Ton J Harries B. th

College of <u>Higher</u>

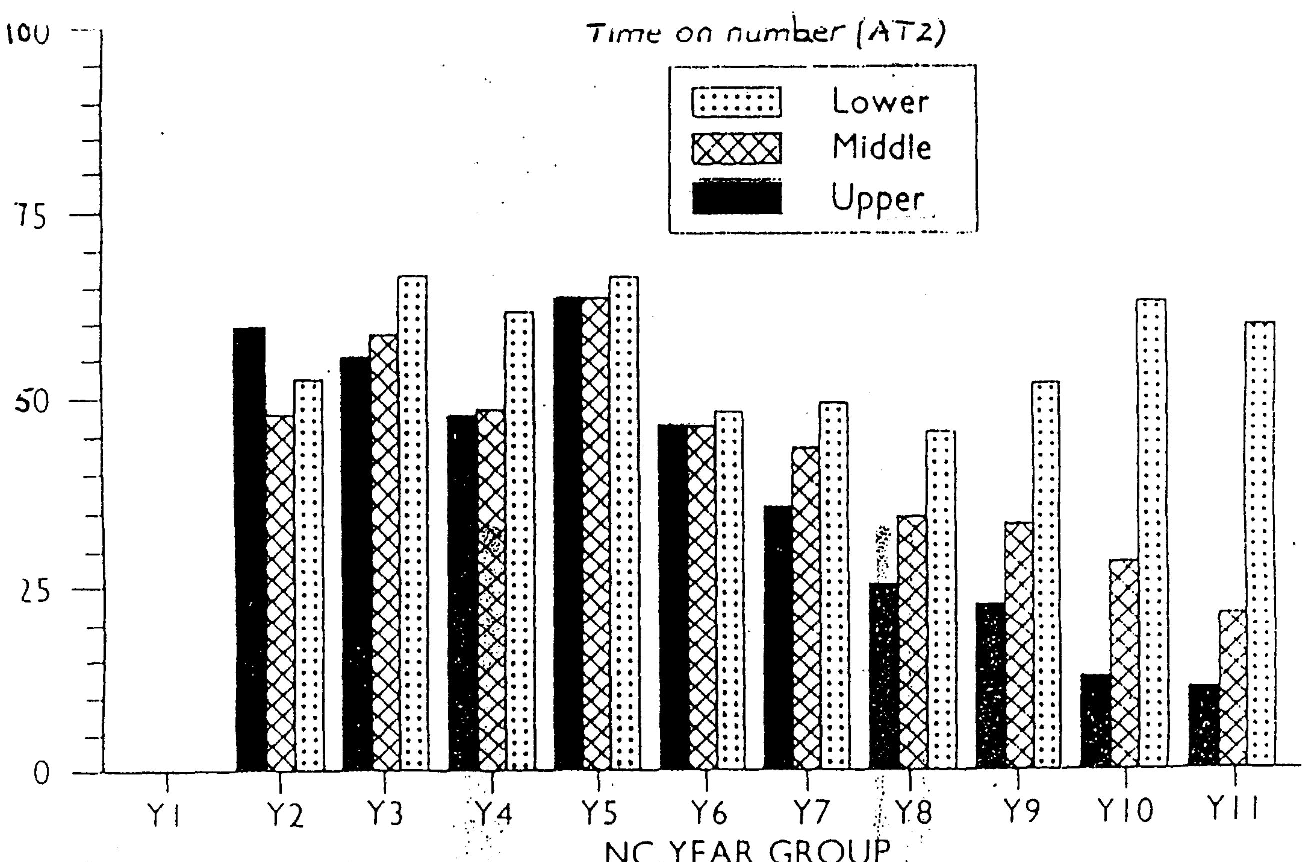
<u>Edu~atioll</u>

<u>Low Attainment and the Pursuit of Alqebra</u>

<u>Introduction</u>

> *I never understood Algebra at school. It never made any sense to me, but I can't say as I've ever felt handicapped since by not being able to do **it.***
>
> *When it comes to algebra and we have to operate with x and y there is a natural desire to know what x and yare. That at least was my feeling: I always thought the teacher knew what they were and wouldn't tell me.*
>
> *(Quoted in Mason 1985)*

Both these quotes from Mason's book Routes to Algebra - the first by a practising teacher and the second by Bertram Russell - illustrate the feelings of many pupils towards the study of algebra. In particular for pupils who are termed low attainers in Mathematics the study of algebra is in many ways considered to be an area of the curriculum that is better not studied. This results in the delivery of a rather narrow Mathematics curriculum being offered to a substantial body of pupils. This is illustrated in the graph below which shows that for low attainers the Mathematics curriculum is often reduced very largely to arithmetic.



