture. This is the thesis. This thesis produces its antithesis which consists of the tension and struggle of the proof and refutations Now the synthesis is the theorem which embodies the respective values of both poles of the antithesis - proof and refutations - on a higher level, without the limitations of both.

Lakatos (1961: 51)

To overcome some of the criticisms directed at Lakatos, this scheme can be generalised to accommodate a broader range of changes, outcomes and responses including theory growth and 'mathematical revolutions' (Ernest forthcoming). Following this scheme, mathematical proofs or other proposals are offered to the appropriate mathematical community as part of a continuing dialogue. They are addressed to an audience, and they are tendered in the expectation of reply, be it acceptance or critique. Such replies may play a part in the development and formulation of new mathematical knowledge. However, such replies, when given by the gatekeepers of institutionalised mathematical knowledge (e.g. journal editors) play the essential warranting role in the acceptance (or rejection) of candidates for new mathematical knowledge. The social acceptance of mathematical knowledge (and hence its status as knowledge) is constituted by this conversation.

Within the contexts of professional research mathematics, individuals use their personal knowledge both to construct mathematical knowledge claims (possibly jointly with others), and to participate in the dialectical process of criticism and warranting of others' mathematical knowledge

claims. In each case, the individual mathematician's symbolic productions are (or are part of) one of the voices in the warranting conversation.

Thus mathematical proof has not only evolved from a dialogical form, but its very function in the mathematical community as an epistemological warrant for items of mathematical knowledge requires the employment of that form. The underlying logic is dialectical.

CONVERSATION IN LEARNING MATHEMATICS AND MIND

conversation and mind

A number of theorists, including Mead and Vygotsky, argue that thought itself is internalized conversation. They claim that thought is constituted and formed by intrapersonal conversation, and that thinking: is internalised conversation with an imagined other. On this basis, mind can be viewed as social and conversational because first of all, individual thinking of any complexity originates with and is formed by internalised conversation; second, all subsequent individual thinking is structured and natured by this origin; and third, some mental functioning is collective (e.g. group problem solving).

Harre has developed a theory of Vygotskian space based on two polarities of thinking or speech, comprising: manifestation or display (public or private) and social location (collective or individual). He combines these in a Cartesian product to make four quadrants which have a cyclic relationship in the development (and location of mind), and which also defines the cycle of the appropriation and testing of knowledge. Harre uses the terms 'appropriation', 'transformation', 'publication', 'conventionalisation' to describe the successive passage of thought and knowledge (and even the construction of personal identity) from one quadrant to the next. This closely resembles the social constructivist theory of the cyclic passage of mathematical knowledge from 'objective' (i.e. public) to subjective and then back to objective knowledge again, shown in Ernest (1991 a: 85, forthcoming). It also fits with theories of socialisation.

society is understood in terms of an ongoing dialectical process composed of the three moments of externalization, objectivation and internalizationIn the life of every individual, therefore, there *is* a temporal sequence, in the course of which he is inducted into the societal dialectic. The beginning point of this process is internalization ...

Berger and Luckmann (1966: 149)

Thus the dialectical process of the formation of mind and of individuality is understood to be dialogical and conversational. This is the model of mind-world relations adopted by social constructivism in the current version (Ernest forthcoming).

Conversation in the Teaching and Learning of Mathematics

Mathematics, like any other area of knowledge, is learned through individuals (learners) participating in language games embedded in forms of life. Personal knowledge or competence in mathematics is acquired through prolonged participation in many socially situated conversations in different contexts with different persons. Initially, the forms of life are domestic and out of school, and these provide an essential set of capabilities for young persons to enter into the novel, formalised learning settings in schools and other educational institutions. Schools, of course, only represent one cluster of contexts and social practices into which young learners enter into and learn from. These are planned teaching and learning situations in which the teaching of mathematics is deliberate. In the context of such intentional forms of mathematics education (in or out of formal institutional settings) certain individuals (teachers) structure mathematical conversations on the basis of their own knowledge, and texts, in order to offer mathematical experiences to learners, with the aim of developing their mathematical competences. They direct, structure and control mathematics learning conversation both to present mathematical knowledge to learners directly or indirectly (i.e. teaching), and to participate in the dialectical process of criticism and warranting of others' mathematical knowledge claims (i.e.

assessment). These two functions are irrevocably intertwined, except in their extreme forms where they are temporally and conventionally separated (e.g. expository lecturing and marking external assessments) .

The learning conversation extends beyond the immediate teacher-pupil interaction. In school contexts, there are attenuated conversations including leamer-textually presented answer interactions, leamer-computer presented answer interactions, learner-peer interactions. In out-of-school contexts there are in addition to the above, leamer-parent and leamer-significant other interactions.