Area of Research

The main aim of the study is to investigate the use and understanding of algebraic concepts of a group of low attaining pupils.

The low attainers in this study are pupils in the bottom Mathematics set out of eight in year 8 of a mixed comprehensive school. Some of the characteristics of the pupils are that they:

show a low level of achievelll.ent in their school Ifathelllatics.
have low esteem - *particularly in relation* to *their ability* to do *Ifathelllatics.*
lack confidence in their capacity to *tackle mathelllatical problems.*
have difficulty in performing basic number tasks.

A number of sub questions for research are:

1. Are these characteristics sufficient for identifying low attainers or are there other characteristics which affect their ability to work mathematically?

What aspects of algebraic thinking are accessible to these pupils and is number competence a pre-requisite?

What are the methods that these pupils use for performing numerical operations and what effect does it have on their ability to generalise?

2. How does the way in which these pupils are introduced to algebraic concepts influence their understanding.

How does the learning environment affect pupils potential for making progress in the learning of algebra?

Does the computer help in the search for an environment in which the ideas of Algebra can be naturally explored?

3. Does using an appropriate environment such as Logo, enhance the algebraic opportunities available to "low attainers"

In particular does the environment provided by Logo create this possibility, and help the student to work in a creative and reflective way?

4. How does working in Logo affect the way in which the pupil views work in other areas of Mathematics?

In what ways does do pupils make links between the symbolic language used in communicating with the computer and working algebraically in other systems

In order to investigate these areas of concern an exploratory pilot study was conducted in the period March to June 1993. It was conducted with a group of 12 low attainers in the bottom

Mathematics set in year 8 at an 11-18 comprehensive school in Bristol. The sessions were all conducted in the computer room where there was a suite of networked RM Nimbus machines. Although the pilot was conducted with the whole group, for the purpose of this pilot a detailed study was made on the learning of one of the pupils.

The aim of the study was three-fold:

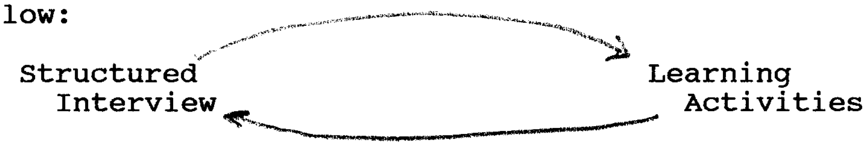
To explore an effective way of working with the pupils.

To refine the research questions and the area of study

To test the validity of some tasks designed to promote the use and understanding of variables.

Plan and Methodology

The methodology for the study is represented by the diagram below:



This was not intended as a pre-test and post-test situation but rather as an opportunity for the pupils to talk about what they had been doing.

In the prepared activities it was intended that the pupils would have the opportunity to use variables in the following ways:

1. variable representing a known physical quantity
    a. use of a single parameter in a procedure
    b. use of >1 parameter in a procedure

2. variable used as an operator
    eg use of scale factors.

3. variable operated on
    eg creating rectangles in which there was a specified relationship between the length and the width.

4. variable used to represent a functional relationship.

3.

Some Aspects of the Interviews

In the initial interview it appeared that the pupils had little experience of using letters. They tentatively tried to make an interpretation of the expressions discussed and tried to maintain some kind of consistency. There seemed to be a vague idea that the letter n represented "any number" but did not really understand what this might mean. They lacked confidence in their ability in that "don't know" was frequently an initial response to a question, and they were easily influenced by the responses of others in the group. In number work they appeared to think in terms of units, tens, hundreds ... millions. There was also a limited view of such things as numbers between 4100 and 4200 only one number was specified 4150.

Two sets of activities were used. The first set of activities were designed to enable the pupils to gain an understanding of some of the basic processes in Logo, create some simple procedures, explore the possibilities for making a shape bigger and smaller, and take a design made up of a series of parts and explore the changes in the shape as different values were given to the variables in different parts of the design. The second set of activities that the pupils were involved in were to do with working with function machines. The acti vi ties invol ved the pupils in working out what particular Logo procedures did, writing their own Logo procedures for particular input/output tables - some of which were provided by the teacher and some of which were generated by themselves. Finally they worked on comparing procedures that produced the same outputs.

In the interviews which were conducted following each set of activities a number of points of interest can be noted.