The public representation of mathematical knowledge within a teaching-learning conversation (including its textual variants) is necessary but not sufficient for such knowledge to become the personally appropriated mathematical knowledge of an individual learner. Sustained two-way participation in such conversations is also necessary to generate, test, correct and validate mathematical performances. Teacher-pupil dialogue (usually asymmetric in classroom forms) typically takes place at two levels: spoken and written. In written 'dialogue' pupils submit texts (written work on set tasks) to the teacher, who responds in a stylised way to its content and form (ticks and crosses, marks awarded represented as fractions, crossings out, brief written comments, etc.). The primary aim of such conversation is that of ensuring that the learner is appropriating collective mathematical knowledge and competences, and not some partial or distorted version. Appropriated mathematical knowledge is potentially unique and idiosyncratic, because of human creativity in sense-making. This possibility also arises because school mathematical knowledge is not something that emerges out of the shared meaning and purpose of a pre-given form of life. Instead it is a set of artificially contrived symbolic practices whose meaning is not already given, but is deferred until the future, or at least a significant part of it is.

What must not be overlooked is that conversation is fundamentally a moral form, not just about exchanging information. For it entails engaging with a speaker or listener as another human being, not just as a source or end-user of information. Thus in education the use of the conversational metaphor in the teaching and learning of mathematics ideally should entail a number of things. For a start:

- Mutual respect and trust between teacher and learner;
- Listening to learners; showing (and feeling) an interest in their views, in their conceptions, and in their sense-making;
- Making teaching into *real* conversation, into a *real* dialogue where there is respect for the leamer's intelligence and where there is space for learner initiative too;
- Treating real subjects and content of mutual interest and of mutual benefit.

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Teachers Assessing Coursework: Themes and Tensions

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The data discussed here has arisen from interviews with teachers carried out as part of a study of the discourse of doing and assessing coursework in secondary schools. In a previous BSRLM session (Morgan, 1992) the features of children's coursework texts were examined and the criteria for what might be 'appropriate' ways of writing considered. The meaning of a text does not, however, reside in the text itself but is constructed through the interaction of the text and its reader within a specific context. Where there are multiple readers (or one reader taking multiple positions) there may be different readings. In particular, features of the children's texts may be interpreted in different ways by a group of teachers, lecturers and researchers at a conference and by a teacher trying to assess a piece of coursework. In order to incorporate the teacher's perspective as reader of children's texts, 'discourse-based' interviews (Odell, 1982) were used in which teachers were asked to read a small number of pieces of children's coursework based on the task 'Inner Triangles' (LEAG, 1991), to talk about what they were reading, to place the pieces of work in rank order, and to explain how they were making their judgements. The pieces of work had been chosen to illustrate contrasting styles of writing, the aim being to identify those features of the children's written work that were considered to be significant and to examine the reasons the teachers gave for their judgements.

Two of the themes identified in this data will be considered here. Both themes will be illustrated by extracts from teachers responses to one page of a piece of coursework written by Robert. Robert's text as a whole contains very few words and consists largely of diagrams, tables of results and formulae. The page under discussion here contains: a table of results (but no indication of where the results came from); a single diagram of a trapezium with its dimensions labelled with variable names; and a formula using these labels to relate the dimensions of a trapezium to the number of unit triangles it contains. All the work is correct and arranged neatly on the page. One of the teachers, Alan, taught Robert himself, but the interview took place two years after the coursework had been completed. Alan's recollections of the boy are, therefore, likely to be of a general nature rather than specifically related to his work on this investigation.

Theme 1: shifting reader positions

Freda's discussion of this page of Robert's work (figure 1) suggests that she does not have a single perspective on his work but reads it from a variety of different points of view.

Several times Freda identifies "problems" with Robert's work. She is reading here in the role of an examiner looking for evidence that the results 'belong' to Robert (lines 1-3), for evidence of "how he's got it" (lines 15-16), and for justification of the results (lines 21-22). Her discomfort in this role, however, is suggested by the use of the word *problem*. There is a tension between the rigour of the examiner, for whom use of the assessment criteria determines .u.n.problematically a decision

about the value of a piece of work, and the wish of a teacher that a pupil should get as high a grade as possible. In this case, Freda reading as teacher/advocate feels that the missing evidence might have been available in the classroom but when she reads as examiner she cannot take account of this possibility. The problems are Robert's problems but they are also Freda's problems in resolving her two roles.

Figure 1

5
Freda *This is a major problem because he's got these results but unless one is there in the class and you're a teacher you don't know whether this is his results or somebody else's. He hasn't shown any diagrams of where these results have come from. He hasn't done any drawings as far as I can see. He's come up with a formula which is Z equals. Z must be the slant height. Is equal to X plus Y equals T . I assume that's right, I don't know.*

CM Yes, I think that's right

10
Freda *That is right is it? Ok . . . but again even that's not, I mean he's given . . . one thing that I think they have to do is when they give a formula they should explain it using quite a few examples and show how it works. The thing that I always look for and I say to the kids is: you write it up as if you're writing it for somebody who's never seen this problem, who's never done that but would be able to understand it if they were to read it. And from this, somebody . . . I don't think it's clear enough for somebody to use it and then work out, I mean he hasn't done even one example of how it works. So I think he's got a major problem here, he hasn't shown how he's got it. If you were in the class, obviously we can award marks when they've done something in class but it's not written down. If you're the teacher in the class and you knew that he's just omitted it by mistake um but also he hasn't included his rough work and that's, when this happens, often you find what you need in the rough work and that can, you know. So none of his results are justified there at all. I think he could have problems with that.*

15

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This tension is expressed again (line 9) when Freda describes "one thing that I think they have to do"; there is an ambiguity in this phrase which is also present in the intonation used: is this her own opinion of what is appropriate or is it her belief about what is expected by the examination board? When she (as teacher) is uncomfortable with the severity of her judgement of the piece of work, she is able to shift the blame to an anonymous authority that lays down what "they have to do". This shifting of the blame is done more explicitly by another teacher, Dave (see figure 2), who referred repeatedly and negatively during his interview to the expectations of the examination board in explaining how he was making his assessments of pupils' work against his own judgement.

At lines 10-11, Freda again expresses a dual role - this time a more comfortable one - as examiner and as teacher/adviser, looking for the specified characteristic of the writing as an assessment criterion and simultaneously advising the pupils of the criterion that is being used. The nature of the criterion she describes, however, forces her to adopt yet another reader position, as an imaginary naive reader "who's never seen this problem, who's never done that but would be able to understand it if they were to read it" (lines 12-14).

Figure 2

5	Dave	With a bright group, I can actually remember a boy doing this and within 20 minutes with a bright group giving me the formula. And he'd worked it out in that time. He'd sussed it out very quickly and so - it actually makes it quite a ponderous kind of activity really cos he'd seen it, he'd got the formula that they're asking for in the generalisation, he knew why it worked, he could explain it and that was all in one at the beginning of the first lesson.
	CM	So where does he go from there?
10	Dave	<i>But he still has to fulfil the criteria on a London piece of work.</i> I mean he was a very amenable lad and he did it but it must have been a bit tedious to say the least.

In this short passage extracted from Freda's interview, she thus adopts a number of reading positions, some of which are potentially contradictory:

- examiner, using externally determined criteria
- examiner, setting and using her own criteria
- teacher/advocate, looking for opportunities to give credit to a pupil
- teacher/adviser, suggesting ways of meeting the criteria
- imaginary naive reader

It is hardly surprising that teachers find the assessment of written reports of investigations difficult. The tension between taking on an examiner role and acting as teacher/advocate is a familiar one for teachers involved in any summative assessment; it is to be expected that one way in which it may be resolved is by appeal to the anonymous authority of the examination board. When the criterion concerned is expressed in terms of suitability for an imaginary audience, however, there is an assumption, not only that the pupil will understand the nature of this hypothetical audience and actually address it, but also that the teacher will successfully adopt the position of the specified audience when reading and judging the pupil's work. Neither of these assumptions are justified (see, for example, Redd-Boyd & Slater, 1989; Gilbert, 1989). Where, as in this case, the teacher is reading as an 'lexpert', judging the mathematical quality, and simultaneously attempting to read as "somebody who's never seen this problem", the tension is less easily resolved. This is particularly the case where the characteristics of the specified imaginary audience are unclear; in Freda's case it appears that "somebody" has some degree of mathematical understanding as they would be able to understand it if they were to read it" and yet they are constructed as needing an example in order to be able to use a simple formula.

Theme 2: shifting roles for algebra

Like Freda, Alan (figure 3) comments on the lack of "evidence" in Robert's work but his teacher/advocate position appears dominant over his examiner role (lines 4-5). He ascribes his "confidence" in the work to his more personal knowledge of the boy concerned. This is confidence that the work is Robert's own effort; Alan is still concerned at the lack of evidence of what Robert can do. One of the key pieces of evidence that is identified as being missing in Robert's work is a generalisation expressed in words. Such generalisation, however, appears to play multiple roles. Initially, Alan claims that he wants to see that Robert "can generalise in words" (line

4.

10); he is looking for evidence of a skill. In the next breath he is suggesting that this would also provide evidence of "understanding". The expression that he uses is, however, ambiguous. "It kind of gives the understanding" (line 11) to whom? Is the understanding given as evidence to an examiner or given to the pupil himself. As Alan shifts between an examiner and a teacher/adviser position, the generalisation in words also shifts from being evidence towards assessment of the pupil to being a pedagogic device for helping the pupil to gain understanding. (Note that at the same time he shifts from referring specifically to Robert to using an unspecific "they" (line 13) and ultimately (lines 20-23) to an explicit generalisation about what "we" tell "the children".)