The pupils seemed to develop a clear understanding of functions as operations and their inverses. This was shown for both single and multiple operations both within and outside the logo environment. They demonstrated an understanding of the use of a variable or parameter as a scale factor and the effect that different values of the parameter had on the shape which had been constructed. It also seemed that they were able to use a variable or parameter as a place-holder for numbers. In addition they showed some evidence of being able to use notation to represent relationships described in words.
In each of case it would seem that the use of Logo has been influential in the subsequent understanding that was demonstrated outside the Logo environment. All the ideas were developed in Logo which meant that the ideas were explored visually on the screen and so there was immediate feedback which immediately confirmed or questioned conjectures that were implicit in the inputs made. In this way the pupils appeared to develop an understanding of concepts such as operations and their inverses, and scale factors and this understanding was used in non-Logo contexts.

Reflections and Issues

In this study the initial objectives were to involve the pupils in problem-solving situations that would allow them to use variables in a number of ways.

To a large extent these objectives were satisfied as has been illustrated in the discussion above. The pupils did use variables in a variety of ways. They were well-motivated. They were on task for whole sessions of more than an hour. They did appear to gain in confidence. They started to consider errors that were made as being interesting things to explore and find out why the errors occurred. But I am still not sure that I am clear about their perception of variables. Further in the work that they did I feel that it was very much teacher directed and did not allow the opportunity for much self-initiated work which could have given a better picture of their understanding.

Keiran (1988) suggests that the development of an understanding of algebra requires the student to develop the concept of a function as a rule or an operator, and also to use the symbolism to build a support structure. The pupils certainly did this in the Logo environment. Most of their work was using functions as operators and the symbolism of Logo was of necessity their support structure. Even so it does not seem to me that these are sufficient conditions for developing an understanding of algebra. To this needs to be added what I believe is the most powerful aspect of algebra - the facility to express generalities, to be able to move from the specific to the general. It is this aspect of algebra that needs to underpin all the initial algebra work with the low attaining pupils as it is this aspect which will empower them to progress further and help them to develop a structure within which they can work.

Interestingly Bell (1993) in an unpublished paper has suggested thet the raw material of Mathematics is Number and Space, and that symbol systems arise as a means of denoting action in these areas. This seems to be a logical idea as from a historical perspective the development of algebra was indeed a means of representing a problem and then acting within that system to solve the problem. It seems to me that this is the way that Logo needs to be viewed for these pupils. It is a succinct, functional language which allows the pupil to represent a problem using symbolic expressions and further it allows the pupil to generalise the problem or situation. Within the system it seems that there are four steps that the pupil needs to go through in order to develop a conceptual undsretanding of the algebra of Logo or for that matter for any other algebra.

- be able and willing to operate with a system of symbolic expressions.
- be able to learn the linguistic aspects of the algebra
- be able to learn to manipulate the symbols of the system
- be able to develop a global view about how to generalise within the system.

So it is important now to specify the aspects of generalisation that are evident within the Logo system. This should then give the base line for the development of appropriate activities. It seems to me that there are a number of aspects as listed below:

Operation with variable/parameter unseen
(FD ... RT ... etc)

Procedure with named variable/parameter
(SQUARE :side)

Using a variable/parameter to create the general from the specific.
(eg scale factor)

Using more than one variable/parameter to create the general from the specific.
(eg squares to regular polygons)

Using a relationship to create a generality

Using multiple procedures to create the general from the specific.

It does seem that in this list the move from the first to the second aspect is a substantial step and as such needs to be considered very carefully.

Future Work

In the initial interviews more work needs to be done on:

pupil ideas on *ordering numbers*
pupil ideas on *numbers between numbers*
pupil ideas on *negative numbers*
pupil methods of doing arithmetic operations in different contexts
pupil ideas on *decimals*

Activities based on the ideas of functionality and generality as specified above need to be developed. Thes activities should also match the algebraic principles discussed above. Some of the follow-up work on functions could also be related to graphs using a graph-plotter.

The work developed need to lend itself to work both at and away from the computer. Linked to this will be work which enables communication possibilities to be developed, since this is an area where these particular pupils have little confidence. The work of Brousseau and Vergnaud and the theory of situations would seem to be a possible vehicle for developing ths possibility. It would also be a means of allowing the pupils to be creative and taking some control over their learning.

Finally both Sfard and Tall have articulated learning structures which emphasize the fact that concepts are learned at two levels - a process level and a structure level. In the work that is done with the low attainers it will be necessary to try to identify those activities which encourage learning at one or other or both of these levels.

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