5.

Figure 3

5	Alan	He jumps straight into a table of results which . . . although there's no evidence for it, that is interesting because I know the way this boy worked, the work was very disorganised, he'd work on pieces of paper thrown everywhere . . . <i>I'm very confident that although there's no evidence of it what he's produced is right and he's done it.</i> It definitely wouldn't be a copy. He was very [..], didn't like working in groups and I am confident that he's produced that table by himself. That's interesting, that can only be a teacher's inside knowledge. Somebody marking that coldly wouldn't be able to state that. He's produced some algebra and the algebra's correct. He's indicated on a diagram what the variables stand for and that's fine. <i>Again I would like to see that he can generalise in words first of all. It kind of gives the understanding, I think, putting it into words the patterns which they see. Then I think it underpins the algebra which they produce later. But nevertheless I'm confident that that's his work, and that he's produced that off his own bat.</i>
10		
15	CM	But if that were a child that you didn't know, would you have doubts about it?
20	Alan	I think you'd have to. When we moderate, we usually do it as a group in the same room usually working in different corners on the scripts that we're moderating and frequently we ask across the room to the teacher concerned where did this come from is that alright and the [..] knowledge, on we go. But there's usually a check of that kind. <i>We try very hard to tell the children generalise in words first of all and we say if you know a pattern, can you tell us about it, tell your friend about it. When they can explain the pattern in words and they write those words down then they're ready to produce the algebra.</i>

Alan appears to be using a criterion (unwritten) that any algebraic generalisation appearing in a piece of coursework should be preceded by a verbal statement of the same generalisation. Stated like this, it is a simple matter to judge whether or not a pupil has fulfilled the criterion. The justification for using this criterion, however, is not so simple; it takes a number of forms:

- it is evidence that the pupil can give a verbal generalisation
- it shows that the pupil understands the algebra
- it proves that the formula 'belongs' to the pupil
- it helps the pupil to understand the pattern
- it prepares the pupil to "produce the algebra"

The discourses of assessment and of pedagogy are intertwined here - perhaps an inevitable consequence of the teacher's role. The suggestion that describing a generalisation in words is a first step towards using algebraic symbols is a common

pedagogic strategy. It seems, however, to have been transformed into a prescriptive algorithm for 'doing investigations'. It may well be that doing a large number of examples helps many children (and adults) to reach a generalisation and that expressing this in words, either spoken or written, helps many to construct a formal algebraic expression. Is this, however, a justification for insisting that those who do not need this help should nevertheless act as if they do?

Implications

In describing the multiple, and in some cases contradictory, positions taken by these teachers in the course of reading and assessing coursework texts, I am not suggesting that the teachers are confused or incompetent. All those quoted here are experienced, thoughtful teachers who have been involved with doing and assessing coursework since GCSE was introduced. The contradictions arise from the nature of the tasks that the teachers are doing and from the multiple discourses within which they are situated. As examiners for a national, summative examination they must be concerned with validity and consistent application of standards. As professionals concerned for the welfare of their pupils and for their own standing as successful teachers they wish to ensure that each pupil's performance is judged as highly as possible. As teachers they are concerned to provide their pupils with pedagogic support and advice that will help them both to learn mathematics and to perform better in assessment tasks. A further possibility that must not be ruled out in looking at these interviews is that the teachers are also positioned as 'interviewees' talking to an interviewer who they know to be working in Higher Education and who had previously worked with some of them in the role of advisory teacher. Alan's decision to describe his general procedures for assessment and pedagogy (figure 3, lines 16-23) suggests such a position.

This analysis leads me to question the practice of teacher assessment of coursework within external examinations in two ways. Firstly, is it possible to resolve the tensions between looking for evidence as an examiner concerned with validity and consistency and as a teacher/advocate concerned with justice for the individual candidate? While moderation processes may go some way towards resolving this, it is clear from Freda's interview that even a teacher who has no connection with the individual pupil whose work is being assessed may take on the teacher/advocate role. Secondly, it appears that assessment criteria related to the formation of generalisations may distort and be distorted by pedagogic practices originally intended to help pupils to develop algebraic thinking. The shift from a pedagogic device to a requirement to express a generalisation in words before expressing it using algebraic notation, like the proliferation of "train spotting" investigations (Hewitt, 1992) , is symptomatic of a need to standardise the form of pupils' investigative work in order to make it amenable to formal examination. Traditional timed written examinations clearly constrained the mathematical experiences of pupils. Coursework examination, greeted by many as a liberation, has introduced its own set of constraints.

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